>

SERIE 2026-2-3 SOLUCIÓN

> *restart*

1) Resuelva

> $Sistema := diff(x(t), t) = x(t) + 2\cdot y(t) + \exp(t),\ diff(y(t), t) = 2\cdot x(t) + y(t) + \exp(3\,t)$:
$Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\,x(t) = x(t) + 2\,y(t) + e^{t}$$

$$\frac{d}{dt}\,y(t) = 2\,x(t) + y(t) + e^{3\,t} \quad (1)$$

> $CondIni := x(0) = 0, y(0) = 0$

$$CondIni := x(0) = 0, y(0) = 0 \quad (2)$$

>

Respuesta 1)

> $AA := array([[1, 2], [2, 1]])$

$$AA := \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad (3)$$

> $BB := array([\exp(t), \exp(3\,t)])$

$$BB := \begin{bmatrix} e^{t} & e^{3\,t} \end{bmatrix} \quad (4)$$

> $Xcero := array([0, 0])$

$$Xcero := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (5)$$

> $with(linalg)$:

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{e^{-t}}{2} + \frac{e^{3\,t}}{2} & \frac{e^{3\,t}}{2} - \frac{e^{-t}}{2} \\ \frac{e^{3\,t}}{2} - \frac{e^{-t}}{2} & \frac{e^{-t}}{2} + \frac{e^{3\,t}}{2} \end{bmatrix} \quad (6)$$

> $SolHom := evalm(MatExp \,\&*\, Xcero)$

$$SolHom := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (7)$$

> $MatExpTau := map(rcurry(eval, t = 't - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} \frac{e^{-t+\tau}}{2} + \frac{e^{3\,t-3\,\tau}}{2} & \frac{e^{3\,t-3\,\tau}}{2} - \frac{e^{-t+\tau}}{2} \\ \frac{e^{3\,t-3\,\tau}}{2} - \frac{e^{-t+\tau}}{2} & \frac{e^{-t+\tau}}{2} + \frac{e^{3\,t-3\,\tau}}{2} \end{bmatrix} \quad (8)$$

> $BBtau := map(rcurry(eval, t = 'tau'), BB)$

$$BBtau := \begin{bmatrix} e^{\tau} & e^{3\,\tau} \end{bmatrix} \quad (9)$$

> $ProdTau := evalm(MatExpTau \,\&*\, BBtau)$

$$ProdTau := \left[\left(\frac{e^{-t+\tau}}{2} + \frac{e^{3t-3\tau}}{2} \right) e^{\tau} + \left(\frac{e^{3t-3\tau}}{2} - \frac{e^{-t+\tau}}{2} \right) e^{3\tau}, \left(\frac{e^{3t-3\tau}}{2} - \frac{e^{-t+\tau}}{2} \right) e^{\tau} + \left(\frac{e^{-t+\tau}}{2} + \frac{e^{3t-3\tau}}{2} \right) e^{3\tau} \right] \quad (10)$$

```
> SolNoHom := map(int, ProdTau, tau = 0 ..t)
```

$$SolNoHom := \left[-\frac{e^{-t}}{8} + \frac{e^{3t}}{8} + \frac{t e^{3t}}{2} \quad \frac{e^{-t}}{8} + \frac{3 e^{3t}}{8} + \frac{t e^{3t}}{2} - \frac{e^{t}}{2} \right] \quad (11)$$

```
> SolPart := x(t) = SolHom[1] + SolNoHom[1], y(t) = SolHom[2] + SolNoHom[2] :
  SolPart[1]; SolPart[2]
```

$$x(t) = -\frac{e^{-t}}{8} + \frac{e^{3t}}{8} + \frac{t e^{3t}}{2}$$

$$y(t) = \frac{e^{-t}}{8} + \frac{3 e^{3t}}{8} + \frac{t e^{3t}}{2} - \frac{e^{t}}{2} \quad (12)$$

```
> ComprobarUno := subs(t = 0, SolPart[1])
```

$$ComprobarUno := x(0) = 0 \quad (13)$$

```
> ComprobarDos := subs(t = 0, SolPart[2])
```

$$ComprobarDos := y(0) = 0 \quad (14)$$

```
> ComprobarTres := simplify(eval(subs(x(t) = rhs(SolPart[1]), y(t) = rhs(SolPart[2]),
  lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))
```

$$ComprobarTres := 0 = 0 \quad (15)$$

```
> ComprobarTres := simplify(eval(subs(x(t) = rhs(SolPart[1]), y(t) = rhs(SolPart[2]),
  lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))
```

$$ComprobarTres := 0 = 0 \quad (16)$$

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>
```

FIN RESPUESTA 1)

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> restart
```

