>

>

EXAMEN 2026-2-2 solución

>

> *restart*

1)

> $Ecua := diff(x(t), t\$2) + 6\cdot diff(x(t), t) + 9\cdot x(t) = 0$

$$Ecua := \frac{d^2}{dt^2}\, x(t) + 6\, \frac{d}{dt}\, x(t) + 9\, x(t) = 0 \quad (1)$$

> $CondIni := x(0) = 20, D(x)(0) = 0$

$$CondIni := x(0) = 20, D(x)(0) = 0 \quad (2)$$

POR TRANSFORMADA DE LAPLACE

> $with(inttrans):$

> $EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTransLap := s^2\, \mathcal{L}(x(t), t, s) - 120 - 20\, s + 6\, s\, \mathcal{L}(x(t), t, s) + 9\, \mathcal{L}(x(t), t, s) = 0 \quad (3)$$

> $SolTransLap := isolate(EcuaTransLap, laplace(x(t), t, s))$

$$SolTransLap := \mathcal{L}(x(t), t, s) = \frac{20\, s + 120}{s^2 + 6\, s + 9} \quad (4)$$

> $SolPart := invlaplace(SolTransLap, s, t)$

$$SolPart := x(t) = 20\, e^{-3\, t}\, (1 + 3\, t) \quad (5)$$

> $SolFinal := simplify(SolPart)$

$$SolFinal := x(t) = (20 + 60\, t)\, e^{-3\, t} \quad (6)$$

> $Ecua$

$$\frac{d^2}{dt^2}\, x(t) + 6\, \frac{d}{dt}\, x(t) + 9\, x(t) = 0 \quad (7)$$

> $ComprobarUno := simplify(eval(subs(x(t) = rhs(SolPart), Ecua)))$

$$ComprobarUno := 0 = 0 \quad (8)$$

> $ComprobarDos := simplify(subs(t = 0, SolPart))$

$$ComprobarDos := x(0) = 20 \quad (9)$$

> $ComprobarTres := D(x)(0) = simplify(subs(t = 0, rhs(diff(SolPart, t))))$

$$ComprobarTres := D(x)(0) = 0 \quad (10)$$

> *restart*

2)

> $Ecua := y(t) + int(tau\cdot \exp(2\cdot tau)\cdot y(t - tau), tau = 0..t) = t\cdot \exp(2\cdot t)$

$$Ecua := y(t) + \int_0^t \tau\, e^{2\,\tau}\, y(t - \tau)\, d\tau = t\, e^{2\, t} \quad (11)$$

POR TRANSFORMADA DE LAPLACE

> $with(inttrans):$

> $EcuaTransLap := laplace(Ecua, t, s)$

(12)

$$EcuaTransLap := \mathcal{L}(y(t), t, s) + \frac{\mathcal{L}(y(t), t, s)}{(s-2)^2} = \frac{1}{(s-2)^2} \quad \textbf{(12)}$$

> $SolTransLap := simplify(isolate(EcuaTransLap, laplace(y(t), t, s)))$

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{1}{s^2 - 4s + 5} \quad \textbf{(13)}$$

> $SolPart := invlaplace(SolTransLap, s, t)$

$$SolPart := y(t) = \sin(t)\, e^{2t} \quad \textbf{(14)}$$

> $Ecua$

$$y(t) + \int_0^t \tau\, e^{2\tau} y(t-\tau)\, d\tau = t\, e^{2t} \quad \textbf{(15)}$$

> $EcuaDos := lhs(Ecua) - \int_0^t \tau\, e^{2\tau} y(t-\tau)\, d\tau - t\, e^{2t} = rhs(Ecua) - \int_0^t \tau\, e^{2\tau} y(t-\tau)\, d\tau - t\, e^{2t}$

$$EcuaDos := y(t) - t\, e^{2t} = -\left(\int_0^t \tau\, e^{2\tau} y(t-\tau)\, d\tau\right) \quad \textbf{(16)}$$

