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SERIE 4 SEMESTRE 2024-2
solución

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1) Resuelva para una constante positiva

> $Ecua := a\cdot diff(u(x,t),t) = diff(u(x,t),x\$2)$

$$Ecua := a\left(\frac{\partial}{\partial t}u(x,t)\right) = \frac{\partial^2}{\partial x^2}u(x,t) \quad (1)$$

> $u(x,t) = F(x)\cdot G(t)$

$$u(x,t) = F(x)\,G(t) \quad (2)$$

> $EcuaSeparable := eval(subs(u(x,t) = F(x)\cdot G(t), Ecua))$

$$EcuaSeparable := a\,F(x)\left(\frac{d}{dt}G(t)\right) = \left(\frac{d^2}{dx^2}F(x)\right)G(t) \quad (3)$$

> $EcuaSeparada := \frac{lhs(EcuaSeparable)}{F(x)\cdot G(t)} = \frac{rhs(EcuaSeparable)}{F(x)\cdot G(t)}$

$$EcuaSeparada := \frac{a\left(\frac{d}{dt}G(t)\right)}{G(t)} = \frac{\frac{d^2}{dx^2}F(x)}{F(x)} \quad (4)$$

> $EcuaX := rhs(EcuaSeparada) = \beta^2;\ EcuaT := lhs(EcuaSeparada) = \beta^2$

$$EcuaX := \frac{\frac{d^2}{dx^2}F(x)}{F(x)} = \beta^2$$

$$EcuaT := \frac{a\left(\frac{d}{dt}G(t)\right)}{G(t)} = \beta^2 \quad (5)$$

> $SolX := dsolve(EcuaX);\ SolT := dsolve(EcuaT)$

$$SolX := F(x) = c_1\,e^{\beta x} + c_2\,e^{-\beta x}$$

$$SolT := G(t) = c_1\,e^{\frac{\beta^2 t}{a}} \quad (6)$$

> $SolGralPos := u(x,t) = rhs(SolX)\cdot subs(c_1 = 1, rhs(SolT))$

$$SolGralPos := u(x,t) = \left(c_1\,e^{\beta x} + c_2\,e^{-\beta x}\right)e^{\frac{\beta^2 t}{a}} \quad (7)$$

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2) Resuelva para una constante igual a 4

> $Ecua := diff(u(x,y),x\$2) - 4\cdot diff(u(x,y),y) = 0$

$$Ecua := \frac{\partial^2}{\partial x^2}u(x,y) - 4\frac{\partial}{\partial y}u(x,y) = 0 \quad (8)$$

> $EcuaSeparable := eval(subs(u(x,y) = F(x)\cdot G(y), Ecua))$

(9)

$$EcuaSeparable := \left(\frac{d^2}{dx^2} F(x)\right) G(y) - 4 F(x) \left(\frac{d}{dy} G(y)\right) = 0 \quad \textbf{(9)}$$

$$> EcuaSeparaDos := lhs(EcuaSeparable) + 4 F(x) \left(\frac{d}{dy} G(y)\right) = rhs(EcuaSeparable) + 4 F(x) \left(\frac{d}{dy} G(y)\right)$$

$$EcuaSeparaDos := \left(\frac{d^2}{dx^2} F(x)\right) G(y) = 4 F(x) \left(\frac{d}{dy} G(y)\right) \quad \textbf{(10)}$$

$$> EcuaSeparada := \frac{lhs(EcuaSeparaDos)}{F(x) \cdot G(y)} = \frac{rhs(EcuaSeparaDos)}{F(x) \cdot G(y)}$$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{4 \left(\frac{d}{dy} G(y)\right)}{G(y)} \quad \textbf{(11)}$$

$$> EcuaX := lhs(EcuaSeparada) = 4; EcuaY := \frac{rhs(EcuaSeparada)}{4} = 1$$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 4$$

