UNAM
FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEMESTRE 2026-2
GRUPO 8
SOLUCIÓN
SERIE TEMA 1

> *restart*

1) Obtener la solución particular del siguiente problema de ecuaciones diferenciales de primer orden con condiciones iniciales

> $Ecua := (1 + e^x)\, y(x) \left(\frac{d}{dx}\, y(x)\right) = e^x$

$$Ecua := (1 + e^x)\, y(x) \left(\frac{d}{dx}\, y(x)\right) = e^x \quad \textbf{(1)}$$

> $CondIni := y(0) = 1$

$$CondIni := y(0) = 1 \quad \textbf{(2)}$$

RESPUESTA 1)

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_separable] \quad \textbf{(3)}$$

> $M := -rhs(Ecua)$

$$M := -e^x \quad \textbf{(4)}$$

> $N := (1 + e^x)\, y$

$$N := (1 + e^x)\, y \quad \textbf{(5)}$$

> $P := e^x;\ Q := -1;\ R := (1 + e^x);\ S := y$

$$P := e^x$$
$$Q := -1$$
$$R := 1 + e^x$$
$$S := y \quad \textbf{(6)}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := \ln(1 + e^x) - \frac{1}{2}\, y^2 = \_C1 \quad \textbf{(7)}$$

> $Para := simplify(subs(x = 0, y = 1, SolGral))$

$$Para := \ln(2) - \frac{1}{2} = \_C1 \quad \textbf{(8)}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$SolPart := \ln(1 + e^x) - \frac{1}{2}\, y^2 = \ln(2) - \frac{1}{2} \quad \textbf{(9)}$$

> $SolFinal := \ln(1+e^x) - \frac{1}{2}y(x)^2 = \ln(2) - \frac{1}{2}$

$$SolFinal := \ln(1+e^x) - \frac{1}{2}y(x)^2 = \ln(2) - \frac{1}{2} \quad \textbf{(10)}$$

> $DerSolFinal := isolate(diff(SolFinal, x), diff(y(x), x))$

$$DerSolFinal := \frac{d}{dx}y(x) = \frac{e^x}{(1+e^x)\,y(x)} \quad \textbf{(11)}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}y(x) = \frac{e^x}{(1+e^x)\,y(x)} \quad \textbf{(12)}$$

> $Comprobar := rhs(DerSolFinal) - rhs(DerEcua) = 0$

$$Comprobar := 0 = 0 \quad \textbf{(13)}$$

>

FIN RESPUESTA 1)

> *restart*

2) Obtener la solución general de la siguiente ecuación diferencial de primer orden

> $Ecuacion := 2\cdot x\cdot(x^2+y^2)\cdot y' - y\cdot(2\cdot x^2+y^2) = 0$

$$Ecuacion := 2\,x\,(x^2+y(x)^2)\left(\frac{d}{dx}y(x)\right) - y(x)\,(2\,x^2+y(x)^2) = 0 \quad \textbf{(14)}$$

RESPUESTA 2)

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert] \quad \textbf{(15)}$$

Se resolverá por coeficientes homogeneos

> $M := -y\,(2\cdot x^2+y^2)$

$$M := -y\,(2\,x^2+y^2) \quad \textbf{(16)}$$

> $N := 2\cdot x\,(x^2+y^2)$

$$N := 2\,x\,(x^2+y^2) \quad \textbf{(17)}$$

> $Comprobar := diff(M, y) \neq diff(N, x)$

$$Comprobar := -2\,x^2 - 3\,y^2 \neq 6\,x^2 + 2\,y^2 \quad \textbf{(18)}$$

> $EcuacionDos := simplify(isolate(eval(subs(y(x) = u(x)\cdot x, Ecuacion)), diff(u(x), x)))$

$$EcuacionDos := \frac{d}{dx}u(x) = -\frac{u(x)^3}{2\,(1+u(x)^2)\,x} \quad \textbf{(19)}$$

