>

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SERIE 2
DE EJERCICIOS DEL TEMA 2
SEMESTRE 2026-2
SOLUCIÓN

2026-03-24

> *restart*

1) OBTENER LA SOLUCIÓN GENERAL DE LA ECUACIÓN DIFERENCIAL SIGUIENTE **(sin utilizar dsolve)**

> $x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{1}{2}\sqrt{x}\,(2+\ln(x))=0$

$$x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{\sqrt{x}\,(2+\ln(x))}{2}=0 \qquad (1)$$

>

SOLUCIÓN 1

> $Ecuacion := x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{1}{2}\sqrt{x}\,(2+\ln(x))=0$

$$Ecuacion := x\ln(x)\left(\frac{d}{dx}y(x)\right)-(1+\ln(x))\,y(x)+\frac{\sqrt{x}\,(2+\ln(x))}{2}=0 \qquad (2)$$

> $EcuaDos := expand\left(\frac{lhs(Ecuacion)}{x\ln(x)}\right)=0$

$$EcuaDos := \frac{d}{dx}y(x)-\frac{y(x)}{x\ln(x)}-\frac{y(x)}{x}+\frac{1}{\sqrt{x}\,\ln(x)}+\frac{1}{2\sqrt{x}}=0 \qquad (3)$$

> $EcuaTres := lhs(EcuaDos)-\left(\frac{1}{\sqrt{x}\,\ln(x)}+\frac{1}{2\sqrt{x}}\right)=rhs(EcuaDos)-\left(\frac{1}{\sqrt{x}\,\ln(x)}+\frac{1}{2\sqrt{x}}\right)$

$$EcuaTres := \frac{d}{dx}y(x)-\frac{y(x)}{x\ln(x)}-\frac{y(x)}{x}=-\frac{1}{\sqrt{x}\,\ln(x)}-\frac{1}{2\sqrt{x}} \qquad (4)$$

> $p := factor\left(-\frac{1}{x\ln(x)}-\frac{1}{x}\right);\ q := factor\left(-\frac{1}{\sqrt{x}\,\ln(x)}-\frac{1}{2\sqrt{x}}\right)$

$$p := -\frac{1+\ln(x)}{x\ln(x)}$$

$$q := -\frac{2+\ln(x)}{2\sqrt{x}\,\ln(x)} \qquad (5)$$

> $IntMasP := simplify(\exp(int(p,x)))$

$$IntMasP := \frac{1}{x\ln(x)} \qquad (6)$$

> $IntMenosP := simplify(\exp(-int(p, x)))$

$$IntMenosP := x \ln(x) \quad (7)$$

> $SolGral := y(x) = \_C1 \cdot IntMenosP + IntMenosP \cdot int(IntMasP \cdot q, x)$

$$SolGral := y(x) = \_C1\, x \ln(x) + \sqrt{x} \quad (8)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobar := 0 = 0 \quad (9)$$

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<<<<<<<<<<<<<<<<<FIN PREGUNTA 1 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> $restart:$

2) DADA LA ECUACIÓN DIFERENCIAL

> $\frac{d^2}{dx^2} y(x) - 4\, y(x) = 0$

$$\frac{d^2}{dx^2} y(x) - 4\, y(x) = 0 \quad (10)$$

a) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL HOMOGENEA **(sin utilizar dsolve)**
b) CON LA SOLUCIÓN GENERAL OBTENIDA EN EL INCISO a) Y DADAS LAS CONDICIONES INICIALES **y(0) = -6 &**
y'(0) = 8 OBTENER LA SOLUCIÓN PARTICULAR **(sin utilizar dsolve)**
c) GRAFIQUE (JUNTAS) LA SOLUCIÓN PARTICULAR OBTENIDA EN EL INCISO b) Y LA PRIMERA DERIVADA DE ÉSTA, CONSIDERNADO UN INTERVALO $0 < x < 1$

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SOLUCIÓN 2

> $Ecua := \frac{d^2}{dx^2} y(x) - 4\, y(x) = 0$

$$Ecua := \frac{d^2}{dx^2} y(x) - 4\, y(x) = 0 \quad (11)$$

> $EcuaCarac := m^2 - 4 = 0$

$$EcuaCarac := m^2 - 4 = 0 \quad (12)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2, -2 \quad (13)$$

> $yy[1] := \exp(Raiz[1] \cdot x);\ yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_1 := e^{2x}$$
$$yy_2 := e^{-2x} \quad (14)$$

> $SolGral := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGral := y(x) = \_C1\, e^{2x} + \_C2\, e^{-2x} \quad (15)$$

