```
> restart
```

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COORDINACIÓN DE CIENCIAS APLICADAS
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1325_26-2_1P
SOLUCIÓN
PRIMER EXAMEN PARCIAL COLEGIADO DE
ECUACIONES DIFERENCIALES TIPO A

SEMESTRE 2026 – 2 Sábado 11 de abril de 2026 DURACIÓN MÁXIMA 2 HORAS

```
>
> restart
```

1) Sea la ecuación diferencial

```
> x·log(x)·y'−y=x^3·(3·log(x)−1)
```

$$x\ln(x)\left(\frac{d}{dx}\,y(x)\right)-y(x)=x^3\,(3\ln(x)-1) \tag{1}$$

```
> Función := y(x)=ln(x) c_1
```

$$Función := y(x)=\ln(x)\,c_1 \tag{2}$$

RESPUESTA

```
> Ecua := x·log(x)·y'−y=x^3·(3·log(x)−1)
```

$$Ecua := x\ln(x)\left(\frac{d}{dx}\,y(x)\right)-y(x)=x^3\,(3\ln(x)-1) \tag{3}$$

```
>
```

a) **V.D. = y(x) ; V.I. = x ; Tipo = EDO(1)L.CV.NH ; Orden = primer orden ; ¿Es lineal? = sí**

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>
```

b) Verifique = ¿la Función es solución de la homogénea asociada?

```
> EcuaDos := expand(lhs(Ecua)/(x·log(x)) = rhs(Ecua)/(x·log(x)))
```

$$EcuaDos := \frac{d}{dx}\,y(x)-\frac{y(x)}{x\ln(x)}=3x^2-\frac{x^2}{\ln(x)} \tag{4}$$

```
> EcuaHom := lhs(EcuaDos)=0
```

$$EcuaHom := \frac{d}{dx}\,y(x)-\frac{y(x)}{x\ln(x)}=0 \tag{5}$$

```
> SolGralHom := y(x)=_C1·log(x)
```

$$SolGralHom := y(x)=\_C1\ln(x) \tag{6}$$

```
> Comprobar := simplify(eval(subs(y(x)=rhs(SolGralHom), EcuaHom)))
```

$$Comprobar := 0=0 \tag{7}$$