> $odeadvisor(EcuacionDos)$

$$[\_separable] \quad \textbf{(20)}$$

> $EcuacionTres := lhs(EcuacionDos)\cdot(2\,(1+u(x)^2)\,x) = rhs(EcuacionDos)\cdot(2\,(1+u(x)^2)\,x)$

$$EcuacionTres := 2\left(\frac{d}{dx}u(x)\right)(1+u(x)^2)\,x = -u(x)^3 \quad \textbf{(21)}$$

> $EcuacionCuatro := lhs(EcuacionTres) - rhs(EcuacionTres) = 0$

$$EcuacionCuatro := 2\left(\frac{d}{dx}u(x)\right)\left(1 + u(x)^2\right)x + u(x)^3 = 0 \quad (22)$$

> $MM := u^3$

$$MM := u^3 \quad (23)$$

> $NN := 2\cdot\left(1 + u^2\right)x$

$$NN := 2\left(u^2 + 1\right)x \quad (24)$$

> $Px := 1; Qu := u^3; Rx := x; Su := 2\left(1 + u^2\right)$

$$Px := 1$$

$$Qu := u^3$$

$$Rx := x$$

$$Su := 2u^2 + 2 \quad (25)$$

> $SolInicial := int\left(\frac{Px}{Rx}, x\right) + int\left(\frac{Su}{Qu}, u\right) = \_C1$

$$SolInicial := \ln(x) + 2\ln(u) - \frac{1}{u^2} = \_C1 \quad (26)$$

> $SolGral := subs\left(u = \frac{y(x)}{x}, SolInicial\right)$

$$SolGral := \ln(x) + 2\ln\left(\frac{y(x)}{x}\right) - \frac{x^2}{y(x)^2} = \_C1 \quad (27)$$

> $Ecuacion$

$$2x\left(x^2 + y(x)^2\right)\left(\frac{d}{dx}y(x)\right) - y(x)\left(2x^2 + y(x)^2\right) = 0 \quad (28)$$

> $DerSolGral := simplify(isolate(diff(SolGral, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}y(x) = \frac{y(x)\left(2x^2 + y(x)^2\right)}{2x\left(x^2 + y(x)^2\right)} \quad (29)$$

> $DerEcua := isolate(Ecuacion, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}y(x) = \frac{y(x)\left(2x^2 + y(x)^2\right)}{2x\left(x^2 + y(x)^2\right)} \quad (30)$$

> $Comprobar := rhs(DerSolGral) - rhs(DerEcua) = 0$

$$Comprobar := 0 = 0 \quad (31)$$

>

FIN RESPUESTA 2)

> $restart$

3) Resuelva la siguiente ecuacion diferencial de primer orden por dos métodos distintos y pruebe que las soluciones generales obtenidas son iguales

> $Ecuacion := x\left(2x^2 + y(x)^2\right) + y(x)\left(x^2 + 2y(x)^2\right)\left(\frac{d}{dx}y(x)\right) = 0$

(32)

$$Ecuacion := x\left(2x^2+y(x)^2\right)+y(x)\left(x^2+2y(x)^2\right)\left(\frac{d}{dx}y(x)\right)=0 \quad (32)$$

RESPUESTA 3)

> with(DEtools) :

> odeadvisor(Ecuacion)

$$[[\_homogeneous, class\ A], \_exact, \_rational, \_dAlembert] \quad (33)$$

>

Por el método de coeficientes homogeneos

> EcuaDos := simplify(isolate(eval(subs(y(x) = x·u(x), Ecuacion)), diff(u(x), x)))

$$EcuaDos := \frac{d}{dx}u(x)=\frac{-2u(x)^4-2u(x)^2-2}{u(x)\left(1+2u(x)^2\right)x} \quad (34)$$