> $EcuaUno := simplify(subs(x=0, rhs(SolGral) = -6))$

$$EcuaUno := \_C1 + \_C2 = -6 \quad (16)$$

> $EcuaDos := simplify(subs(x=0, rhs(diff(SolGral, x)) = 8))$

$$EcuaDos := 2\ \_C1 - 2\ \_C2 = 8 \quad (17)$$

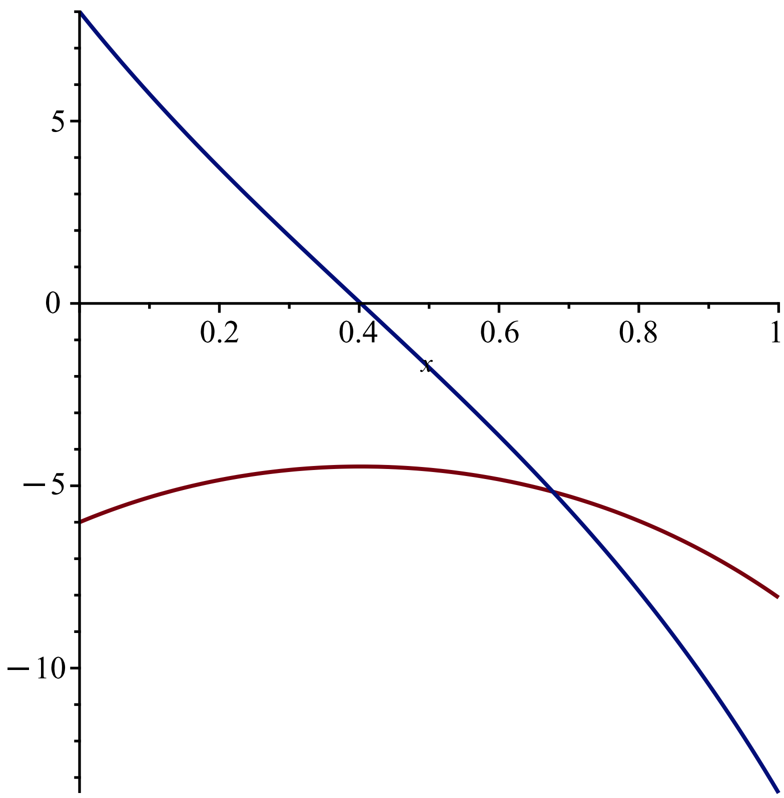
> $Para := solve(\{EcuaUno, EcuaDos\})$

$$Para := \{\_C1 = -1, \_C2 = -5\} \quad (18)$$

> $SolPart := subs(\_C1 = rhs(Para[1]), \_C2 = rhs(Para[2]), SolGral)$

$$SolPart := y(x) = -e^{2x} - 5\,e^{-2x} \quad (19)$$

> $plot([rhs(SolPart), rhs(diff(SolPart, x))], x = 0..1)$



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<<<<<<<<<<<<<<<<<<<<<<<FIN PREGUNTA 2 >>>>>>>>>>>>>>>>>>>>>>>>

> $restart:$

3) OBTENGA Y GRAFIQUE { EN EL INTERVALO - 1..1 } LA SOLUCIÓN PARTICULAR DE LOS SIGUIENTES PROBLEMAS **(sin utilizar dsolve):**

a) CON CONDICIONES EN LA FRONTERA

> $\frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0; y(0) = -3; y\left(\frac{1}{2}\pi\right) = 3; y\left(\frac{3}{2}\pi\right) = 9$

$$\frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0$$

$$y(0) = -3$$

$$y\left(\frac{1}{2}\pi\right)=3$$

$$y\left(\frac{3}{2}\pi\right)=9 \quad \textbf{(20)}$$

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SOLUCIÓN 3a)

> $Ecuacion := \frac{d^3}{dx^3}y(x)+\frac{d^2}{dx^2}y(x)+\frac{d}{dx}y(x)+y(x)=0$

$$Ecuacion := \frac{d^3}{dx^3}y(x)+\frac{d^2}{dx^2}y(x)+\frac{d}{dx}y(x)+y(x)=0 \quad \textbf{(21)}$$

> $EcuaCarac := m^3+m^2+m+1=0$

$$EcuaCarac := m^3+m^2+m+1=0 \quad \textbf{(22)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1, \mathrm{I}, -\mathrm{I} \quad \textbf{(23)}$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \cos(\mathrm{Im}(Raiz[2])\cdot x); yy[3] := \sin(\mathrm{Im}(Raiz[2])\cdot x)$

$$yy_1 := e^{-x}$$

$$yy_2 := \cos(x)$$

$$yy_3 := \sin(x) \quad \textbf{(24)}$$

> $SolGral := y(x) = \_C1\cdot yy[1]+\_C2\cdot yy[2]+\_C3\cdot yy[3]$

$$SolGral := y(x) = \_C1\, e^{-x}+\_C2\cos(x)+\_C3\sin(x) \quad \textbf{(25)}$$

> $EcuaUno := simplify(subs(x=0, rhs(SolGral)=-3))$

$$EcuaUno := \_C1+\_C2=-3 \quad \textbf{(26)}$$

> $EcuaDos := simplify\left(subs\left(x=\frac{Pi}{2}, rhs(SolGral)=3\right)\right)$

$$EcuaDos := \_C1\, e^{-\frac{1}{2}\pi}+\_C3=3 \quad \textbf{(27)}$$

> $EcuaTres := simplify\left(subs\left(x=\frac{3\cdot Pi}{2}, rhs(SolGral)=9\right)\right)$