$$EcuaY := \frac{\frac{d}{dy} G(y)}{G(y)} = 1 \quad \textbf{(12)}$$

$$> SolX := dsolve(EcuaX); SolY := dsolve(EcuaY)$$

$$SolX := F(x) = c_1 e^{-2x} + c_2 e^{2x}$$

$$SolY := G(y) = c_1 e^{y} \quad \textbf{(13)}$$

$$> SolGralCuatro := u(x, y) = rhs(SolX) \cdot subs(c_1 = 1, rhs(SolY))$$

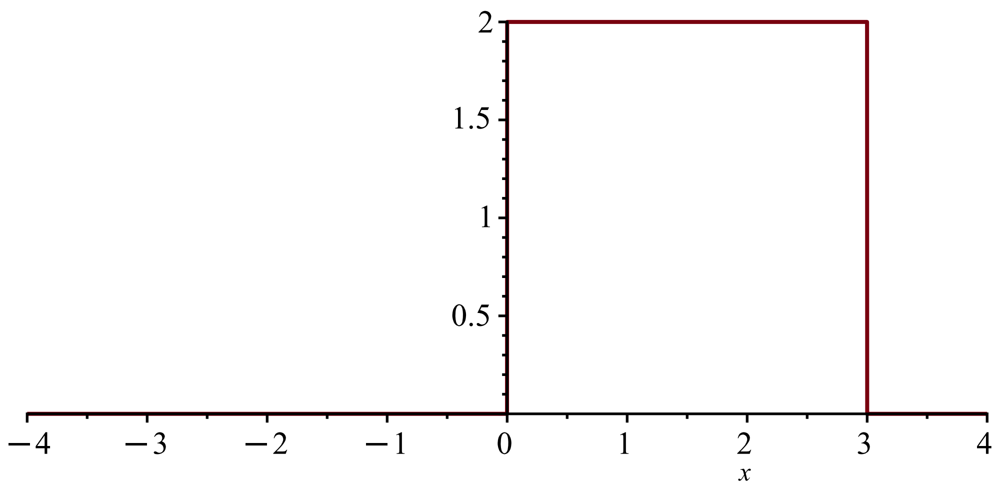
$$SolGralCuatro := u(x, y) = \left(c_1 e^{-2x} + c_2 e^{2x}\right) e^{y} \quad \textbf{(14)}$$

> *restart*

3) Obtener la serie seno de Fourier

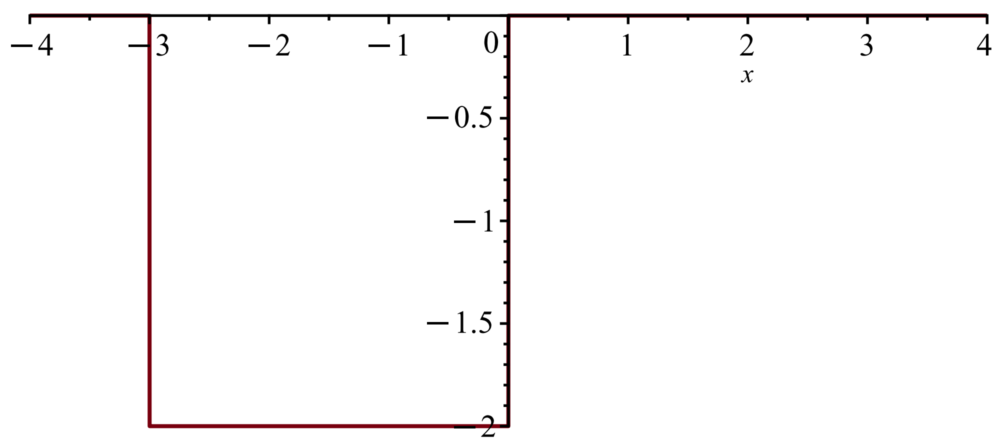
$$> f := 2 \cdot \text{Heaviside}(x) - 2 \cdot \text{Heaviside}(x - 3); plot(f, x = -4..4)$$