2) Resuelva

```
> Ecua := diff(y(t), t$2) + 6·diff(y(t), t) + 5·y(t) = exp(t)·Dirac(t - 1)
```

$$Ecua := \frac{d^2}{dt^2} y(t) + 6 \frac{d}{dt} y(t) + 5 y(t) = e^{t} \, \mathrm{Dirac}(t - 1) \quad (17)$$

```
> CondIni := y(0) = 0, D(y)(0) = 4
```

$$CondIni := y(0) = 0, D(y)(0) = 4 \quad (18)$$

```
>
```

Respuesta 2)

```
> with(inttrans) :
> EcuaTL := subs(CondIni, laplace(Ecua, t, s))
```

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) - 4 + 6 s \mathcal{L}(y(t), t, s) + 5 \mathcal{L}(y(t), t, s) = e^{1-s} \quad (19)$$

```
> SolTL := isolate(EcuaTL, laplace(y(t), t, s))
```

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{e^{1-s} + 4}{s^2 + 6 s + 5} \quad (20)$$

> $SolPart := simplify(invlaplace(SolTL, s, t))$

$$SolPart := y(t) = \frac{\text{Heaviside}(t-1)\,\sinh(2t-2)\,e^{4-3t}}{2} + 2\,e^{-3t}\sinh(2t) \quad (21)$$

> $ComprobarUno := simplify(eval(subs(t=0, SolPart)))$

$$ComprobarUno := y(0) = 0 \quad (22)$$

> $ComprobarDos := D(y)(0) = simplify(eval(subs(t=0, rhs(diff(SolPart, t)))))$

$$ComprobarDos := D(y)(0) = 4 \quad (23)$$

> $Ecua$

$$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 5\,y(t) = e^t\,\text{Dirac}(t-1) \quad (24)$$

> $ComprobarTres := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarTres := 0 = 0 \quad (25)$$

>

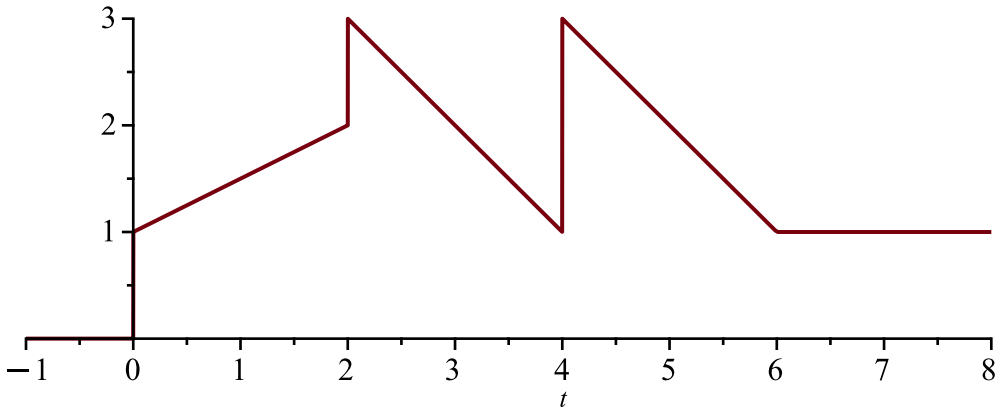
FIN RESPUESTA 2)

> $restart$

3) Obtenga la TL

> $f := \text{Heaviside}(t) + \frac{(t\cdot\text{Heaviside}(t))}{2} - \frac{((t-2)\cdot\text{Heaviside}(t-2))}{2} + \text{Heaviside}(t-2) - (t-2)\cdot\text{Heaviside}(t-2) + (t-4)\cdot\text{Heaviside}(t-4) + 2\cdot\text{Heaviside}(t-4) - (t-4)\cdot\text{Heaviside}(t-4) + (t-6)\cdot\text{Heaviside}(t-6); plot(f, t=-1..8, scaling = CONSTRAINED)$

$$f := \text{Heaviside}(t) + \frac{t\,\text{Heaviside}(t)}{2} - \frac{3\,(t-2)\,\text{Heaviside}(t-2)}{2} + \text{Heaviside}(t-2) + 2\,\text{Heaviside}(t-4) + (t-6)\,\text{Heaviside}(t-6)$$




```
> with(inttrans) :
> F := laplace(f, t, s)
```

$$F := \frac{e^{-2s}(2s-3) + 2e^{-6s} + 1}{2s^2} + \frac{1 + 2e^{-4s}}{s} \quad (26)$$

```
>
```

FIN RESPUESTA 3)

```
> restart
```

4) Resuelva

```
> Ecua := diff(y(t), t$2) - 2·diff(y(t), t) + y(t) = Heaviside(t - 2)
```

$$Ecua := \frac{d^2}{dt^2} y(t) - 2\frac{d}{dt} y(t) + y(t) = \text{Heaviside}(t-2) \quad (27)$$

```
> CondIni := y(0) = 0, D(y)(0) = 0
```

$$CondIni := y(0) = 0, D(y)(0) = 0 \quad (28)$$

```
>
```

Respuesta 4)

```
> with(inttrans) :
> EcuaTL := subs(CondIni, laplace(Ecua, t, s))
```