$$EcuaTres := \_C1\, e^{-\frac{3}{2}\pi}-\_C3=9 \quad \textbf{(28)}$$

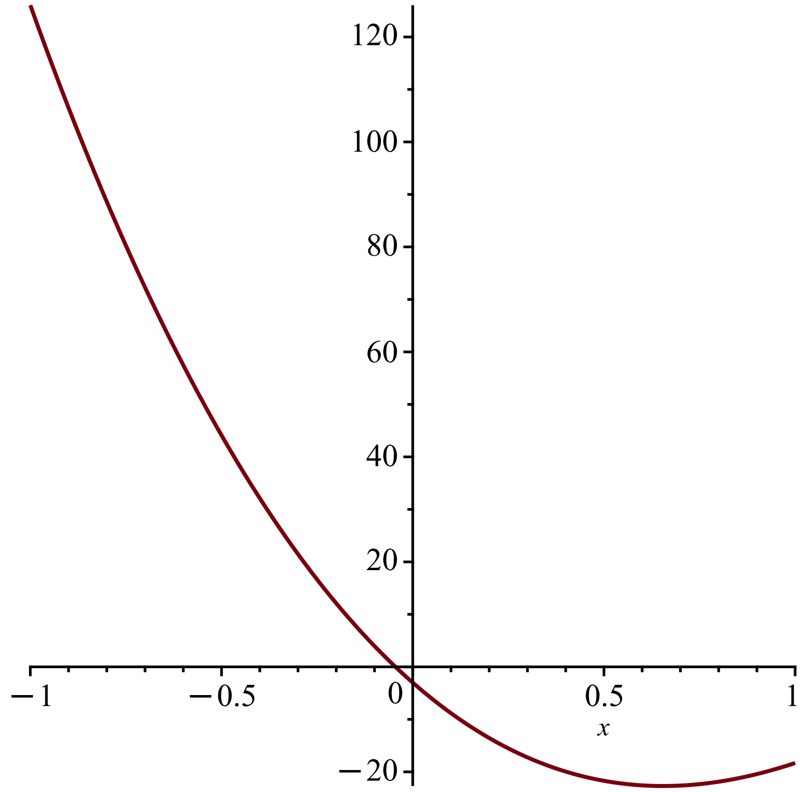
> $Para := solve(\{EcuaUno, EcuaDos, EcuaTres\})$

$$Para := \left\{\_C1=\frac{12}{e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}}, \_C2=-\frac{3\left(e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}+4\right)}{e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}}, \_C3=\frac{3\left(e^{-\frac{3}{2}\pi}-3\,e^{-\frac{1}{2}\pi}\right)}{e^{-\frac{3}{2}\pi}+e^{-\frac{1}{2}\pi}}\right\} \quad \textbf{(29)}$$

> $SolPart := subs(\_C1 = rhs(Para[1]), \_C2 = rhs(Para[2]), \_C3 = rhs(Para[3]), SolGral) :$
> $evalf(\%, 3)$

$$y(x) = 55.3\, \mathrm{e}^{-1.\, x} - 58.2 \cos(x) - 8.49 \sin(x) \qquad (30)$$

> $plot(rhs(SolPart), x = -1 ..1)$



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> $restart :$

b) CON CONDICIONES INICIALES

> $\frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t) - 7 \left(\frac{\mathrm{d}}{\mathrm{d}t} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2; x(1) = 2; \mathrm{D}(x)(1) = -2$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t) - 7 \left(\frac{\mathrm{d}}{\mathrm{d}t} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2$$

$$x(1) = 2$$

$$\mathrm{D}(x)(1) = -2 \qquad (31)$$

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SOLUCIÓN 3b)

> $Ecua := \frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t) - 7 \left(\frac{\mathrm{d}}{\mathrm{d}t} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2$

$$Ecua := \frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t) - 7 \left(\frac{\mathrm{d}}{\mathrm{d}t} x(t)\right) + 12\, x(t) = \cos(3\, t) + t^2 \qquad (32)$$

> $EcuaHom := lhs(Ecua) = 0; Q := rhs(Ecua)$

$$EcuaHom := \frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12\, x(t) = 0$$

$$Q := \cos(3\,t) + t^2 \quad \textbf{(33)}$$

> $EcuaCarac := m^2 - 7 \cdot m + 12 = 0$

$$EcuaCarac := m^2 - 7\,m + 12 = 0 \quad \textbf{(34)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 4, 3 \quad \textbf{(35)}$$

> $xx[1] := \exp(Raiz[1] \cdot t); xx[2] := \exp(Raiz[2] \cdot t)$

$$xx_1 := e^{4\,t}$$

$$xx_2 := e^{3\,t} \quad \textbf{(36)}$$

> $SolGralHom := x(t) = \_C1 \cdot xx[1] + \_C2 \cdot xx[2]$

$$SolGralHom := x(t) = \_C1\, e^{4\,t} + \_C2\, e^{3\,t} \quad \textbf{(37)}$$

> $SolGralNoHom := x(t) = AA \cdot xx[1] + BB \cdot xx[2]$

$$SolGralNoHom := x(t) = AA\, e^{4\,t} + BB\, e^{3\,t} \quad \textbf{(38)}$$

> $with(linalg):$

> $WW := wronskian([xx[1], xx[2]], t)$

$$WW := \begin{bmatrix} e^{4\,t} & e^{3\,t} \\ 4\,e^{4\,t} & 3\,e^{3\,t} \end{bmatrix} \quad \textbf{(39)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \cos(3\,t) + t^2 \end{bmatrix} \quad \textbf{(40)}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} e^{-4\,t} \left(\cos(3\,t) + t^2\right) & -e^{-3\,t} \left(\cos(3\,t) + t^2\right) \end{bmatrix} \quad \textbf{(41)}$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := e^{-4\,t} \left(\cos(3\,t) + t^2\right)$$

$$Bprima := -e^{-3\,t} \left(\cos(3\,t) + t^2\right) \quad \textbf{(42)}$$

> $AA := simplify(int(Aprima, t) + \_C10)$

$$AA := \frac{1}{800} \left(384 \cos(t)^2 \sin(t) - 512 \cos(t)^3 - 96 \sin(t) + 384 \cos(t) + 800\, \_C10\, e^{4\,t} - 200\, t^2 - 100\, t - 25\right) e^{-4\,t} \quad \textbf{(43)}$$