> $SolUno := y(t - \mathrm{tau}) = \sin(t - \mathrm{tau})\, e^{2\cdot(t-\mathrm{tau})}$

$$SolUno := y(t-\tau) = \sin(t-\tau)\, e^{2t-2\tau} \quad \textbf{(17)}$$

> $SolFinal := simplify\left(isolate\left(y(t) - t\, e^{2t} = -eval\left(subs\left(y(t-\mathrm{tau}) = rhs(SolUno), \int_0^t \tau\, e^{2\tau} y(t - \tau)\, d\tau\right)\right), y(t)\right)\right)$

$$SolFinal := y(t) = \sin(t)\, e^{2t} \quad \textbf{(18)}$$

SE COMPRUEBA QUE ES LA SOLUCIÓN PARTICULAR

>

> $restart$

3)

> $Sistema := diff(x(t), t\$2) + 3\cdot diff(y(t), t) + 3\cdot y(t) = 0, diff(x(t), t\$2) + 3\cdot y(t) = t\cdot \exp(-t) : Sistema[1]; Sistema[2]$

$$\frac{d^2}{dt^2} x(t) + 3 \frac{d}{dt} y(t) + 3\, y(t) = 0$$

$$\frac{d^2}{dt^2} x(t) + 3\, y(t) = t\, e^{-t} \quad \textbf{(19)}$$

> $CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0$

$$CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0 \quad \textbf{(20)}$$

MEDIANTE TRANSFORMADA DE LAPLACE

> $with(inttrans) :$

> $SistTransLap := subs(CondIni, laplace(Sistema[1], t, s)), subs(CondIni, laplace(Sistema[2], t, s)) : SistTransLap[1]; SistTransLap[2]$

$$s^2 \mathcal{L}(x(t), t, s) + 3\, s\, \mathcal{L}(y(t), t, s) + 3\, \mathcal{L}(y(t), t, s) = 0$$

(21)

$$s^2 \mathcal{L}(x(t), t, s) + 3\,\mathcal{L}(y(t), t, s) = \frac{1}{(1+s)^2} \quad \textbf{(21)}$$

> *VarTrans* := *solve*({*SistTransLap*}, {*laplace*(x(t), t, s), *laplace*(y(t), t, s)})

$$VarTrans := \left\{\mathcal{L}(x(t), t, s) = \frac{1}{(1+s)\,s^3},\ \mathcal{L}(y(t), t, s) = -\frac{1}{3\,s\,(s^2+2\,s+1)}\right\} \quad \textbf{(22)}$$

> *Solucion* := *invlaplace*(*VarTrans*[1], s, t), *invlaplace*(*VarTrans*[2], s, t) : *Solucion*[1]; *Solucion*[2]

$$x(t) = \frac{t^2}{2} - t + 1 - \mathrm{e}^{-t}$$

$$y(t) = -\frac{1}{3} + \frac{\mathrm{e}^{-t}\,(t+1)}{3} \quad \textbf{(23)}$$

> *ComprobarUno* := *simplify*(*eval*(*subs*(x(t) = *rhs*(*Solucion*[1]), y(t) = *rhs*(*Solucion*[2]), *lhs*(*Sistema*[1]) − *rhs*(*Sistema*[1]) = 0)))

$$ComprobarUno := 0 = 0 \quad \textbf{(24)}$$

> *ComprobarDos* := *simplify*(*eval*(*subs*(x(t) = *rhs*(*Solucion*[1]), y(t) = *rhs*(*Solucion*[2]), *lhs*(*Sistema*[2]) − *rhs*(*Sistema*[2]) = 0)))

$$ComprobarDos := 0 = 0 \quad \textbf{(25)}$$

MEDIANTE MATRIZ EXPONENCIAL

> *Sist* := *diff*(x(t), t) = z(t), *diff*(y(t), t) = −(1/3)·t·exp(−t), *diff*(z(t), t) = −3·y(t) + t·exp(−t) : *Sist*[1]; *Sist*[2]; *Sist*[3]

$$\frac{\mathrm{d}}{\mathrm{d}t}\,x(t) = z(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,y(t) = -\frac{t\,\mathrm{e}^{-t}}{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,z(t) = -3\,y(t) + t\,\mathrm{e}^{-t} \quad \textbf{(26)}$$