```
>
```

La Función sí es solución de la homogénea asociada

>

c) Resuélvala

> $p := -\frac{1}{x\ln(x)}$

$$p := -\frac{1}{x\ln(x)} \quad (8)$$

> $q := rhs(EcuaDos)$

$$q := 3x^2 - \frac{x^2}{\ln(x)} \quad (9)$$

> $IntPpos := \exp(int(p, x))$

$$IntPpos := \frac{1}{\ln(x)} \quad (10)$$

> $IntPneg := \exp(int(-p, x))$

$$IntPneg := \ln(x) \quad (11)$$

> $SolGralNoHom := y(x) = \_C1 \cdot IntPneg + IntPneg \cdot int(IntPpos \cdot q, x)$

$$SolGralNoHom := y(x) = \_C1\ln(x) + x^3 \quad (12)$$

> $Comprobar := dsolve(Ecua)$

$$Comprobar := y(x) = \ln(x)\,c_1 + x^3 \quad (13)$$

> $restart$

>

2. Use el método de coeficientes homogéneos, para determinar la solución explícita del problema de valor inicial dado por:

> $4\cdot x^2 - x\cdot y + y^2 + (x^2 - x\cdot y + 4\cdot y^2)\cdot y' = 0$

$$4x^2 - x\,y(x) + y(x)^2 + \left(x^2 - x\,y(x) + 4\,y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (14)$$

> $y(1) = -3$

$$y(1) = -3 \quad (15)$$

SOLUCIÓN

> $Ecua := 4\cdot x^2 - x\cdot y + y^2 + (x^2 - x\cdot y + 4\cdot y^2)\cdot y' = 0$

$$Ecua := 4x^2 - x\,y(x) + y(x)^2 + \left(x^2 - x\,y(x) + 4\,y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (16)$$

> $with(DEtools)$:

> $odeadvisor(Ecua)$

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert] \quad (17)$$

> $EcuaDos := simplify(isolate(simplify(eval(subs(y(x) = u(x)\cdot x, Ecua))), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx}\,u(x) = \frac{-4\,u(x)^3 - 4}{x\left(1 - u(x) + 4\,u(x)^2\right)} \quad (18)$$

> $odeadvisor(EcuaDos)$

$$[\_separable] \quad (19)$$

> $M := simplify\left(-\left(\frac{-4\,u^3 - 4}{(1 - u + 4\,u^2)} \right) \right)$

$$M := \frac{4\,u^3 + 4}{4\,u^2 - u + 1} \tag{20}$$

> $N := x$

$$N := x \tag{21}$$

> $P := 1; Q := M; R := N; S := 1$

$$P := 1$$
$$Q := \frac{4\,u^3 + 4}{4\,u^2 - u + 1}$$
$$R := x$$
$$S := 1 \tag{22}$$

> $SolGralU := int\left(\frac{P}{R}, x \right) + int\left(\frac{S}{Q}, u \right) = \_C1$

$$SolGralU := \ln(x) + \frac{\ln(u^2 - u + 1)}{4} + \frac{\ln(u + 1)}{2} = \_C1 \tag{23}$$

> $SolGralUdos := simplify(\exp(lhs(SolGralU))) = \_C1$

$$SolGralUdos := x\,(u^2 - u + 1)^{1/4}\sqrt{u + 1} = \_C1 \tag{24}$$

> $SolGralFinal := simplify\left(subs\left(u = \frac{y(x)}{x}, lhs(SolGralUdos) \right) \right)^4 = \_C1$

$$SolGralFinal := \left(y(x)^2 - x\,y(x) + x^2\right)\,(y(x) + x)^2 = \_C1 \tag{25}$$

> $Ecua$

$$4\,x^2 - x\,y(x) + y(x)^2 + \left(x^2 - x\,y(x) + 4\,y(x)^2\right)\left(\frac{d}{dx}\,y(x) \right) = 0 \tag{26}$$

> $DerSolFinal := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = \frac{-4\,x^2 + x\,y(x) - y(x)^2}{x^2 - x\,y(x) + 4\,y(x)^2} \tag{27}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{-4\,x^2 + x\,y(x) - y(x)^2}{x^2 - x\,y(x) + 4\,y(x)^2} \tag{28}$$

> $ComprobarUno := rhs(DerEcua) - rhs(DerSolFinal) = 0$

$$ComprobarUno := 0 = 0 \tag{29}$$

> $SolAlterna := \left(y^2 - x\,y + x^2\right)\,(y + x)^2 = \_C1$

$$SolAlterna := \left(x^2 - x\,y + y^2\right)\,(y + x)^2 = \_C1 \tag{30}$$

> $Para := subs(x = 1, y = -3, SolAlterna)$