$$f := 2\ \text{Heaviside}(x) - 2\ \text{Heaviside}(x - 3)$$

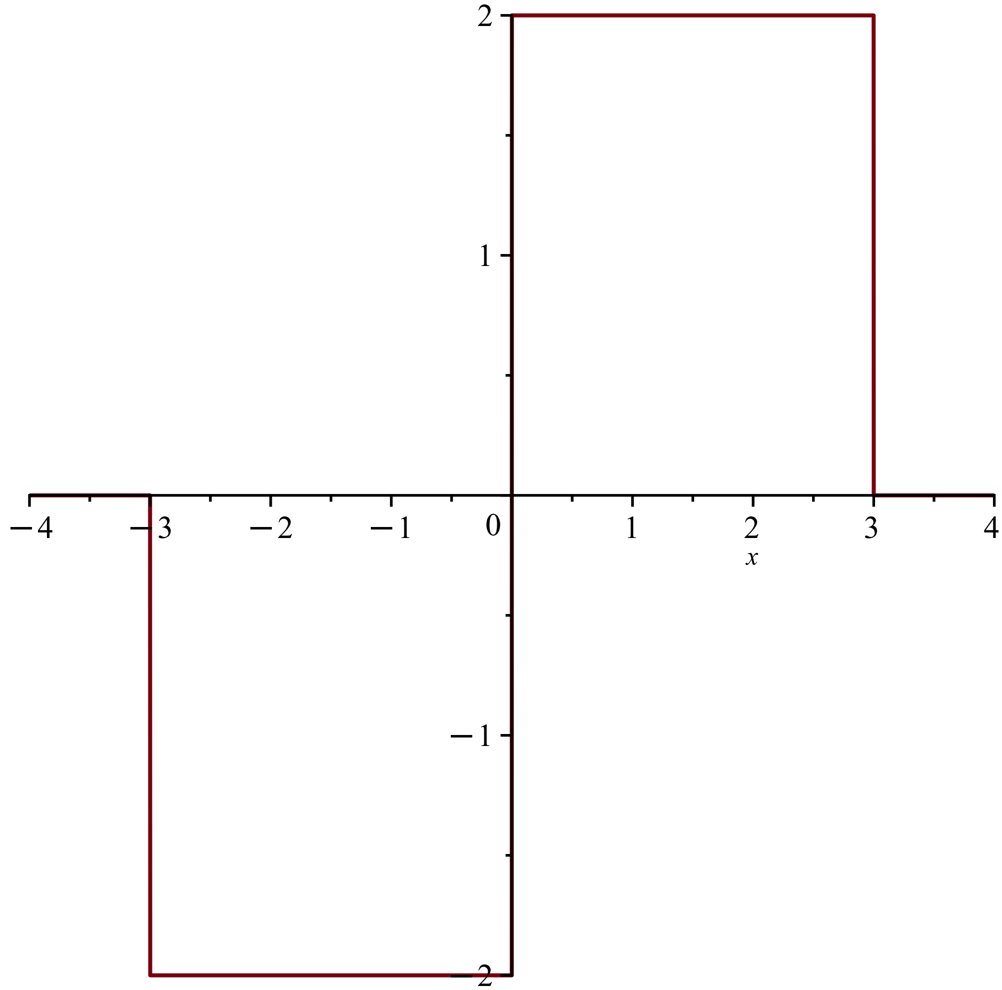


> $g := -2\cdot \text{Heaviside}(x+3) + 2\cdot \text{Heaviside}(x);\ plot(g, x = -4..4)$

$$g := -2\ \text{Heaviside}(x+3) + 2\ \text{Heaviside}(x)$$



> $h := f + g : plot(h, x = -4..4)$



> $L := 4$

$$L := 4 \tag{15}$$

> $a[0] := \frac{1}{L} \cdot int(h, x = -L..L)$

$$a_0 := 0 \tag{16}$$

> $a[n] := simplify\left(\frac{1}{L} \cdot int\left(h \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} x\right), x = -L..L\right)\right)$

