SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL (TEMAS 4 Y 5)
SEMESTRE 2013-2

2013 MAYO 24

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1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE **(sin usar dsolve)**:

a) **(15/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS

b) **(15/100 puntos)** GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA; PARA UN INTERVALO DE 0 < t < 3

$$\frac{d^2}{dt^2} y(t) + 4\, y(t) = 64\, (t-1)\, \mathrm{Heaviside}(t-1) \sin(2t-2)$$

$$y(0) = 2$$

$$D(y)(0) = 0 \qquad (1)$$

RESPUESTA 1a)

> $Ecuacion := \frac{d^2}{dt^2} y(t) + 4\, y(t) = 64\, (t-1)\, \mathrm{Heaviside}(t-1) \sin(2t-2);$

$$Ecuacion := \frac{d^2}{dt^2} y(t) + 4\, y(t) = 64\, (t-1)\, \mathrm{Heaviside}(t-1) \sin(2t-2) \qquad (2)$$

> $Condiciones := y(0) = 2, D(y)(0) = 0$

$$Condiciones := y(0) = 2, D(y)(0) = 0 \qquad (3)$$

> $with(inttrans):$

> $TransLapEcua := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcua := s^2\, laplace(y(t), t, s) - 2\, s + 4\, laplace(y(t), t, s) = \frac{256\, e^{-s}\, s}{(s^2+4)^2} \qquad (4)$$

> $TransLapSolucion := simplify(isolate(TransLapEcua, laplace(y(t), t, s)))$

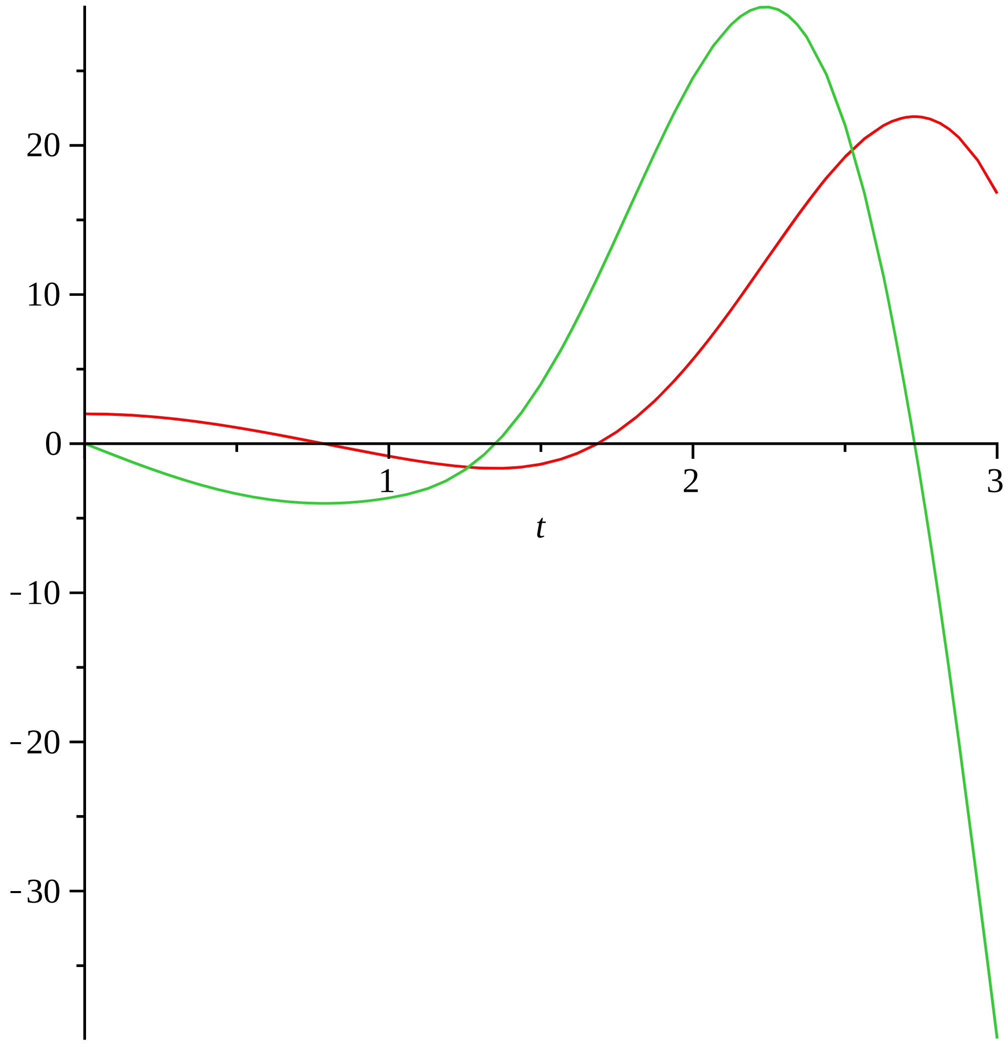
$$TransLapSolucion := laplace(y(t), t, s) = \frac{2\, s\, (128\, e^{-s} + s^4 + 8\, s^2 + 16)}{(s^2+4)^3} \qquad (5)$$

