

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEMESTRE 2013-2
SEGUNDO EXAMEN PARCIAL

ABRIL 22 DE 2013

> restart

1) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left(\frac{d}{dx} y(x) \right) = 0 \quad (1)$$

RESPUESTA 1)

> Ecuacion := $x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left(\frac{d}{dx} y(x) \right) = 0$

$$Ecuacion := x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left(\frac{d}{dx} y(x) \right) = 0 \quad (2)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

[_Bernoulli] (3)

> FactInt := intfactor(Ecuacion)

$$FactInt := \frac{1}{x^4} \quad (4)$$

> M := $x^4 \ln(x) - 2xy^3$; N := $3x^2y^2$;

$$M := x^4 \ln(x) - 2xy^3$$
$$N := 3y^2x^2 \quad (5)$$

> comprobacion₂ := simplify(diff(M, y) - diff(N, x)) = 0

$$comprobacion_2 := -12y^2x = 0 \quad (6)$$

> MM := expand(FactInt·M); NN := simplify(FactInt·N);

$$MM := \ln(x) - \frac{2y^3}{x^3}$$
$$NN := \frac{3y^2}{x^2} \quad (7)$$

> comprobacion₃ := simplify(diff(MM, y) - diff(NN, x)) = 0

$$comprobacion_3 := 0 = 0 \quad (8)$$

> IntNNy := int(NN, y)

$$IntNNy := \frac{y^3}{x^2} \quad (9)$$

> SolucionGeneral := IntNNy + int((MM - diff(IntNNy, x)), x) = C₁

$$SolucionGeneral := \frac{y^3}{x^2} + x \ln(x) - x = C_1 \quad (10)$$

FIN RESPUESTA 1)

> restart

2) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$y(x) (y(x)^2 + 2x^2) - 2x (x^2 + y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (11)$$

RESPUESTA 2)

> Ecuacion := y(x) (y(x)^2 + 2x^2) - 2x (x^2 + y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0

$$Ecuacion := y(x) (y(x)^2 + 2x^2) - 2x (x^2 + y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (12)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

[[_homogeneous, class A], _rational, _dAlembert] (13)

> EcuacionSeparable := factor(isolate(simplify(eval(subs(y(x) = x*u(x), Ecuacion))), diff(u(x), x)))

$$EcuacionSeparable := \frac{d}{dx} u(x) = -\frac{1}{2} \frac{u(x)^3}{x(1+u(x)^2)} \quad (14)$$

> EcuacionSeparada := lhs(EcuacionSeparable) - rhs(EcuacionSeparable) = 0

$$EcuacionSeparada := \frac{d}{dx} u(x) + \frac{1}{2} \frac{u(x)^3}{x(1+u(x)^2)} = 0 \quad (15)$$

> odeadvisor(EcuacionSeparada)

[_separable] (16)

> M := \frac{1}{2} \frac{u^3}{x(1+u^2)}; N := 1;

$$M := \frac{1}{2} \frac{u^3}{x(1+u^2)} \\ N := 1 \quad (17)$$

> P := \frac{1}{x}; Q := \frac{1}{2} \frac{u^3}{(1+u^2)}; R := 1; S := 1;

$$P := \frac{1}{x} \\ Q := \frac{1}{2} \frac{u^3}{1+u^2} \\ R := 1 \\ S := 1 \quad (18)$$

> Solucion := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = C_1

$$Solucion := \ln(x) + 2 \ln(u) - \frac{1}{u^2} = C_1 \quad (19)$$

> SolucionDos := expand\left(subs\left(u = \frac{y}{x}, Solucion\right)\right)

$$\text{SolucionDos} := \ln(x) + 2 \ln\left(\frac{y}{x}\right) - \frac{x^2}{y^2} = C_1 \quad (20)$$

$$> \text{SolucionTres} := \text{lhs}(\text{SolucionDos}) + \frac{x^2}{y^2} = \text{rhs}(\text{SolucionDos}) + \frac{x^2}{y^2}$$

$$\text{SolucionTres} := \ln(x) + 2 \ln\left(\frac{y}{x}\right) = C_1 + \frac{x^2}{y^2} \quad (21)$$

$$> \text{SolucionCuatro} := \text{expand}(\exp(\text{lhs}(\text{SolucionTres}))) = \text{expand}(\exp(\text{rhs}(\text{SolucionTres})))$$