$$Para := 52 = \_C1 \tag{31}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGralFinal)$

$$SolPart := \left(y(x)^2 - x\,y(x) + x^2\right)\,(y(x) + x)^2 = 52 \tag{32}$$

> *restart*

>

3. Determine el factor integrante que transforme la ecuación diferencial dada en una diferencial exacta y, obtenga la solución general de la ecuación resultante

> $(x^2+y^2+x)+x\cdot y\cdot y'=0$

$$x^2+y(x)^2+x+x\,y(x)\left(\frac{d}{dx}\,y(x)\right)=0 \tag{33}$$

>

RESPUESTA

> $Ecua := (x^2+y^2+x)+x\cdot y\cdot y'=0$

$$Ecua := x^2+y(x)^2+x+x\,y(x)\left(\frac{d}{dx}\,y(x)\right)=0 \tag{34}$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecua*)

$$[\_rational, \_Bernoulli] \tag{35}$$

> $FactInt := intfactor(Ecua)$

$$FactInt := x \tag{36}$$

> $EcuaDos := expand(FactInt\cdot Ecua)$

$$EcuaDos := x^3+x\,y(x)^2+x^2+x^2\,y(x)\left(\frac{d}{dx}\,y(x)\right)=0 \tag{37}$$

> *odeadvisor*(*EcuaDos*)

$$[\_exact, \_rational, \_Bernoulli] \tag{38}$$

> $M := x^3+x\,y^2+x^2$

$$M := x^3+y^2\,x+x^2 \tag{39}$$

> $N := x^2\,y$

$$N := x^2\,y \tag{40}$$

> $IntMx := int(M, x)$

$$IntMx := \frac{1}{4}\,x^4+\frac{1}{2}\,x^2\,y^2+\frac{1}{3}\,x^3 \tag{41}$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGral := \frac{1}{4}\,x^4+\frac{1}{2}\,x^2\,y^2+\frac{1}{3}\,x^3=\_C1 \tag{42}$$

> $SolFinal := \frac{1}{4}\,x^4+\frac{1}{2}\,x^2\,y(x)^2+\frac{1}{3}\,x^3=\_C1$

$$SolFinal := \frac{x^4}{4}+\frac{x^2\,y(x)^2}{2}+\frac{x^3}{3}=\_C1 \tag{43}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x)=\frac{-x^2-y(x)^2-x}{x\,y(x)} \tag{44}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{-x^2 - y(x)^2 - x}{x\, y(x)} \tag{45}$$

> $Comprobar := rhs(DerSolFinal) - rhs(DerEcua) = 0$

$$Comprobar := 0 = 0 \tag{46}$$

> $restart$

4. Resolver la ecuación diferencial

> $y'' - 8 \cdot y' + 16 \cdot y = (1 - x)\exp(4x)$

$$\frac{d^2}{dx^2}\, y(x) - 8\, \frac{d}{dx}\, y(x) + 16\, y(x) = (1 - x)\, e^{4x} \tag{47}$$

>

RESPUESTA

> $Ecua := y'' - 8 \cdot y' + 16 \cdot y = (1 - x)\exp(4x)$

$$Ecua := \frac{d^2}{dx^2}\, y(x) - 8\, \frac{d}{dx}\, y(x) + 16\, y(x) = (1 - x)\, e^{4x} \tag{48}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) - 8\, \frac{d}{dx}\, y(x) + 16\, y(x) = 0 \tag{49}$$

> $Q := rhs(Ecua)$

$$Q := (1 - x)\, e^{4x} \tag{50}$$

> $EcuaCarac := m^2 - 8 \cdot m + 16 = 0$

$$EcuaCarac := m^2 - 8\, m + 16 = 0 \tag{51}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 4, 4 \tag{52}$$

> $yy[1] := \exp(Raiz[1] \cdot x); yy[2] := x \cdot \exp(Raiz[1] \cdot x)$

$$yy_1 := e^{4x}$$

$$yy_2 := x\, e^{4x} \tag{53}$$

> $with(linalg)$:

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{4x} & x\, e^{4x} \\ 4\, e^{4x} & e^{4x} + 4\, x\, e^{4x} \end{bmatrix} \tag{54}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & (1 - x)\, e^{4x} \end{bmatrix} \tag{55}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} x\,(x - 1) & 1 - x \end{bmatrix} \tag{56}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := x\,(x - 1)$$

$$Bprima := 1 - x \tag{57}$$

> $IntAA := int(Aprima, x)$