> $BB := simplify(int(Bprima, t) + \_C20)$

$$BB := -\frac{1}{54} \left(36 \cos(t)^2 \sin(t) - 36 \cos(t)^3 - 9 \sin(t) + 27 \cos(t) - 54\, \_C20\, e^{3\,t} - 18\, t^2 - 12\, t - 4\right) e^{-3\,t} \quad \textbf{(44)}$$

> $SolGralNoHom$

(45)

$$x(t) = \frac{1}{800}\left(384\cos(t)^2\sin(t) - 512\cos(t)^3 - 96\sin(t) + 384\cos(t) + 800\,\_C10\,e^{4t} - 200t^2 - 100t - 25\right)e^{-4t}e^{4t} - \frac{1}{54}\left(36\cos(t)^2\sin(t) - 36\cos(t)^3 - 9\sin(t) + 27\cos(t) - 54\,\_C20\,e^{3t} - 18t^2 - 12t - 4\right)e^{-3t}e^{3t} \quad \mathbf{(45)}$$

> *EcuaUno* := *subs*($t=1$, *rhs*(*SolGralNoHom*) = 2) : *evalf*(%, 2)

$$0.24 + 53.\,\_C10 + 21.\,\_C20 = 2. \quad \mathbf{(46)}$$

> *EcuaDos* := *subs*($t=1$, *rhs*(*diff*(*SolGralNoHom*, t)) = −2) : *evalf*(%, 2)

$$0.42 + 220.\,\_C10 + 61.\,\_C20 = -2. \quad \mathbf{(47)}$$

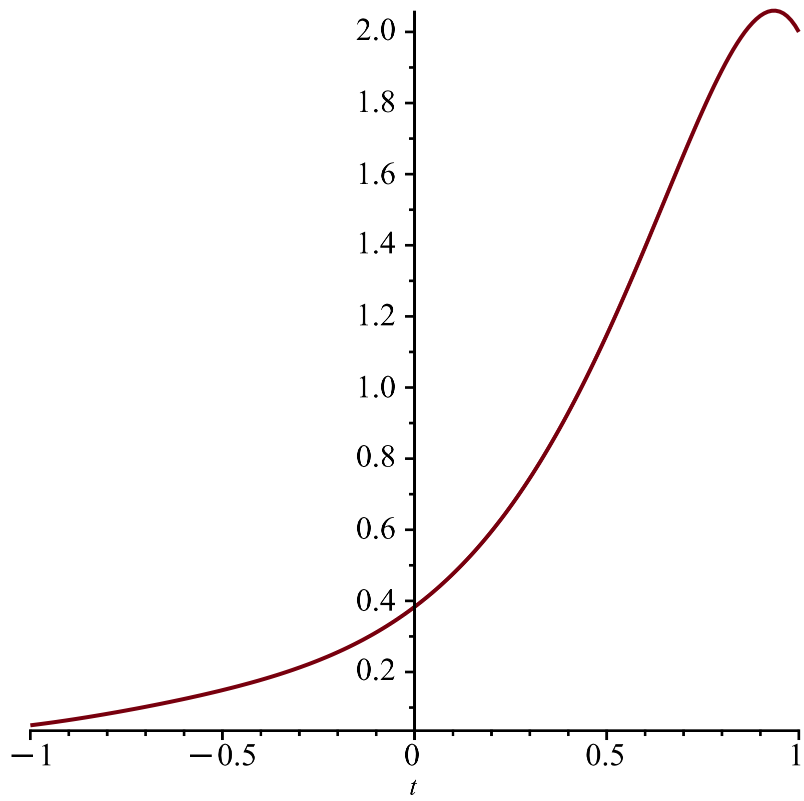
> *Param* := *solve*({*EcuaUno*, *EcuaDos*}) : *evalf*(%, 2)

$$\{\_C10 = -0.15,\ \_C20 = 0.52\} \quad \mathbf{(48)}$$

> *SolPart* := *simplify*(*subs*(_C10 = *rhs*(*Param*[1]), _C20 = *rhs*(*Param*[2]), *SolGralNoHom*)) : *evalf*(%, 2)

$$x(t) = -0.0069\,e^{3.+4.t} + 0.0087\,e^{4.+3.t} - 0.18\cos(t)^2\sin(t) + 0.027\cos(t)^3 + 0.083t^2 + 0.046\sin(t) - 0.020\cos(t) + 0.097t + 0.042 \quad \mathbf{(49)}$$

> *plot*(*rhs*(*SolPart*), $t = -1..1$)



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> *restart* :

c) CON CONDICIONES INICIALES

> $\frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x}; y(0) = -5; D(y)(0) = 8$

$$\frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x}$$
$$y(0) = -5$$
$$D(y)(0) = 8 \quad (50)$$

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SOLUCIÓN 3c)

> $Ecua := \frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x}$

$$Ecua := \frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 3 e^{2x} \quad (51)$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 2\left(\frac{d}{dx} y(x)\right) + 2 y(x) = 0 \quad (52)$$

> $Q := rhs(Ecua)$

$$Q := 3 e^{2x} \quad (53)$$

> $EcuaCarac := m^2 + 2m + 2 = 0$

$$EcuaCarac := m^2 + 2m + 2 = 0 \quad (54)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1 + I, -1 - I \quad (55)$$

> $yy[1] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \cos(\mathrm{Im}(Raiz[1]) \cdot x); yy[2] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := e^{-x} \cos(x)$$
$$yy_2 := e^{-x} \sin(x) \quad (56)$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGralHom := y(x) = \_C1\, e^{-x} \cos(x) + \_C2\, e^{-x} \sin(x) \quad (57)$$

