>

solución

Ecuaciones Diferenciales
Primer Examen Final
Semestre 2013-2

2013 Mayo 27

> *restart*

1) Resuelva la ecuación diferencial

> $y' = \frac{2\cdot x\cdot y}{x\cdot\cdot 2 - y\cdot\cdot 2}$

$$\frac{d}{dx}\, y(x) = \frac{2\,x\,y(x)}{x^2 - y(x)^2} \quad \textbf{(1)}$$

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RESPUESTA 1)

> $Ecuacion := \frac{d}{dx}\, y(x) = \frac{2\,x\,y(x)}{x^2 - y(x)^2}$

$$Ecuacion := \frac{d}{dx}\, y(x) = \frac{2\,x\,y(x)}{x^2 - y(x)^2} \quad \textbf{(2)}$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_dAlembert] \quad \textbf{(3)}$$

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Coeficientes Homogéneos

> $EcuacionDos := isolate(simplify(eval(subs(y(x) = x\cdot u(x), Ecuacion))), diff(u(x), x))$

$$EcuacionDos := \frac{d}{dx}\, u(x) = \frac{-\frac{2\,u(x)}{-1 + u(x)^2} - u(x)}{x} \quad \textbf{(4)}$$

> $P := x;\ Q := simplify\left(-\left(-\frac{2\,u}{-1+u^2} - u\right)\right)$

$$P := x$$

$$Q := \frac{u\,(1+u^2)}{-1+u^2} \quad \textbf{(5)}$$

> $SolucionDos := simplify\left(int\left(\frac{1}{P}, x\right) + int\left(\frac{1}{Q}, u\right)\right) = C_1$

$$SolucionDos := \ln(x) - \ln(u) + \ln(1+u^2) = C_1 \quad \textbf{(6)}$$

> $SolucionTres := simplify\left(subs\left(u = \frac{y}{x}, SolucionDos\right)\right)$

$$SolucionTres := \ln(x) - \ln\left(\frac{y}{x}\right) + \ln\left(\frac{x^2+y^2}{x^2}\right) = C_1 \quad \textbf{(7)}$$

> $SolucionGeneral := simplify(\exp(lhs(SolucionTres))) = C_1$

$$SolucionGeneral := \frac{x^2+y^2}{y} = C_1 \quad \textbf{(8)}$$

> $SolucionDerivable := \frac{x^2+y(x)^2}{y(x)} = C_1$

$$SolucionDerivable := \frac{x^2+y(x)^2}{y(x)} = C_1 \quad \textbf{(9)}$$

> $DerivadaSolucion := isolate(diff(SolucionDerivable, x), diff(y(x), x))$

$$DerivadaSolucion := \frac{d}{dx} y(x) = -\frac{2\, x\, y(x)}{y(x)^2 - x^2} \quad \textbf{(10)}$$

> $Ecuacion$

$$\frac{d}{dx} y(x) = \frac{2\, x\, y(x)}{x^2 - y(x)^2} \quad \textbf{(11)}$$

> $Comprobacion := simplify(rhs(Ecuacion) - rhs(DerivadaSolucion)) = 0$

$$Comprobacion := 0 = 0 \quad \textbf{(12)}$$

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FIN RESPUESTA 1)

> $restart$

2) Resuelva la ecuación diferencial

> $x\cdot\cdot 3\cdot y'' - 3\cdot x\cdot\cdot 2\cdot y' + 3\cdot x\cdot y = x\cdot\cdot 5 + 2\cdot x\cdot\cdot 3$

$$x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3\, x^2 \left(\frac{d}{dx} y(x)\right) + 3\, x\, y(x) = x^5 + 2\, x^3 \quad \textbf{(13)}$$

si un conjunto de soluciones de la ecuación homogénea asociada

> $x\cdot\cdot 3\cdot y'' - 3\cdot x\cdot\cdot 2\cdot y' + 3\cdot x\cdot y = 0;\ \{x, x\cdot\cdot 3, 2\cdot x\cdot\cdot 3 - x\}$

$$x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3\, x^2 \left(\frac{d}{dx} y(x)\right) + 3\, x\, y(x) = 0$$

$$\{x, x^3, 2\, x^3 - x\} \quad \textbf{(14)}$$

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RESPUESTA 2)