> odeadvisor(EcuaDos)

$$[\_separable] \quad (35)$$

> $M := 2\left(u^4+u^2+1\right)$

$$M := 2u^4+2u^2+2 \quad (36)$$

> $N := x\,u\left(2u^2+1\right)$

$$N := u\left(2u^2+1\right)x \quad (37)$$

> $P := 2;\ Q := u^4+u^2+1;\ R := x;\ S := expand\left(u\left(2u^2+1\right)\right)$

$$P := 2$$

$$Q := u^4+u^2+1$$

$$R := x$$

$$S := 2u^3+u \quad (38)$$

> $SolGralUno := int\left(\frac{P}{R}, x\right)+int\left(\frac{S}{Q}, u\right)=\_C1$

$$SolGralUno := 2\ln(x)+\frac{\ln\left(u^4+u^2+1\right)}{2}=\_C1 \quad (39)$$

> SolGralDos := simplify(exp(lhs(SolGralUno))) = _C1

$$SolGralDos := x^2\sqrt{u^4+u^2+1}=\_C1 \quad (40)$$

> $SolGralTres := lhs(SolGralDos)^2=\_C1$

$$SolGralTres := x^4\left(u^4+u^2+1\right)=\_C1 \quad (41)$$

> $SolGralHomogeneos := expand\left(subs\left(u=\frac{y(x)}{x}, SolGralTres\right)\right)$

$$SolGralHomogeneos := y(x)^4+x^2y(x)^2+x^4=\_C1 \quad (42)$$

> Ecuacion

$$x\left(2x^2+y(x)^2\right)+y(x)\left(x^2+2y(x)^2\right)\left(\frac{d}{dx}y(x)\right)=0 \quad (43)$$

> DerEcua := simplify(isolate(Ecuacion, diff(y(x), x)))

(44)

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{-2\,x^3 - x\,y(x)^2}{y(x)\,\left(x^2 + 2\,y(x)^2\right)} \quad (44)$$

> *DerSolGralHomogeneos* := *simplify*(*isolate*(*diff*(*SolGralHomogeneos*, *x*), *diff*(*y*(*x*), *x*)))

$$DerSolGralHomogeneos := \frac{d}{dx}\, y(x) = \frac{-2\,x^3 - x\,y(x)^2}{y(x)\,\left(x^2 + 2\,y(x)^2\right)} \quad (45)$$

>

Por el método de exacta

> *Ecuacion*

$$x\,\left(2\,x^2 + y(x)^2\right) + y(x)\,\left(x^2 + 2\,y(x)^2\right)\left(\frac{d}{dx}\, y(x)\right) = 0 \quad (46)$$

> $M := x\,(2\,x^2 + y^2)$

$$M := x\,\left(2\,x^2 + y^2\right) \quad (47)$$

> $N := y\,(x^2 + 2\,y^2)$

$$N := y\,\left(x^2 + 2\,y^2\right) \quad (48)$$

> *Comprobar* := *diff*(*M*, *y*) = *diff*(*N*, *x*)

$$Comprobar := 2\,x\,y = 2\,x\,y \quad (49)$$

> *IntM* := *int*(*M*, *x*)

$$IntM := \frac{\left(2\,x^2 + y^2\right)^2}{8} \quad (50)$$

> *SolGralExacta* := *IntM* + *int*((*N* − *diff*(*IntM*, *y*)), *y*) = _C1

$$SolGralExacta := \frac{\left(2\,x^2 + y^2\right)^2}{8} + \frac{3\,y^4}{8} = \_C1 \quad (51)$$

> *SolGralFinalExacta* := *expand*(*lhs*(*SolGralExacta*) · 2) = _C1

$$SolGralFinalExacta := x^4 + x^2\,y^2 + y^4 = \_C1 \quad (52)$$

> $SolFinalExacta := x^4 + x^2 \cdot y(x)^2 + y(x)^4 = \_C1$

$$SolFinalExacta := y(x)^4 + x^2\,y(x)^2 + x^4 = \_C1 \quad (53)$$

> *DerSolFinalExacta* := *simplify*(*isolate*(*diff*(*SolFinalExacta*, *x*), *diff*(*y*(*x*), *x*)))

$$DerSolFinalExacta := \frac{d}{dx}\, y(x) = \frac{-2\,x^3 - x\,y(x)^2}{y(x)\,\left(x^2 + 2\,y(x)^2\right)} \quad (54)$$

> *DerSolGralHomogeneos*

$$\frac{d}{dx}\, y(x) = \frac{-2\,x^3 - x\,y(x)^2}{y(x)\,\left(x^2 + 2\,y(x)^2\right)} \quad (55)$$