> $SolucionParticular := invlaplace(TransLapSolucion, s, t)$

$$SolucionParticular := y(t) = 2\cos(2t) + 4\, (t-1)\, (\sin(2t-2) - 2\cos(2t-2)\, (t-1))\, \mathrm{Heaviside}(t-1) \qquad (6)$$

RESPUESTA 1b)

> $plot([rhs(SolucionParticular), rhs(diff(SolucionParticular, t))], t = 0..3)$



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FIN PREGUNTA 1)

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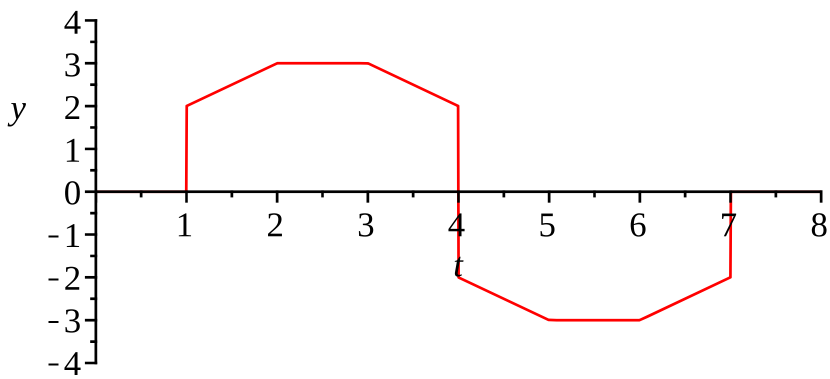
2) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:



a) **(15/100 puntos)** OBTENER SU TRANSFORMADA DE LAPLACE.
b) **(25/100 puntos)** OBTENER Y GRAFICAR SU SERIE COSENO DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