> $Ecuacion := x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3\, x^2 \left(\frac{d}{dx} y(x)\right) + 3\, x\, y(x) = x^5 + 2\, x^3$

$$Ecuacion := x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3\, x^2 \left(\frac{d}{dx} y(x)\right) + 3\, x\, y(x) = x^5 + 2\, x^3 \quad \textbf{(15)}$$

> $EcuacionNormal := expand\left(\frac{lhs(Ecuacion)}{x\cdot\cdot 3}\right) = simplify\left(\frac{rhs(Ecuacion)}{x\cdot\cdot 3}\right)$

$$EcuacionNormal := \frac{d^2}{dx^2} y(x) - \frac{3\left(\frac{d}{dx} y(x)\right)}{x} + \frac{3\, y(x)}{x^2} = x^2 + 2 \quad \textbf{(16)}$$

> $EcuacionHom := lhs(Ecuacion) = 0$

$$EcuacionHom := x^3 \left(\frac{d^2}{dx^2} y(x)\right) - 3\, x^2 \left(\frac{d}{dx} y(x)\right) + 3\, x\, y(x) = 0 \quad \textbf{(17)}$$

> $Q := rhs(EcuacionNormal)$

$$Q := x^2 + 2 \quad \textbf{(18)}$$

> $SolUno := y(x) = x;\ SolDos := y(x) = x\cdot\cdot 3$

$$SolUno := y(x) = x$$
$$SolDos := y(x) = x^3 \quad \mathbf{(19)}$$

> $SolucionHom := y(x) = C_1 \cdot rhs(SolUno) + C_2 \cdot \mathrm{rhs}(\mathrm{SolDos})$

$$SolucionHom := y(x) = C_1\, x + C_2\, x^3 \quad \mathbf{(20)}$$

> $Comprobacion_0 := simplify(eval(subs(y(x) = rhs(SolucionHom), EcuacionHom)))$

$$Comprobacion_0 := 0 = 0 \quad \mathbf{(21)}$$

> $SolucionNoHom := y(x) = A \cdot rhs(SolUno) + B \cdot rhs(SolDos)$

$$SolucionNoHom := y(x) = A\, x + B\, x^3 \quad \mathbf{(22)}$$

> $with(linalg):$

> $WW := wronskian([rhs(SolUno), rhs(SolDos)], x)$

$$WW := \begin{bmatrix} x & x^3 \\ 1 & 3\,x^2 \end{bmatrix} \quad \mathbf{(23)}$$

> $AA := array([0, Q])$

$$AA := \begin{bmatrix} 0 & x^2 + 2 \end{bmatrix} \quad \mathbf{(24)}$$

> $SOL := linsolve(WW, AA)$

$$SOL := \begin{bmatrix} -\frac{1}{2}\, x^2 - 1 & \frac{1}{2}\, \frac{x^2 + 2}{x^2} \end{bmatrix} \quad \mathbf{(25)}$$

> $Aprima := SOL_1; Bprima := SOL_2$

$$Aprima := -\frac{1}{2}\, x^2 - 1$$
$$Bprima := \frac{1}{2}\, \frac{x^2 + 2}{x^2} \quad \mathbf{(26)}$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2$

$$A := -\frac{1}{6}\, x^3 - x + C_1$$
$$B := \frac{1}{2}\, x - \frac{1}{x} + C_2 \quad \mathbf{(27)}$$

> $SolucionNoHomogenea := expand(SolucionNoHom)$

$$SolucionNoHomogenea := y(x) = \frac{1}{3}\, x^4 - 2\, x^2 + C_1\, x + C_2\, x^3 \quad \mathbf{(28)}$$

> $Ecuacion$

$$x^3 \left(\frac{d^2}{dx^2}\, y(x) \right) - 3\, x^2 \left(\frac{d}{dx}\, y(x) \right) + 3\, x\, y(x) = x^5 + 2\, x^3 \quad \mathbf{(29)}$$

> $Comprobacion_1 := simplify(eval(subs(y(x) = rhs(SolucionNoHomogenea), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_1 := 0 = 0 \quad \mathbf{(30)}$$

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FIN RESPUESTA 2)

> restart

3) Resuelva el sistema de ecuaciones diferenciales

> $diff(x(t),t) - 5\cdot x(t) + 2\cdot y(t) = 0;\ diff(y(t),t) - 4\cdot x(t) + y(t) = 0$

$$\frac{d}{dt} x(t) - 5\,x(t) + 2\,y(t) = 0$$
$$\frac{d}{dt} y(t) - 4\,x(t) + y(t) = 0 \quad \textbf{(31)}$$