> *AA* := *array*([[0, 0, 1], [0, 0, 0,], [0, −3, 0]])

$$AA := \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -3 & 0 \end{bmatrix} \quad \textbf{(27)}$$

> *BB* := *array*([0, −(t·exp(−t))/3, t·exp(−t)])

$$BB := \begin{bmatrix} 0 & -\frac{t\,\mathrm{e}^{-t}}{3} & t\,\mathrm{e}^{-t} \end{bmatrix} \quad \textbf{(28)}$$

> *with*(*linalg*) :

> *MatExp* := *exponential*(*AA*, t)

(29)

$$MatExp := \begin{bmatrix} 1 & -\frac{3\,t^2}{2} & t \\ 0 & 1 & 0 \\ 0 & -3\,t & 1 \end{bmatrix} \quad (29)$$

> $Xcero := array([0, 0, 0])$

$$Xcero := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad (30)$$

> $MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} 1 & -\frac{3\,(t-\tau)^2}{2} & t-\tau \\ 0 & 1 & 0 \\ 0 & -3\,t+3\,\tau & 1 \end{bmatrix} \quad (31)$$

> $BBtau := map(rcurry(eval, t='tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & -\frac{\tau\,e^{-\tau}}{3} & \tau\,e^{-\tau} \end{bmatrix} \quad (32)$$

> $ProdTau := evalm(MatExpTau \&* BBtau)$

$$ProdTau := \begin{bmatrix} \frac{(t-\tau)^2\,\tau\,e^{-\tau}}{2} + (t-\tau)\,\tau\,e^{-\tau} & -\frac{\tau\,e^{-\tau}}{3} & -\frac{(-3\,t+3\,\tau)\,\tau\,e^{-\tau}}{3} + \tau\,e^{-\tau} \end{bmatrix} \quad (33)$$

> $SolPartNoHom := map(int, ProdTau, tau = 0..t)$

$$SolPartNoHom := \begin{bmatrix} \frac{t^2}{2} - t + 1 - e^{-t} & -\frac{1}{3} + \frac{t\,e^{-t}}{3} + \frac{e^{-t}}{3} & t - 1 + e^{-t} \end{bmatrix} \quad (34)$$

> $x(t) = SolPartNoHom[1]; y(t) = SolPartNoHom[2]; z(t) = SolPartNoHom[3]$

$$x(t) = \frac{t^2}{2} - t + 1 - e^{-t}$$

$$y(t) = -\frac{1}{3} + \frac{t\,e^{-t}}{3} + \frac{e^{-t}}{3}$$

$$z(t) = t - 1 + e^{-t} \quad (35)$$

> $Solucion[1]$

$$x(t) = \frac{t^2}{2} - t + 1 - e^{-t} \quad (36)$$

> $Solucion[2]$

$$y(t) = -\frac{1}{3} + \frac{e^{-t}\,(t+1)}{3} \quad (37)$$

SOLUCIÓN ALTERNA (CON SIGNO NEGATIVO EL SEGUBDO TÉRMINO DE LA PRIMERA ECUACIÓN)

> $restart$

> $Sistema := diff(x(t), t\$2) - 3\cdot diff(y(t), t) + 3\cdot y(t) = 0, diff(x(t), t\$2) + 3\cdot y(t) = t\cdot \exp($

$-t)$: $Sistema[1]$; $Sistema[2]$

$$\frac{d^2}{dt^2} x(t) - 3 \frac{d}{dt} y(t) + 3 y(t) = 0$$

$$\frac{d^2}{dt^2} x(t) + 3 y(t) = t e^{-t} \quad \textbf{(38)}$$

> $SolX := x(t) = 3 \cdot t - 5 - \frac{t^2}{2} + \exp(-t) \cdot (5 + 2t)$

$$SolX := x(t) = 3t - 5 - \frac{t^2}{2} + e^{-t}(5 + 2t) \quad \textbf{(39)}$$

> $SolY := y(t) = \frac{1}{3} - \frac{\exp(-t) \cdot (1 + t)}{3}$

$$SolY := y(t) = \frac{1}{3} - \frac{e^{-t}(1 + t)}{3} \quad \textbf{(40)}$$

> $ComprobarUno := simplify(eval(subs(x(t) = rhs(SolX), y(t) = rhs(SolY), lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))$

$$ComprobarUno := 0 = 0 \quad \textbf{(41)}$$

> $ComprobarDos := simplify(eval(subs(x(t) = rhs(SolX), y(t) = rhs(SolY), lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))$

$$ComprobarDos := 0 = 0 \quad \textbf{(42)}$$

> $restart$

>

4)