$$\text{SolucionCuatro} := \frac{y^2}{x} = e^{C_1} e^{\frac{x^2}{y^2}} \quad (22)$$

$$> \text{SolucionCinco} := \text{isolate}(\text{SolucionCuatro}, \exp(C_1))$$

$$\text{SolucionCinco} := e^{C_1} = \frac{y^2}{x e^{\frac{x^2}{y^2}}} \quad (23)$$

$$> \text{SolucionGeneral} := \frac{1}{\text{rhs}(\text{SolucionCinco})} = C_1$$

$$\text{SolucionGeneral} := \frac{x e^{\frac{x^2}{y^2}}}{y^2} = C_1 \quad (24)$$

FIN RESPUESTA 2)

> restart

3) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0 \quad (25)$$

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RESPUESTA 3)

$$> \text{Ecuacion} := 1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0$$

$$\text{Ecuacion} := 1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0 \quad (26)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

[_rational, [_1st_order, _with_symmetry_[F(x),G(x)]], [_Abel, 2nd type, class B]] (27)

> FactInt := infactor(Ecuacion)

$$\text{FactInt} := \frac{1}{x^2} \quad (28)$$

> M := 1 - x^2 · y; N := x^2 (y - x)

$$M := 1 - x^2 y$$

$$N := x^2 (y - x) \quad (29)$$

> MM := expand(FactInt · M); NN := expand(FactInt · N)

$$MM := \frac{1}{x^2} - y$$

$$NN := y - x \quad (30)$$

$$> \text{comprobacion} := \text{simplify}(\text{diff}(MM, y) - \text{diff}(NN, x)) = 0$$

$$\text{comprobacion} := 0 = 0 \quad (31)$$

$$> \text{IntNNy} := \text{int}(NN, y)$$

$$\text{IntNNy} := \frac{1}{2} y^2 - x y \quad (32)$$

$$> \text{SolucionGeneral} := \text{IntNNy} + \text{int}((MM - \text{diff}(\text{IntNNy}, x)), x) = C_1$$

$$\text{SolucionGeneral} := \frac{1}{2} y^2 - x y - \frac{1}{x} = C_1 \quad (33)$$

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FIN RESPUESTA 3)

> restart

4) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$x y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2 x y(x) + x^2 + 2 y(x) - 2 x + 2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (34)$$

RESPUESTA 4)

$$> \text{Ecuacion} := x y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2 x y(x) + x^2 + 2 y(x) - 2 x + 2) \left(\frac{d}{dx} y(x) \right) = 0$$

$$\text{Ecuacion} := x y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2 x y(x) + x^2 + 2 y(x) - 2 x + 2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (35)$$

> with(DEtools) :

$$> \text{odeadvisor}(\text{Ecuacion})$$

$$[_{\text{separable}}] \quad (36)$$

$$> M := \text{factor}(x y^2 - y^2 + x - 1); N := \text{factor}(x^2 y - 2 x y + x^2 + 2 y - 2 x + 2)$$

$$M := (y^2 + 1) (x - 1)$$

$$N := (x^2 - 2 x + 2) (1 + y) \quad (37)$$

$$> P := x - 1; Q := y \cdot 2 + 1; R := x \cdot 2 - 2 \cdot x + 2; S := y + 1;$$

$$P := x - 1$$

$$Q := y^2 + 1$$

$$R := x^2 - 2 x + 2$$

$$S := 1 + y \quad (38)$$

$$> \text{SolucionGeneral} := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, y\right) = C_1$$

$$\text{SolucionGeneral} := \frac{1}{2} \ln(x^2 - 2 x + 2) + \frac{1}{2} \ln(y^2 + 1) + \arctan(y) = C_1 \quad (39)$$

$$> \text{Solucion2} := \text{lhs}(\text{SolucionGeneral}) \cdot 2 = C_1$$

$$\text{Solucion2} := \ln(x^2 - 2 x + 2) + \ln(y^2 + 1) + 2 \arctan(y) = C_1 \quad (40)$$

$$\begin{aligned} > \text{Solucion3} := \text{simplify}(\exp(\text{lhs}(\text{Solucion2}))) = C_1 \\ \text{Solucion3} := (x^2 - 2x + 2)(y^2 + 1)e^{2\arctan(y)} = C_1 \end{aligned} \quad (41)$$