Eso muestra que la solución general por ambos métodos resulta igual

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>

FIN RESPUESTA 3)

> *restart*

4) Obtener la solución general de la siguiente ecuación diferencial de primer orden

> $Ecuacion := 2\,x^2\,y(x) + 2\,y(x) + 5 + \left(2\,x^3 + 2\,x\right)\left(\frac{d}{dx}\,y(x)\right) = 0$

$$Ecuacion := 2\,x^2\,y(x) + 2\,y(x) + 5 + \left(2\,x^3 + 2\,x\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(56)}$$

>

RESPUESTA 4)

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[\_linear] \quad \textbf{(57)}$$

> $IntFact := intfactor(Ecuacion)$

$$IntFact := \frac{1}{x^2 + 1} \quad \textbf{(58)}$$

> $M := 2\,x^2\,y + 2\,y + 5$

$$M := 2\,x^2\,y + 2\,y + 5 \quad \textbf{(59)}$$

> $N := \left(2\,x^3 + 2\,x\right)$

$$N := 2\,x^3 + 2\,x \quad \textbf{(60)}$$

> $MM := IntFact \cdot M$

$$MM := \frac{2\,x^2\,y + 2\,y + 5}{x^2 + 1} \quad \textbf{(61)}$$

> $MMx := 2 \cdot y + \frac{5}{x^2 + 1}$

$$MMx := 2\,y + \frac{5}{x^2 + 1} \quad \textbf{(62)}$$

> $NN := simplify(IntFact \cdot N)$

$$NN := 2\,x \quad \textbf{(63)}$$

> $Comprobar := simplify(diff(MM, y) = diff(NN, x))$

$$Comprobar := 2 = 2 \quad \textbf{(64)}$$

Por lo tanto es exacta

> $IntNNy := int(NN, y)$

$$IntNNy := 2\,x\,y \quad \textbf{(65)}$$

> $SolGral := IntNNy + int((MMx - diff(IntNNy, x)), x) = \_C1$

$$SolGral := 2\,x\,y + 5\,\arctan(x) = \_C1 \quad \textbf{(66)}$$

> $SolGralFinal := 2\,x\,y(x) + 5\,\arctan(x) = \_C1$

$$SolGralFinal := 2\,x\,y(x) + 5\,\arctan(x) = \_C1 \quad \textbf{(67)}$$

> $DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\,y(x) = -\frac{1}{2}\,\frac{2\,x^2\,y(x) + 2\,y(x) + 5}{x\left(x^2 + 1\right)} \quad \textbf{(68)}$$

> $DerEcua := isolate(Ecuacion, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{-2\,x^2\,y(x) - 2\,y(x) - 5}{2\,x^3 + 2\,x} \quad \textbf{(69)}$$

> $Comprobar := simplify(rhs(DerSolGral) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \quad \textbf{(70)}$$

>

FIN RESPUESTA 4)

> $restart$

5) Obtener la solución de la ecuación de primer orden lineal

> $Ecuacion := x\,(x^3 + 1)\left(\frac{d}{dx}\, y(x)\right) + (2\,x^3 - 1)\,y(x) = 0$

$$Ecuacion := x\,(x^3 + 1)\left(\frac{d}{dx}\, y(x)\right) + (2\,x^3 - 1)\,y(x) = 0 \quad \textbf{(71)}$$

>

RESPUESTA 5)

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[\_separable] \quad \textbf{(72)}$$

> $M := (2\,x^3 - 1)\,y$

$$M := y\,(2\,x^3 - 1) \quad \textbf{(73)}$$

> $N := expand(x\,(x^3 + 1))$

$$N := x^4 + x \quad \textbf{(74)}$$

> $P := (2\,x^3 - 1);\, Q := y;\, R := x^4 + x;\, S := 1$

$$P := 2\,x^3 - 1$$
$$Q := y$$
$$R := x^4 + x$$
$$S := 1 \quad \textbf{(75)}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := \ln(x^2 - x + 1) + \ln(x + 1) - \ln(x) + \ln(y) = \_C1 \quad \textbf{(76)}$$