$$a_n := 0 \tag{17}$$

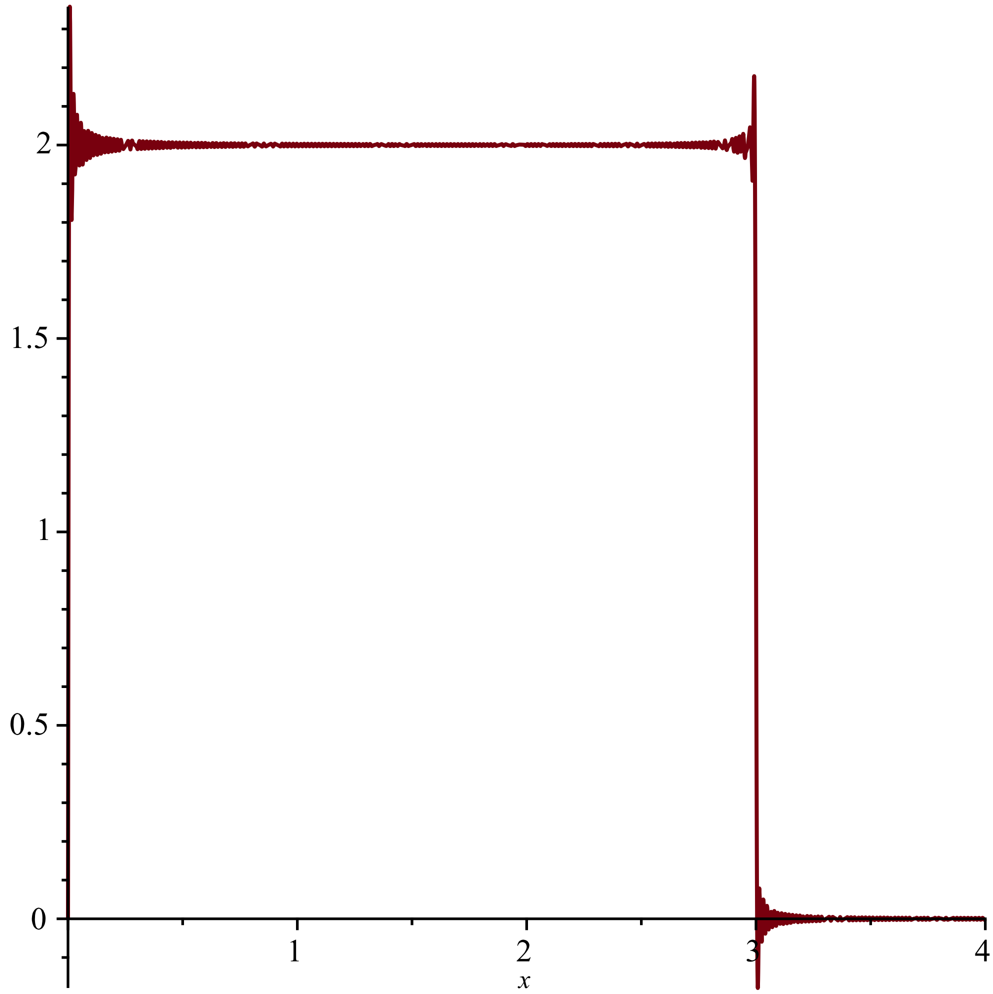
> $b[n] := simplify\left(\frac{1}{L} \cdot int\left(h \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} x\right), x = -L..L\right)\right)$

$$b_n := \frac{4 - 4\cos\left(\frac{3\, n\, \pi}{4}\right)}{n\, \pi} \tag{18}$$

> $STF := Sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..\mathrm{infinity}\right)$

$$STF := \sum_{n=1}^{\infty} \frac{\left(4 - 4\cos\left(\frac{3\, n\, \pi}{4}\right)\right) \sin\left(\frac{n\, \pi\, x}{4}\right)}{n\, \pi} \tag{19}$$

> $STF500 := sum\left(b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..500\right) : plot(STF500, x = 0..4)$



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4) Determinar ecuacion en derivadas parciales cuya solución general