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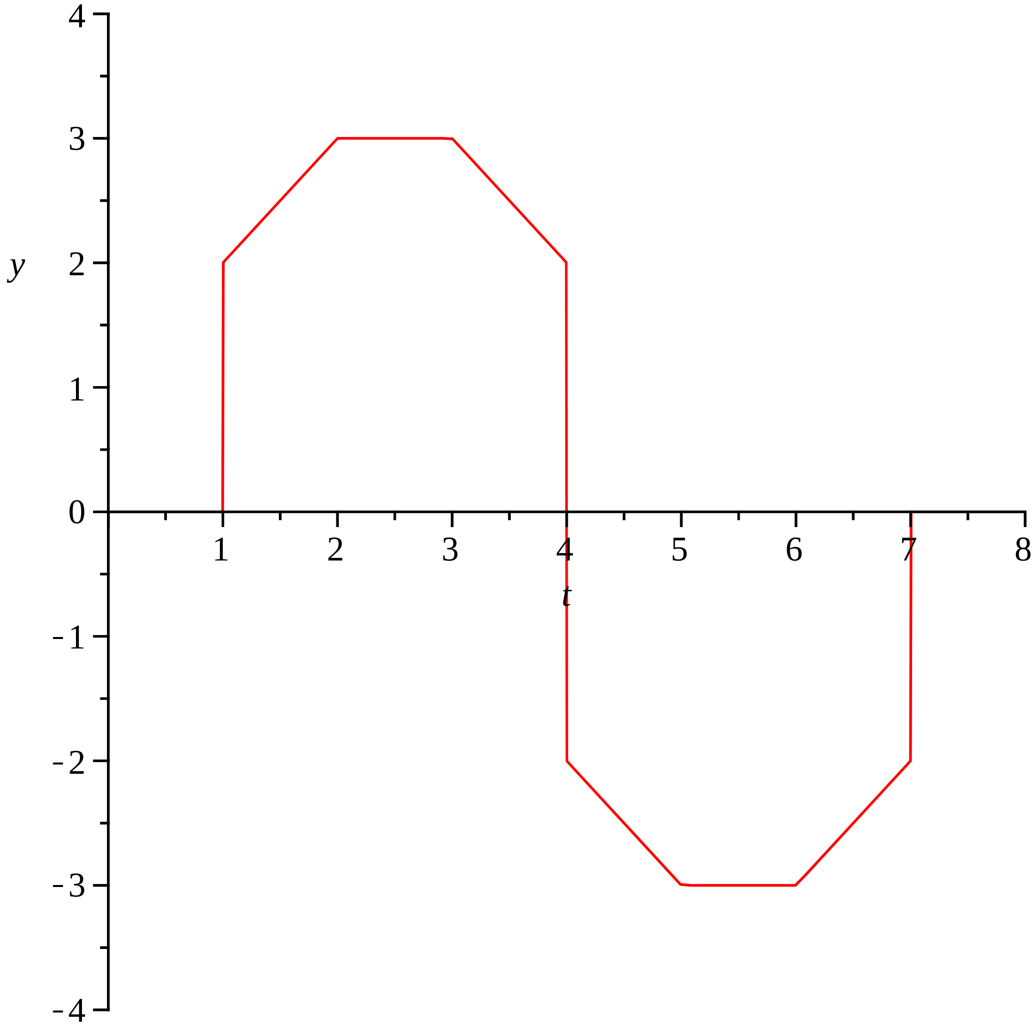
RESPUESTA 2a)

> $f := 2\cdot \text{Heaviside}(t-1) + (t-1)\cdot\text{Heaviside}(t-1) - (t-2)\cdot\text{Heaviside}(t-2) - (t-3)\cdot\text{Heaviside}(t-3) + (t-4)\cdot\text{Heaviside}(t-4) - 4\cdot\text{Heaviside}(t-4) - (t-4)\cdot\text{Heaviside}(t-4) + (t-5)\cdot\text{Heaviside}(t-5) + (t-6)\cdot\text{Heaviside}(t-6) - (t-7)\cdot\text{Heaviside}(t-7) + 2\cdot\text{Heaviside}(t-7) : plot(f, t=0..8, y=-4..4)$



> $with(inttrans):$

> $F := laplace(f, t, s)$

$$F := \frac{e^{-s} - e^{-7s} + e^{-6s} + e^{-5s} - e^{-3s} - e^{-2s}}{s^2} + \frac{2\left(e^{-s} + e^{-7s} - 2\,e^{-4s}\right)}{s} \quad \mathbf{(7)}$$

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RESPUESTA 2b)

> $L := 4;\ a_0 := \left(\frac{1}{L}\right)\cdot int(f, t=0..2\cdot L);\ C := \frac{a_0}{2}$

$$L := 4$$
$$a_0 := 0$$
$$C := 0 \quad \mathbf{(8)}$$

> $a_n := simplify\left(\left(\frac{1}{L}\right)\cdot int\left(f\cdot\cos\left(\frac{n\cdot \text{Pi}\cdot t}{L}\right), t=0..2\cdot L\right)\right)$