> $SolGralNoHom := y(x) = AA \cdot yy[1] + BB \cdot yy[2]$

$$SolGralNoHom := y(x) = AA\, e^{-x} \cos(x) + BB\, e^{-x} \sin(x) \quad (58)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-x} \cos(x) & e^{-x} \sin(x) \\ -e^{-x} \cos(x) - e^{-x} \sin(x) & -e^{-x} \sin(x) + e^{-x} \cos(x) \end{bmatrix} \quad (59)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 3 e^{2x} \end{bmatrix} \quad (60)$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} -3\,e^{3x}\sin(x) & 3\,e^{3x}\cos(x) \end{bmatrix} \tag{61}$$

> $Aprima := Para[1];\ Bprima := Para[2]$

$$Aprima := -3\,e^{3x}\sin(x)$$

$$Bprima := 3\,e^{3x}\cos(x) \tag{62}$$

> $AA := simplify(int(Aprima, x) + \_C10) : evalf(\%, 2)$

$$0.30\,e^{3.x}\cos(x) - 0.90\,e^{3.x}\sin(x) + \_C10 \tag{63}$$

> $BB := simplify(int(Bprima, x) + \_C20) : evalf(\%, 2)$

$$0.90\,e^{3.x}\cos(x) + 0.30\,e^{3.x}\sin(x) + \_C20 \tag{64}$$

> $SolGralNoHom$

$$y(x) = \left(\frac{3}{10}\,e^{3x}\cos(x) - \frac{9}{10}\,e^{3x}\sin(x) + \_C10\right)e^{-x}\cos(x) + \left(\frac{9}{10}\,e^{3x}\cos(x) + \frac{3}{10}\,e^{3x}\sin(x) + \_C20\right)e^{-x}\sin(x) \tag{65}$$

> $EcuaUno := subs(x = 0, rhs(SolGralNoHom) = -5) : evalf(\%, 2)$

$$0.30 + \_C10 = -5. \tag{66}$$

> $EcuaDos := subs(x = 0, rhs(diff(SolGralNoHom, x)) = 8) : evalf(\%, 2)$

$$0.60 - 1.\ \_C10 + \_C20 = 8. \tag{67}$$

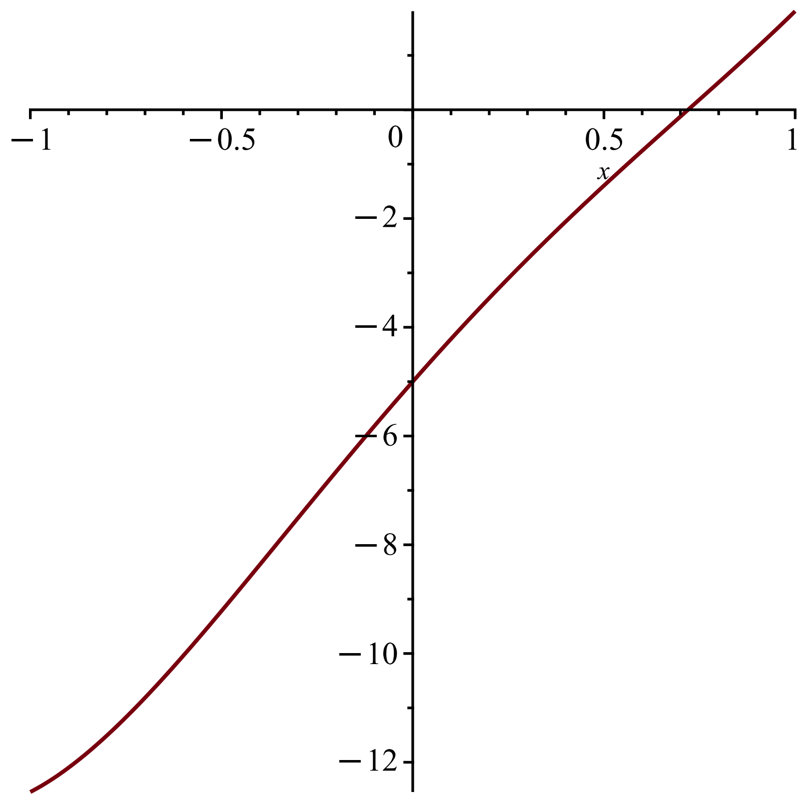
> $Param := solve(\{EcuaUno, EcuaDos\}) : evalf(\%, 2)$

$$\{\_C10 = -5.3,\ \_C20 = 2.1\} \tag{68}$$

> $SolPart := simplify(subs(\_C10 = rhs(Param[1]), \_C20 = rhs(Param[2]), SolGralNoHom)) :$
> $evalf(\%, 2)$

$$y(x) = 0.10\,e^{-1.x}\left(3.\,e^{3.x} - 53.\cos(x) + 21.\sin(x)\right) \tag{69}$$

> $plot(rhs(SolPart), x = -1..1)$



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<<<<<<<<<<<<<<<<<<< FIN PREGUNTA 3 >>

> *restart* :

4) DADO EL SIGUIENTE PROBLEMA DE CONDICIONES INICIALES & UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**

> $\frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-3t} \cos(2\,t); y(0) = -2; D(y)(0) = 0; D^{(2)}(y)(0) = 7; D^{(3)}(y)(0) = -5$

$$\frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-3t} \cos(2\,t)$$
$$y(0) = -2$$
$$D(y)(0) = 0$$
$$D^{(2)}(y)(0) = 7$$
$$D^{(3)}(y)(0) = -5 \quad \textbf{(70)}$$

a) OBTENER SU SOLUCIÓN PARTICULAR

b) GRAFICAR EL RESULTADO DEL INCISO a) EN UN INTERVALO 0 < t < 1

>

SOLUCION 4a)