> $SolGral := u(x, y) = f(x)\cdot\exp(x\cdot y) + g(x)\cdot\exp(-x\cdot y) + \frac{\exp(y)}{1 - x^2}$

$$SolGral := u(x, y) = f(x)\, e^{x y} + g(x)\, e^{-x y} + \frac{e^{y}}{-x^2 + 1} \quad \textbf{(20)}$$

> $SolHom := u(x, y) = f(x)\, e^{x y} + g(x)\, e^{-x y}$

$$SolHom := u(x, y) = f(x)\, e^{x y} + g(x)\, e^{-x y} \quad \textbf{(21)}$$

> $SolNoHom := u(x, y) = \frac{\exp(y)}{1 - x^2}$

$$SolNoHom := u(x, y) = \frac{e^{y}}{-x^2 + 1} \quad \textbf{(22)}$$

>

> $DerSolHom := diff(u(x, y), y) = diff(rhs(SolHom), y)$

$$DerSolHom := \frac{\partial}{\partial y}\, u(x, y) = f(x)\, x\, e^{x y} - g(x)\, x\, e^{-x y} \quad \textbf{(23)}$$

> $DerDerSolHom := diff(u(x,y), y\$2) = diff(rhs(SolHom), y\$2)$

$$DerDerSolHom := \frac{\partial^2}{\partial y^2} u(x,y) = f(x)\, x^2\, e^{xy} + g(x)\, x^2\, e^{-xy} \quad (24)$$

> $EcuaHom := simplify(DerDerSolHom - SolHom \cdot x^2)$

$$EcuaHom := -x^2\, u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) = 0 \quad (25)$$

> $Q := simplify(eval(subs(u(x,y) = rhs(SolNoHom), lhs(EcuaHom))))$

$$Q := e^{y} \quad (26)$$

> $EcuaFinal := lhs(EcuaHom) = Q$

$$EcuaFinal := -x^2\, u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) = e^{y} \quad (27)$$

> $SolSol := pdsolve(EcuaFinal)$

$$SolSol := u(x,y) = e^{-xy} f_2(x) + e^{xy} f_1(x) - \frac{e^{y}}{x^2 - 1} \quad (28)$$

> restart

5) Resuelva para una constante negativa

> $Ecua := k \cdot diff(u(x,t), x\$2) = diff(u(x,t), t)$

$$Ecua := k \left(\frac{\partial^2}{\partial x^2} u(x,t) \right) = \frac{\partial}{\partial t} u(x,t) \quad (29)$$

> $EcuaSeparable := eval(subs(u(x,t) = F(x) \cdot G(t), Ecua))$

$$EcuaSeparable := k \left(\frac{d^2}{dx^2} F(x) \right) G(t) = F(x) \left(\frac{d}{dt} G(t) \right) \quad (30)$$

> $EcuaSeparada := \frac{lhs(EcuaSeparable)}{k \cdot F(x) \cdot G(t)} = \frac{rhs(EcuaSeparable)}{k \cdot F(x) \cdot G(t)}$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d}{dt} G(t)}{k\, G(t)} \quad (31)$$

> $EcuaX := lhs(EcuaSeparada) = -\beta^2;\ EcuaT := rhs(EcuaSeparada) = -\beta^2$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2$$

$$EcuaT := \frac{\frac{d}{dt} G(t)}{k\, G(t)} = -\beta^2 \quad (32)$$

> $SolX := dsolve(EcuaX);\ SolT := dsolve(EcuaT)$

$$SolX := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x)$$

$$SolT := G(t) = c_1\, e^{-\beta^2 k t} \quad (33)$$

> $SolGralNeg := u(x,t) = rhs(SolX) \cdot subs(c_1 = 1, rhs(SolT))$

(34)

$$SolGralNeg := u(x, t) = \left(c_1 \sin(\beta x) + c_2 \cos(\beta x)\right) e^{-\beta^2 k t} \quad \textbf{(34)}$$

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FIN SERIE 4

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