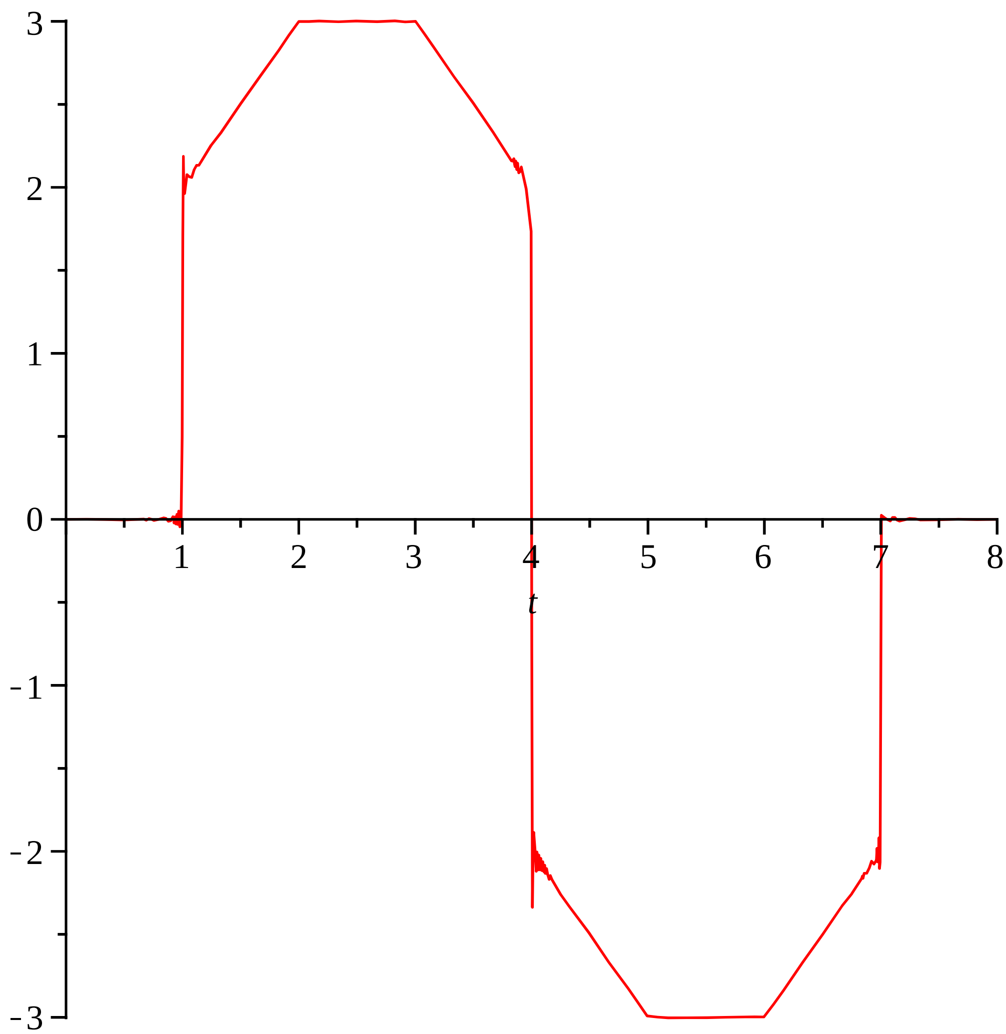
$$a_n := -\frac{1}{n^2\pi^2}\left(2\left(n\pi\sin\left(\frac{1}{4}n\pi\right)+2\cos\left(\frac{1}{4}n\pi\right)-2\cos\left(\frac{1}{2}n\pi\right)-2\cos\left(\frac{3}{4}n\pi\right)\right.\right.$$
$$-2\sin(n\pi)\,n\pi+2\cos\left(\frac{5}{4}n\pi\right)+2\cos\left(\frac{3}{2}n\pi\right)-2\cos\left(\frac{7}{4}n\pi\right)$$
$$\left.\left.+n\pi\sin\left(\frac{7}{4}n\pi\right)\right)\right) \tag{9}$$