> $Ecua := \frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-3t} \cos(2\,t)$

(71)

$$Ecua := \frac{d^4}{dt^4} y(t) + 5 \left(\frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t) \quad \textbf{(71)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^4}{dt^4} y(t) + 5 \left(\frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 0 \quad \textbf{(72)}$$

> $Q := rhs(Ecua)$

$$Q := 5 e^{-3t} \cos(2t) \quad \textbf{(73)}$$

> $EcuaCarac := m^4 + 5 \cdot m^2 - 4 = 0$

$$EcuaCarac := m^4 + 5 m^2 - 4 = 0 \quad \textbf{(74)}$$

> $Raiz := solve(EcuaCarac) : evalf(\%, 2)$

$$2.4\, I, -2.4\, I, 0.85, -0.85 \quad \textbf{(75)}$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot t) : evalf(\%, 2); yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot t) : evalf(\%, 2);$
$yy[3] := \exp(Raiz[3] \cdot t) : evalf(\%, 2); yy[4] := \exp(Raiz[4] \cdot t) : evalf(\%, 2)$

$$\cos(2.4\, t)$$
$$\sin(2.4\, t)$$
$$e^{0.85\, t}$$
$$e^{-0.85\, t} \quad \textbf{(76)}$$

> $SolGralHom := y(t) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2] + \_C3 \cdot yy[3] + \_C4 \cdot yy[4] : evalf(\%, 2)$

$$y(t) = \_C1 \cos(2.4\, t) + \_C2 \sin(2.4\, t) + \_C3\, e^{0.85\, t} + \_C4\, e^{-0.85\, t} \quad \textbf{(77)}$$

> $SolGralNoHom := y(t) = AA \cdot yy[1] + BB \cdot yy[2] + DD \cdot yy[3] + EE \cdot yy[4] : evalf(\%, 2)$

$$y(t) = AA \cos(2.4\, t) + BB \sin(2.4\, t) + DD\, e^{0.85\, t} + EE\, e^{-0.85\, t} \quad \textbf{(78)}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2], yy[3], yy[4]], t) : evalf(\%, 2)$

$$\begin{bmatrix} \cos(2.4\, t) & \sin(2.4\, t) & e^{0.85\, t} & e^{-0.85\, t} \\ -2.4 \sin(2.4\, t) & 2.4 \cos(2.4\, t) & 0.85\, e^{0.85\, t} & -0.85\, e^{-0.85\, t} \\ -5.8 \cos(2.4\, t) & -5.8 \sin(2.4\, t) & 0.75\, e^{0.85\, t} & 0.75\, e^{-0.85\, t} \\ 13. \sin(2.4\, t) & -13. \cos(2.4\, t) & 0.59\, e^{0.85\, t} & -0.59\, e^{-0.85\, t} \end{bmatrix} \quad \textbf{(79)}$$

> $BB := array([0, 0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 0 & 5 e^{-3t} \cos(2t) \end{bmatrix} \quad \textbf{(80)}$$

> $Para := simplify(linsolve(WW, BB)) : evalf(\%, 2)$

$$[0.31 \sin(2.4\, t)\, e^{-3.\, t} \cos(2.\, t), -0.31 \cos(2.4\, t)\, e^{-3.\, t} \cos(2.\, t), 0.46\, e^{-3.8\, t} \cos(2.\, t), -0.46\, e^{-2.2\, t} \cos(2.\, t)] \quad \textbf{(81)}$$

> $Aprima := Para[1] : evalf(\%, 2); Bprima := Para[2] : evalf(\%, 2); Dprima := Para[3] :$
$evalf(\%, 2); Eprima := Para[4] : evalf(\%, 2)$

$$0.31 \sin(2.4\, t)\, e^{-3.\, t} \cos(2.\, t)$$
$$-0.31 \cos(2.4\, t)\, e^{-3.\, t} \cos(2.\, t)$$

$$0.46\, e^{-3.8\, t} \cos(2.\, t)$$
$$-0.46\, e^{-2.2\, t} \cos(2.\, t) \qquad \textbf{(82)}$$

> $AA := simplify(int(Aprima, t) + \_C10) : evalf(\%, 2); BB := simplify(int(Bprima, t) + \_C20) : evalf(\%, 2); DD := simplify(int(Dprima, t) + \_C30) : evalf(\%, 2); EE := simplify(int(Eprima, t) + \_C40) : evalf(\%, 2)$

$$-0.0069 \cos(0.40\, t)\, e^{-3.\, t} - 0.025 \cos(4.4\, t)\, e^{-3.\, t} - 0.017 \sin(4.4\, t)\, e^{-3.\, t} - 0.054 \sin(0.40\, t)\, e^{-3.\, t} + 1.0\, \_C10$$
$$-0.0072 \sin(0.40\, t)\, e^{-3.\, t} - 0.026 \sin(4.4\, t)\, e^{-3.\, t} + 1.0\, \_C20 + 0.054 \cos(0.40\, t)\, e^{-3.\, t} + 0.018 \cos(4.4\, t)\, e^{-3.\, t}$$
$$-0.091\, e^{-3.8\, t} \cos(2.\, t) + 0.050\, e^{-3.8\, t} \sin(2.\, t) + 1.0\, \_C30$$
$$0.12\, e^{-2.2\, t} \cos(2.\, t) - 0.11\, e^{-2.2\, t} \sin(2.\, t) + 1.2\, \_C40 \qquad \textbf{(83)}$$