> $Ecua := x^2 \cdot diff(u(x, y), x\$2) + diff(u(x, y), y\$2) = 0$

$$Ecua := x^2 \left(\frac{\partial^2}{\partial x^2} u(x, y)\right) + \frac{\partial^2}{\partial y^2} u(x, y) = 0 \quad \textbf{(43)}$$

> alpha := 0

$$\alpha := 0 \quad \textbf{(44)}$$

POR MÉTODO DE SEPARACIÓN DE VARIABLES

> $EcuaDos := eval(subs(u(x, y) = F(x) \cdot G(y), Ecua))$

$$EcuaDos := x^2 \left(\frac{d^2}{dx^2} F(x)\right) G(y) + F(x) \left(\frac{d^2}{dy^2} G(y)\right) = 0 \quad \textbf{(45)}$$

> $EcuaSeparada := \frac{lhs(EcuaDos) - F(x) \left(\frac{d^2}{dy^2} G(y)\right)}{F(x) \cdot G(y)} = \frac{rhs(EcuaDos) - F(x) \left(\frac{d^2}{dy^2} G(y)\right)}{F(x) \cdot G(y)}$

$$EcuaSeparada := \frac{x^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = -\frac{\frac{d^2}{dy^2} G(y)}{G(y)} \quad \textbf{(46)}$$

> $EcuaX := lhs(EcuaSeparada) = \text{alpha}$

$$EcuaX := \frac{x^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = 0 \quad \textbf{(47)}$$

> $EcuaY := rhs(EcuaSeparada) = \text{alpha}$

$$EcuaY := -\frac{\frac{d^2}{dy^2} G(y)}{G(y)} = 0 \quad \textbf{(48)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 x + c_2 \quad \textbf{(49)}$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = c_1 y + c_2 \quad \textbf{(50)}$$

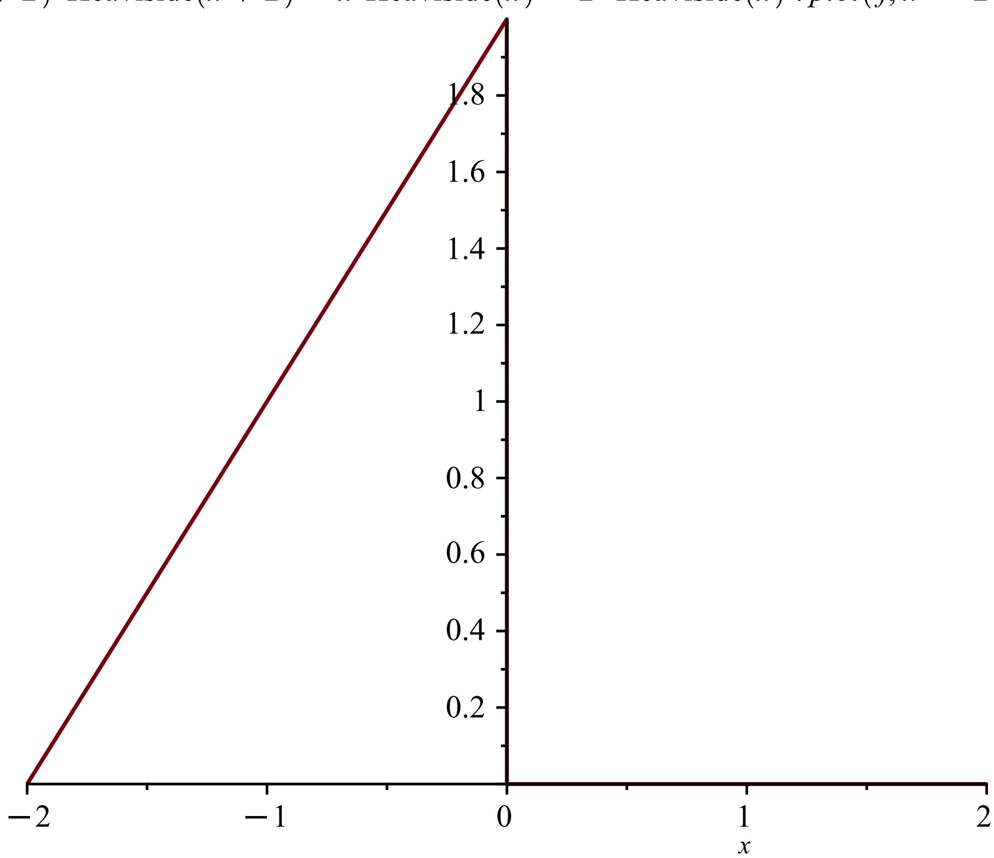
> $SolGral := u(x, y) = rhs(SolX) \cdot subs(c_1 = 1, c_2 = 1, rhs(SolY))$