> $b_n := simplify\left(\left(\frac{1}{L}\right)\cdot int\left(f\cdot\sin\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), t=0..2\cdot L\right)\right)$

$$b_n := \frac{1}{n^2\pi^2}\left(2\left(\cos\left(\frac{1}{4}n\pi\right)n\pi-2\sin\left(\frac{1}{4}n\pi\right)+2\sin\left(\frac{1}{2}n\pi\right)+2\sin\left(\frac{3}{4}n\pi\right)\right.\right.$$
$$-2\cos(n\pi)\,n\pi-2\sin\left(\frac{5}{4}n\pi\right)-2\sin\left(\frac{3}{2}n\pi\right)+2\sin\left(\frac{7}{4}n\pi\right)$$
$$\left.\left.+\cos\left(\frac{7}{4}n\pi\right)n\pi\right)\right) \tag{10}$$

> $STF_{500} := C+sum\left(a_n\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right)+b_n\cdot\sin\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), n=1..500\right):$

> $plot(STF_{500}, t=0..8)$



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FIN PREGUNTA 2)

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3) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA **CONSTANTE DE SEPARACIÓN NEGATIVA**:

$$\frac{\partial^2}{\partial t^2} y(x,t) + \frac{\partial}{\partial t} y(x,t) = x^2 \left(\frac{\partial}{\partial x} y(x,t) \right) \tag{11}$$

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RESPUESTA 3)

> $Ecuacion := \frac{\partial^2}{\partial t^2} y(x,t) + \frac{\partial}{\partial t} y(x,t) = x^2 \left(\frac{\partial}{\partial x} y(x,t) \right)$

$$Ecuacion := \frac{\partial^2}{\partial t^2} y(x,t) + \frac{\partial}{\partial t} y(x,t) = x^2 \left(\frac{\partial}{\partial x} y(x,t) \right) \tag{12}$$

> $EcuacionUno := eval(subs(y(x,t) = F(x) \cdot G(t), Ecuacion))$

$$EcuacionUno := F(x) \left(\frac{d^2}{dt^2} G(t) \right) + F(x) \left(\frac{d}{dt} G(t) \right) = x^2 \left(\frac{d}{dx} F(x) \right) G(t) \tag{13}$$

> $EcuacionDos := simplify\left(\frac{lhs(EcuacionUno)}{F(x) \cdot G(t)} \right) = simplify\left(\frac{rhs(EcuacionUno)}{F(x) \cdot G(t)} \right)$

$$EcuacionDos := \frac{\frac{d^2}{dt^2} G(t) + \frac{d}{dt} G(t)}{G(t)} = \frac{x^2 \left(\frac{d}{dx} F(x) \right)}{F(x)} \tag{14}$$

> $EcuacionT := lhs(EcuacionDos) = -\text{beta} \cdot{} \cdot 2; EcuacionX := rhs(EcuacionDos) = -\text{beta} \cdot{} \cdot 2$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t) + \frac{d}{dt} G(t)}{G(t)} = -\beta^2$$

$$EcuacionX := \frac{x^2 \left(\frac{d}{dx} F(x) \right)}{F(x)} = -\beta^2 \tag{15}$$

> $SolucionT := dsolve(EcuacionT); SolucionX := dsolve(EcuacionX)$

$$SolucionT := G(t) = \_C1\, e^{\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1-4\beta^2} \right) t} + \_C2\, e^{\left(-\frac{1}{2} - \frac{1}{2}\sqrt{1-4\beta^2} \right) t}$$

$$SolucionX := F(x) = \_C1\, e^{\frac{\beta^2}{x}} \tag{16}$$

> $SolucionGeneral := y(x,t) = subs(\_C1 = 1, rhs(SolucionX)) \cdot rhs(SolucionT)$

$$SolucionGeneral := y(x,t) = e^{\frac{\beta^2}{x}} \left(\_C1\, e^{\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1-4\beta^2} \right) t} + \_C2\, e^{\left(-\frac{1}{2} - \frac{1}{2}\sqrt{1-4\beta^2} \right) t} \right) \tag{17}$$

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FIN PREGUNTA 3)

> *restart*

FIN DEL EXAMEN

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