> $SolGralNoHom : evalf(\%, 2)$

$$y(t) = -0.0000049\, \left(1400.\, \cos(0.40\, t)\, e^{-3.\, t} + 5100.\, \cos(4.4\, t)\, e^{-3.\, t} + 3500.\, \sin(4.4\, t)\, e^{-3.\, t} + 11000.\, \sin(0.40\, t)\, e^{-3.\, t} - 2.1\, 10^{5}\, \_C10\right) \cos(2.4\, t) + 0.0000050\, \left(-1400.\, \sin(0.40\, t)\, e^{-3.\, t} - 5100.\, \sin(4.4\, t)\, e^{-3.\, t} + 2.1\, 10^{5}\, \_C20 + 11000.\, \cos(0.40\, t)\, e^{-3.\, t} + 3500.\, \cos(4.4\, t)\, e^{-3.\, t}\right) \sin(2.4\, t) - 0.00038\, \left(240.\, e^{-3.8\, t} \cos(2.\, t) - 130.\, e^{-3.8\, t} \sin(2.\, t) - 2600.\, \_C30\right) e^{0.85\, t} - 0.00084\, \left(-140.\, e^{-2.2\, t} \cos(2.\, t) + 130.\, e^{-2.2\, t} \sin(2.\, t) - 1400.\, \_C40\right) e^{-0.85\, t} \qquad \textbf{(84)}$$

> $EcuaUno := subs(t = 0, rhs(SolGralNoHom) = -2) : evalf(\%, 2)$

$$1.0\, \_C10 + 1.0\, \_C30 + 1.2\, \_C40 = -2. \qquad \textbf{(85)}$$

> $EcuaDos := subs(t = 0, rhs(diff(SolGralNoHom, t)) = 0) : evalf(\%, 2)$

$$-0.01 + 2.5\, \_C20 + 0.84\, \_C30 - 1.0\, \_C40 = 0. \qquad \textbf{(86)}$$

> $EcuaTres := subs(t = 0, rhs(diff(SolGralNoHom, t\$2)) = 7) : evalf(\%, 2)$

$$0.28 - 5.8\, \_C10 + 0.73\, \_C30 + 0.85\, \_C40 = 7. \qquad \textbf{(87)}$$

> $EcuaCuatro := subs(t = 0, rhs(diff(SolGralNoHom, t\$3)) = -5) : evalf(\%, 2)$

$$-1.5 - 15.\, \_C20 + 0.63\, \_C30 - 0.75\, \_C40 = -5. \qquad \textbf{(88)}$$

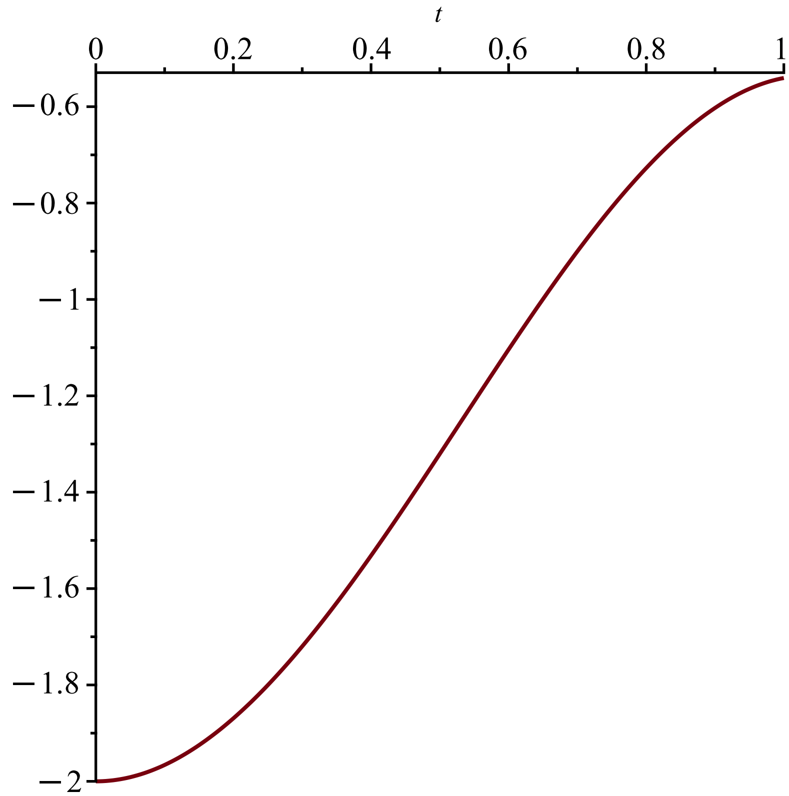
> $Param := solve(\{EcuaUno, EcuaDos, EcuaTres, EcuaCuatro\}) : evalf(\%, 2)$

$$\{\_C10 = -1.3, \_C20 = 0.27, \_C30 = -1.1, \_C40 = -0.28\} \qquad \textbf{(89)}$$