$$IntAA := \frac{1}{3}x^3 - \frac{1}{2}x^2 \quad (58)$$

> $IntBB := int(Bprima, x)$

$$IntBB := x - \frac{1}{2}x^2 \quad (59)$$

> $SolGral := y(x) = expand(simplify((\_C1 + IntAA)\cdot yy[1] + (\_C2 + IntBB)\cdot yy[2]))$

$$SolGral := y(x) = -\frac{(e^x)^4 x^3}{6} + (e^x)^4 x\,\_C2 + \frac{(e^x)^4 x^2}{2} + (e^x)^4\,\_C1 \quad (60)$$

> $SolFinal := y(x) = \_C1\cdot\exp(4x) + \_C2\cdot x\cdot\exp(4x) + \frac{x^2\cdot\exp(4x)}{2} - \frac{x^3\cdot\exp(4x)}{6}$

$$SolFinal := y(x) = e^{4x}\,\_C1 - \frac{e^{4x}x^3}{6} + \frac{e^{4x}x^2}{2} + x\,e^{4x}\,\_C2 \quad (61)$$

> $Ecua$

$$\frac{d^2}{dx^2}y(x) - 8\frac{d}{dx}y(x) + 16\,y(x) = (1-x)\,e^{4x} \quad (62)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (63)$$

> $restart$

>

5. Resolver la ecuación diferencial

> $x\cdot y'' - (2\cdot x + 1)\cdot y' + (x+1)\cdot y = (x^2 + x - 1)\cdot\exp(2x)$

$$x\left(\frac{d^2}{dx^2}y(x)\right) - (2x+1)\left(\frac{d}{dx}y(x)\right) + (x+1)\,y(x) = (x^2+x-1)\,e^{2x} \quad (64)$$

>

cuyo conjunto fundamental de soluciones de su homogénea asociada es :

> $yy[1] := e^x; yy[2] := e^x x^2$

$$yy_1 := e^x$$

$$yy_2 := e^x x^2 \quad (65)$$

RESPUESTA

> $Ecua := x\cdot y'' - (2\cdot x + 1)\cdot y' + (x+1)\cdot y = (x^2 + x - 1)\cdot\exp(2x)$

$$Ecua := x\left(\frac{d^2}{dx^2}y(x)\right) - (2x+1)\left(\frac{d}{dx}y(x)\right) + (x+1)\,y(x) = (x^2+x-1)\,e^{2x} \quad (66)$$

> $EcuaDos := expand\left(\frac{Ecua}{x}\right)$

$$EcuaDos := \frac{d^2}{dx^2}y(x) - 2\frac{d}{dx}y(x) + y(x) - \frac{\frac{d}{dx}y(x)}{x} + \frac{y(x)}{x} = x(e^x)^2 + (e^x)^2 - \frac{(e^x)^2}{x} \quad (67)$$

> $EcuaDosHom := lhs(EcuaDos) = 0$

$$EcuaDosHom := \frac{d^2}{dx^2} y(x) - 2 \frac{d}{dx} y(x) + y(x) - \frac{\frac{d}{dx} y(x)}{x} + \frac{y(x)}{x} = 0 \quad (68)$$

> $Q := rhs(EcuaDos)$

$$Q := x \left(e^x\right)^2 + \left(e^x\right)^2 - \frac{\left(e^x\right)^2}{x} \quad (69)$$

> $yy[1] := \exp(x); yy[2] := x^2 \cdot \exp(x)$

$$yy_1 := e^x$$

$$yy_2 := e^x x^2 \quad (70)$$

> $with(linalg)$:

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & e^x x^2 \\ e^x & e^x x^2 + 2 e^x x \end{bmatrix} \quad (71)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x \left(e^x\right)^2 + \left(e^x\right)^2 - \frac{\left(e^x\right)^2}{x} \end{bmatrix} \quad (72)$$

> $ParaVar := expand(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{e^x \left(x^2 + x - 1\right)}{2} & \frac{e^x \left(x^2 + x - 1\right)}{2 x^2} \end{bmatrix} \quad (73)$$

> $Aprima := expand(ParaVar[1]); Bprima := expand(ParaVar[2])$

$$Aprima := -\frac{e^x x^2}{2} - \frac{e^x x}{2} + \frac{e^x}{2}$$

$$Bprima := \frac{e^x}{2} + \frac{e^x}{2x} - \frac{e^x}{2x^2} \quad (74)$$

> $IntAA := int(Aprima, x)$

$$IntAA := -\frac{e^x x (x - 1)}{2} \quad (75)$$

> $IntBB := int(Bprima, x)$

$$IntBB := \frac{e^x (x + 1)}{2x} \quad (76)$$

> $SolGral := y(x) = expand(simplify((\_C1 + IntAA) \cdot yy[1] + (\_C2 + IntBB) \cdot yy[2]))$

$$SolGral := y(x) = c_2 e^x x^2 + x \left(e^x\right)^2 + c_1 e^x \quad (77)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (78)$$

> $restart$

>