RESPUESTA ALTERNA 4)

$$\begin{aligned} > \text{intfactor}(\text{Ecuacion}) \\ \frac{1}{e^{-2\arctan(y(x))}} \end{aligned} \quad (42)$$

$$\begin{aligned} > \text{FactInt} := \frac{1}{e^{-2\arctan(y)}} \\ \text{FactInt} := \frac{1}{e^{-2\arctan(y)}} \end{aligned} \quad (43)$$

$$\begin{aligned} > M := x^2y^2 - y^2 + x - 1; N := x^2y - 2xy + x^2 + 2y - 2x + 2 \\ M := x^2y^2 - y^2 + x - 1 \\ N := x^2y - 2xy + x^2 + 2y - 2x + 2 \end{aligned} \quad (44)$$

$$\begin{aligned} > MM := \text{simplify}(\text{FactInt} \cdot M); NN := \text{simplify}(\text{FactInt} \cdot N); \\ MM := e^{2\arctan(y)}(x^2y^2 - y^2 + x - 1) \\ NN := e^{2\arctan(y)}(x^2y - 2xy + x^2 + 2y - 2x + 2) \end{aligned} \quad (45)$$

$$\begin{aligned} > \text{comprobacion1} := \text{simplify}(\text{diff}(MM, y) - \text{diff}(NN, x)) = 0 \\ \text{comprobacion1} := 0 = 0 \end{aligned} \quad (46)$$

$$\begin{aligned} > \text{IntMMx} := \text{int}(MM, x) \\ \text{IntMMx} := e^{2\arctan(y)} \left(\frac{1}{2} x^2 y^2 - x y^2 + \frac{1}{2} x^2 - x \right) \end{aligned} \quad (47)$$

$$\begin{aligned} > \text{SolucionAlterna} := \text{simplify}(\text{IntMMx} + \text{int}((NN - \text{diff}(\text{IntMMx}, y)), y)) = C_1 \\ \text{SolucionAlterna} := \frac{1}{2} e^{2\arctan(y)}(x^2y^2 - 2xy^2 + x^2 - 2x + 2y^2 + 2) = C_1 \end{aligned} \quad (48)$$

$$\begin{aligned} > \text{SolucionFinal} := \text{factor}(\text{lhs}(\text{SolucionAlterna}) \cdot 2) = C_1 \\ \text{SolucionFinal} := (x^2 - 2x + 2)(y^2 + 1)e^{2\arctan(y)} = C_1 \end{aligned} \quad (49)$$

FIN RESPUESTA 4)

> restart

5) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$\sin(xy(x)) + xy(x)\cos(xy(x)) + x^2\cos(xy(x)) \left(\frac{d}{dx} y(x) \right) = 0 \quad (50)$$

RESPUESTA 5)

$$\begin{aligned} > \text{Ecuacion} := \sin(xy(x)) + xy(x)\cos(xy(x)) + x^2\cos(xy(x)) \left(\frac{d}{dx} y(x) \right) = 0 \\ \text{Ecuacion} := \sin(xy(x)) + xy(x)\cos(xy(x)) + x^2\cos(xy(x)) \left(\frac{d}{dx} y(x) \right) = 0 \end{aligned} \quad (51)$$

> with(DEtools) :

$$\begin{aligned} > \text{odeadvisor}(\text{Ecuacion}) \\ [[_homogeneous, class G], _exact] \end{aligned} \quad (52)$$

$$> M := \sin(xy) + xy\cos(xy); N := x^2\cos(xy);$$

$$M := \sin(x y) + x y \cos(x y)$$

$$N := x^2 \cos(x y) \quad (53)$$

$$\text{> } comprobacion_1 := \text{simplify}(\text{diff}(M, y) - \text{diff}(N, x)) = 0$$

$$comprobacion_1 := 0 = 0 \quad (54)$$

$$\text{> } IntMx := \text{simplify}(\text{int}(M, x))$$

$$IntMx := \sin(x y) x \quad (55)$$

$$\text{> } SolucionGeneral := IntMx + \text{int}((N - \text{diff}(IntMx, y)), y) = C_1$$

$$SolucionGeneral := \sin(x y) x = C_1 \quad (56)$$

FIN RESPUESTA 5)

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> restart

FIN DEL EXAMEN