> $SolPart := simplify(subs(\_C10 = rhs(Param[1]), \_C20 = rhs(Param[2]), \_C30 = rhs(Param[3]), \_C40 = rhs(Param[4]), SolGralNoHom)) : evalf(\%, 2)$

$$y(t) = -0.020 \cos(0.40\, t)\, e^{-3.\, t} \cos(2.4\, t) - 0.020\, e^{-3.\, t} \cos(4.4\, t) \cos(2.4\, t) - 0.054\, e^{-3.\, t} \sin(4.4\, t) \cos(2.4\, t) - 0.054\, e^{-3.\, t} \sin(0.40\, t) \cos(2.4\, t) - 0.015\, e^{-3.\, t} \sin(4.4\, t) \sin(2.4\, t) - 0.020\, e^{-3.\, t} \sin(0.40\, t) \sin(2.4\, t) + 0.049 \cos(0.40\, t)\, e^{-3.\, t} \sin(2.4\, t) + 0.059\, e^{-3.\, t} \cos(4.4\, t) \sin(2.4\, t) - 0.49\, e^{0.85\, t} + 0.49\, e^{-0.85\, t} + 0.24 \sin(2.4\, t) - 1.5 \cos(2.4\, t) - 0.049\, e^{-3.\, t} \sin(2.\, t) \qquad \textbf{(90)}$$

> $plot(rhs(SolPart), t = 0 ..1)$



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<<<<<<<<<<<<<<<<<<<<<<<<<FIN PREGUNTA 4 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> *restart*

5) SI SABEMOS QUE LA SOLUCIÓN GENERAL

> $y(x) = \frac{C_1}{x^2} + C_2 x$

$$y(x) = \frac{C_1}{x^2} + C_2 x \tag{91}$$

SATISFACE LA ECUACIÓN DIFERENCIAL HOMOGÉNEA SIGUIENTE

> $-2\,y(x) + \left(\frac{d^2}{dx^2}\,y(x)\right)x^2 + 2\,x\left(\frac{d}{dx}\,y(x)\right) = 0$

$$-2\,y(x) + \left(\frac{d^2}{dx^2}\,y(x)\right)x^2 + 2\,x\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{92}$$

(93)

OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN NO HOMOGÉNEA UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**

> $-2\,y(x) + \left(\frac{d^2}{dx^2}\,y(x)\right)x^2 + 2\,x\left(\frac{d}{dx}\,y(x)\right) = 32\,x^2$

$$-2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2 \quad \textbf{(94)}$$

>

SOLUCIÓN 5

> $SolGral := y(x) = \frac{C_1}{x^2} + C_2\,x$

$$SolGral := y(x) = \frac{C_1}{x^2} + C_2\,x \quad \textbf{(95)}$$

> $EcuaHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=0$

$$EcuaHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=0 \quad \textbf{(96)}$$

> $EcuaNoHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2$

$$EcuaNoHom := -2\,y(x)+\left(\frac{d^2}{dx^2}\,y(x)\right)x^2+2\,x\left(\frac{d}{dx}\,y(x)\right)=32\,x^2 \quad \textbf{(97)}$$

> $EcuaHomStandard := expand\left(\frac{lhs(EcuaHom)}{x^2}\right)=0$

$$EcuaHomStandard := -\frac{2\,y(x)}{x^2}+\frac{d^2}{dx^2}\,y(x)+\frac{2\left(\frac{d}{dx}\,y(x)\right)}{x}=0 \quad \textbf{(98)}$$

> $EcuaNoHomStandard := expand\left(\frac{lhs(EcuaNoHom)}{x^2}\right)=expand\left(\frac{rhs(EcuaNoHom)}{x^2}\right)$

$$EcuaNoHomStandard := -\frac{2\,y(x)}{x^2}+\frac{d^2}{dx^2}\,y(x)+\frac{2\left(\frac{d}{dx}\,y(x)\right)}{x}=32 \quad \textbf{(99)}$$

> $Q := rhs(EcuaNoHomStandard)$

$$Q := 32 \quad \textbf{(100)}$$

> $ComprobarUno := simplify(eval(subs(y(x)=rhs(SolGral), EcuaHomStandard)))$

$$ComprobarUno := 0 = 0 \quad \textbf{(101)}$$

> $SolNoHom := y(x) = \frac{AA}{x^2} + BB\cdot x$

$$SolNoHom := y(x) = \frac{AA}{x^2} + BB\,x \quad \textbf{(102)}$$

> $yy[1] := \frac{1}{x^2};\, yy[2] := x$

$$yy_1 := \frac{1}{x^2}$$

(103)

$$yy_2 := x \quad (103)$$

> with(linalg) :

> WW := wronskian([yy[1], yy[2]], x)

$$WW := \begin{bmatrix} \frac{1}{x^2} & x \\ -\frac{2}{x^3} & 1 \end{bmatrix} \quad (104)$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & 32 \end{bmatrix} \quad (105)$$

> Para := simplify(linsolve(WW, BB))

$$Para := \begin{bmatrix} -\frac{32}{3}x^3 & \frac{32}{3} \end{bmatrix} \quad (106)$$

> Aprima := Para[1]; Bprima := Para[2]

$$Aprima := -\frac{32}{3}x^3$$

$$Bprima := \frac{32}{3} \quad (107)$$

> AA := simplify(int(Aprima, x) + _C1)

$$AA := -\frac{8}{3}x^4 + \_C1 \quad (108)$$

> BB := simplify(int(Bprima, x) + _C2)

$$BB := \frac{32}{3}x + \_C2 \quad (109)$$

> SolFinal := expand(SolNoHom)

$$SolFinal := y(x) = 8x^2 + \frac{\_C1}{x^2} + x\,\_C2 \quad (110)$$

> comprobarDos := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(EcuaNoHom) − rhs(EcuaNoHom) = 0)))

$$comprobarDos := 0 = 0 \quad (111)$$

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FINAL SERIE 2

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