(29)

$$EcuaTL := s^2\,\mathcal{L}(y(t),t,s) - 2\,s\,\mathcal{L}(y(t),t,s) + \mathcal{L}(y(t),t,s) = \frac{e^{-2\,s}}{s} \quad (29)$$

> $SolTL := isolate(EcuaTL, laplace(y(t),t,s))$

$$SolTL := \mathcal{L}(y(t),t,s) = \frac{e^{-2\,s}}{s\,(s^2 - 2\,s + 1)} \quad (30)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \text{Heaviside}(t-2)\,\left(1 + e^{t-2}\,(t-3)\right) \quad (31)$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (32)$$

>

FIN RESPUESTA 4)

> $restart$

5) Resolver

> $Ecua := diff(y(t),t) - y(t) = (t-1)\cdot\text{Heaviside}(t-3)$

$$Ecua := \frac{d}{dt}\,y(t) - y(t) = (t-1)\,\text{Heaviside}(t-3) \quad (33)$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \quad (34)$$

> $with(inttrans)$:

> $EcuaTL := subs(CondIni, laplace(Ecua,t,s))$

$$EcuaTL := s\,\mathcal{L}(y(t),t,s) - \mathcal{L}(y(t),t,s) = \frac{e^{-3\,s}\,(2\,s+1)}{s^2} \quad (35)$$

> $SolTL := isolate(EcuaTL, laplace(y(t),t,s))$

$$SolTL := \mathcal{L}(y(t),t,s) = \frac{e^{-3\,s}\,(2\,s+1)}{s^2\,(s-1)} \quad (36)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = -\text{Heaviside}(t-3)\,\left(-3\,e^{t-3} + t\right) \quad (37)$$

> $ComprobarUno := simplify(subs(t=0, SolPart))$

$$ComprobarUno := y(0) = 0 \quad (38)$$

> $ComprobarDos := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \quad (39)$$

>

FIN RESPUESTA 5)

> $restart$

6) Obtener

> $F := \frac{s}{(2\cdot s^2 - 4\cdot s + 20)}$

$$F := \frac{s}{2\,s^2 - 4\,s + 20} \quad (40)$$

```
> with(inttrans) :
> f := invlaplace(F, s, t)
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$$f := \frac{e^{t}\,(3\cos(3\,t) + \sin(3\,t))}{6} \tag{41}$$

```
> G := 4/(exp(2·s)·(s − 3))
```

$$G := \frac{4}{e^{2\,s}\,(s-3)} \tag{42}$$

```
> g := invlaplace(G, s, t)
```

$$g := 4\,\mathrm{Heaviside}(t-2)\,e^{3\,t-6} \tag{43}$$

```
>
FIN RESPUESTA 6)
> restart
7) Resolver
> Ecua := h(t) = t·exp(t) + int(v·h(t − v), v = 0..t)
```

$$Ecua := h(t) = t\,e^{t} + \int_{0}^{t} v\,h(t-v)\,dv \tag{44}$$

```
> with(inttrans) :
> EcuaTL := laplace(Ecua, t, s)
```

$$EcuaTL := \mathcal{L}(h(t), t, s) = \frac{1}{(s-1)^{2}} + \frac{\mathcal{L}(h(t), t, s)}{s^{2}} \tag{45}$$

```
> SolTL := simplify(isolate(EcuaTL, laplace(h(t), t, s)))
```

$$SolTL := \mathcal{L}(h(t), t, s) = \frac{s^{2}}{(s-1)^{3}\,(s+1)} \tag{46}$$

```
> SolPart := invlaplace(SolTL, s, t)
```

$$SolPart := h(t) = -\frac{e^{-t}}{8} + \frac{e^{t}\,(2\,t^{2} + 6\,t + 1)}{8} \tag{47}$$

```
>
FIN RESPUESTA 7)
> restart
8) Obtenga
> with(inttrans) :
> f := t·Dirac(t − 1)· Heaviside(t − 3)
```

$$f := t\,\mathrm{Dirac}(t-1)\,\mathrm{Heaviside}(t-3) \tag{48}$$

```
> F := laplace(f, t, s)
```

$$F := 0 \tag{49}$$

```
> H := s·exp(−3·s)/(s^2 + 4·s + 5)
```

$$H := \frac{s\,e^{-3\,s}}{s^{2} + 4\,s + 5} \tag{50}$$

> $h := invlaplace(H, s, t)$

$$h := \text{Heaviside}(t-3)\, e^{-2t+6}\, (\cos(t-3) - 2\sin(t-3)) \quad \textbf{(51)}$$

>

FIN RESPUESTA 8)

> *restart*

9)

> $f := \cos(2\cdot t)\cdot \text{Heaviside}(t - \text{Pi})$

$$f := \cos(2t)\, \text{Heaviside}(t-\pi) \quad \textbf{(52)}$$

> *with*(*inttrans*) :

> $F := laplace(f, t, s)$

$$F := \frac{e^{-s\pi}\, s}{s^2+4} \quad \textbf{(53)}$$

>