> $SolGralDos := expand(\exp(lhs(SolGral))) = \_C1$

$$SolGralDos := y\,x^2 + \frac{y}{x} = \_C1 \quad \textbf{(77)}$$

> $SolGralTres := y(x)x^2 + \frac{y(x)}{x} = \_C1$

$$SolGralTres := y(x)\,x^2 + \frac{y(x)}{x} = \_C1 \quad \textbf{(78)}$$

> $DerSolGral := simplify(isolate(diff(lhs(SolGralTres), x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = -\frac{y(x)\,(2\,x^3 - 1)}{x\,(x^3 + 1)} \quad \textbf{(79)}$$

> $DerEcua := isolate(Ecuacion, diff(y(x), x))$

$$DerEcua := \frac{d}{dx} y(x) = -\frac{y(x)\,(2x^3 - 1)}{x\,(x^3 + 1)} \quad \textbf{(80)}$$

> $Comprobacion := simplify(rhs(DerSolGral) - rhs(DerEcua)) = 0$

$$Comprobacion := 0 = 0 \quad \textbf{(81)}$$

>

OTRA RESPUESTA COMO LINEAL

> $Ecuacion$

$$x\,(x^3 + 1)\left(\frac{d}{dx} y(x)\right) + (2x^3 - 1)\,y(x) = 0 \quad \textbf{(82)}$$

> $EcuacionCero := diff(y(x), x) + \frac{(2x^3 - 1)\,y(x)}{x\,(x^3 + 1)} = 0$

$$EcuacionCero := \frac{d}{dx} y(x) + \frac{y(x)\,(2x^3 - 1)}{x\,(x^3 + 1)} = 0 \quad \textbf{(83)}$$

> $p := \frac{(2x^3 - 1)}{x\,(x^3 + 1)}$

$$p := \frac{2x^3 - 1}{x\,(x^3 + 1)} \quad \textbf{(84)}$$

> $SolGral := expand(isolate(y(x) = expand(\_C1 \cdot \exp(-int(p, x))), \_C1))$

$$SolGral := \_C1 = y(x)\,x^2 + \frac{y(x)}{x} \quad \textbf{(85)}$$

> $SolGralTres$

$$y(x)\,x^2 + \frac{y(x)}{x} = \_C1 \quad \textbf{(86)}$$

>

FIN RESPUESTA 5)

> $restart$

6) Obtener la solución particular de ecuación de primer orden lineal con condiciones iniciales

> $Ecuacion := \frac{d}{dx} y(x) + y(x)\cos(x) = \sin(x)\cos(x)$

$$Ecuacion := \frac{d}{dx} y(x) + y(x)\cos(x) = \sin(x)\cos(x) \quad \textbf{(87)}$$

> $CondIni := y(0) = 1$

$$CondIni := y(0) = 1 \quad \textbf{(88)}$$

>

RESPUESTA 6)

> $p := \cos(x)$

$$p := \cos(x) \quad \textbf{(89)}$$

> $q := \sin(x) \cdot \cos(x)$

$$q := \sin(x)\cos(x) \quad \textbf{(90)}$$

> $SolGral := y(x) = expand(\_C1 \cdot \exp(int(-p, x)) + \exp(int(-p, x)) \cdot int(\exp(int(p, x)) \cdot q, x))$

$$SolGral := y(x) = \frac{\_C1}{e^{\sin(x)}} + \sin(x) - 1 \quad \textbf{(91)}$$

> $Para := simplify(isolate(subs(x = 0, rhs(SolGral) = 1), \_C1))$

$$Para := \_C1 = 2 \quad \textbf{(92)}$$

> $SolPart := subs(\_C1 = rhs(Para), SolGral)$

$$SolPart := y(x) = \frac{2}{e^{\sin(x)}} + \sin(x) - 1 \quad \textbf{(93)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolPart), Ecuacion)))$

$$Comprobar := \sin(x)\cos(x) = \sin(x)\cos(x) \quad \textbf{(94)}$$

>

FIN RESPUESTA 6)

> *restart*

FIN SERIE 1)

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