$$SolGral := u(x, y) = (c_1 x + c_2)(1 + y) \quad \textbf{(51)}$$

> *restart*

5)

> $f := (x + 2) \cdot \text{Heaviside}(x + 2) - x \cdot \text{Heaviside}(x) - 2 \cdot \text{Heaviside}(x) : plot(f, x = -2..2)$



POR SERIE TRIGONOMÉTRICA DE FOURIER

> $L := 2$

$$L := 2 \quad \textbf{(52)}$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := 1 \quad \textbf{(53)}$$

> $a[n] := subs\left(\cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

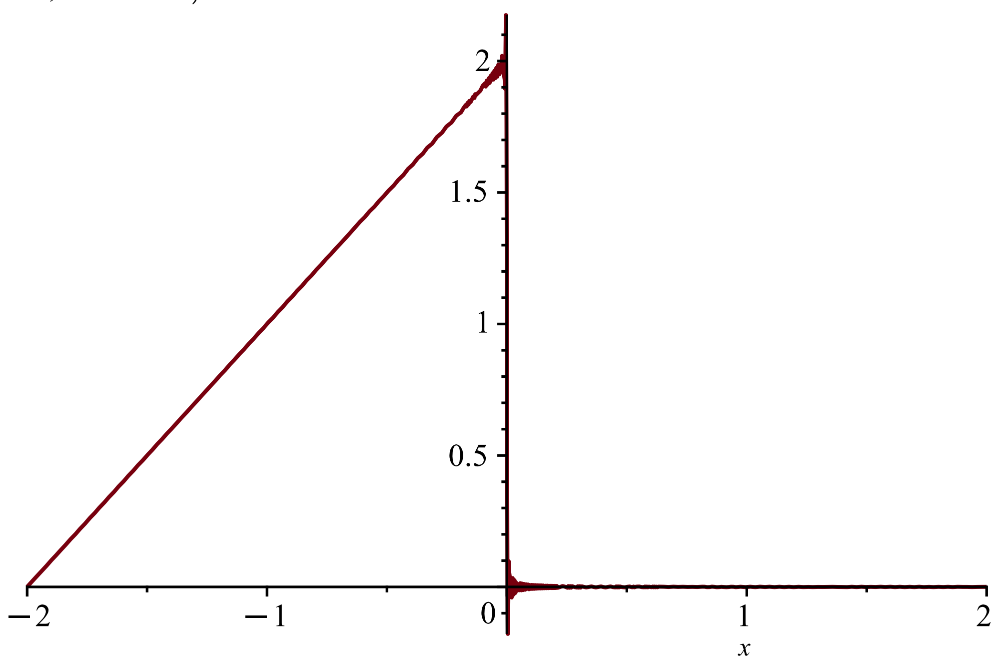
$$a_n := \frac{-4(-1)^n + 4}{2 n^2 \pi^2} \quad \textbf{(54)}$$

> $b[n] := subs\left(\sin(n \cdot \text{Pi}) = 0, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$b_n := -\frac{2}{n \pi} \quad \textbf{(55)}$$

> $STF500 := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..500\right):$

> $plot(STF500, x = -2..2)$



> restart

FIN EXAMEN

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