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RESPUESTA 3)

> $Sistema := \frac{d}{dt} x(t) = 5\,x(t) - 2\,y(t),\ \frac{d}{dt} y(t) = 4\,x(t) - y(t) : Sistema_1;\ Sistema_2;$

$$\frac{d}{dt} x(t) = 5\,x(t) - 2\,y(t)$$
$$\frac{d}{dt} y(t) = 4\,x(t) - y(t) \quad \textbf{(32)}$$

> $AA := array([[5,-2],[4,-1]])$

$$AA := \begin{bmatrix} 5 & -2 \\ 4 & -1 \end{bmatrix} \quad \textbf{(33)}$$

> $with(linalg):$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} -e^{t} + 2\,e^{3t} & -e^{3t} + e^{t} \\ 2\,e^{3t} - 2\,e^{t} & 2\,e^{t} - e^{3t} \end{bmatrix} \quad \textbf{(34)}$$

> $Xcero := array([C_1, C_2])$

$$Xcero := \begin{bmatrix} C_1 & C_2 \end{bmatrix} \quad \textbf{(35)}$$

> $Solucion := evalm(MatExp\ \&*\ Xcero) : SolucionGeneralUno := x(t) = simplify(Solucion_1);$
> $SolucionGeneralDos := y(t) = simplify(Solucion_2)$

$$SolucionGeneralUno := x(t) = -C_1\,e^{t} + 2\,C_1\,e^{3t} - C_2\,e^{3t} + C_2\,e^{t}$$
$$SolucionGeneralDos := y(t) = 2\,C_1\,e^{3t} - 2\,C_1\,e^{t} + 2\,C_2\,e^{t} - C_2\,e^{3t} \quad \textbf{(36)}$$

> $Comprobacion_1 := simplify(eval(subs(x(t) = rhs(SolucionGeneralUno), y(t)$
> $= rhs(SolucionGeneralDos), lhs(Sistema_1) - rhs(Sistema_1) = 0)))$

$$Comprobacion_1 := 0 = 0 \quad \textbf{(37)}$$

> $Comprobacion_2 := simplify(eval(subs(x(t) = rhs(SolucionGeneralUno), y(t)$
> $= rhs(SolucionGeneralDos), lhs(Sistema_2) - rhs(Sistema_2) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad \textbf{(38)}$$

> $SolucionGeneralDiez := x(t) = simplify(subs(C_1 = C_{10} + C_{20}, C_2 = C_{10} + 2\cdot C_{20},$
> $rhs(SolucionGeneralUno)))$

$$SolucionGeneralDiez := x(t) = e^{t}\,C_{20} + e^{3t}\,C_{10} \quad \textbf{(39)}$$

> $SolucionGeneralVeinte := y(t) = simplify\left(subs\left(C_1 = C_{10} + C_{20}, C_2 = C_{10} + 2 \cdot C_{20}, rhs(SolucionGeneralDos)\right)\right)$

$$SolucionGeneralVeinte := y(t) = e^{3t} C_{10} + 2\, e^{t} C_{20} \quad \textbf{(40)}$$

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FIN RESPUESTA 3)

> *restart*

4) Aplique Transformada de Laplace para resolver el problema del valor inicial

> $diff(x(t), t\$2) + 9 \cdot x(t) = \sin(2 \cdot t); x(0) = 0; D(x)(0) = 0$

$$\frac{d^2}{dt^2}\, x(t) + 9\, x(t) = \sin(2\, t)$$

$$x(0) = 0$$

$$D(x)(0) = 0 \quad \textbf{(41)}$$

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RESPUESTA 4)

> $Ecuacion := \frac{d^2}{dt^2}\, x(t) + 9\, x(t) = \sin(2\, t);$

$$Ecuacion := \frac{d^2}{dt^2}\, x(t) + 9\, x(t) = \sin(2\, t) \quad \textbf{(42)}$$

> $Condiciones := x(0) = 0, D(x)(0) = 0$

$$Condiciones := x(0) = 0, D(x)(0) = 0 \quad \textbf{(43)}$$

> $with(inttrans):$

> $TransLapEcua := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcua := s^2\, laplace(x(t), t, s) + 9\, laplace(x(t), t, s) = \frac{2}{s^2 + 4} \quad \textbf{(44)}$$

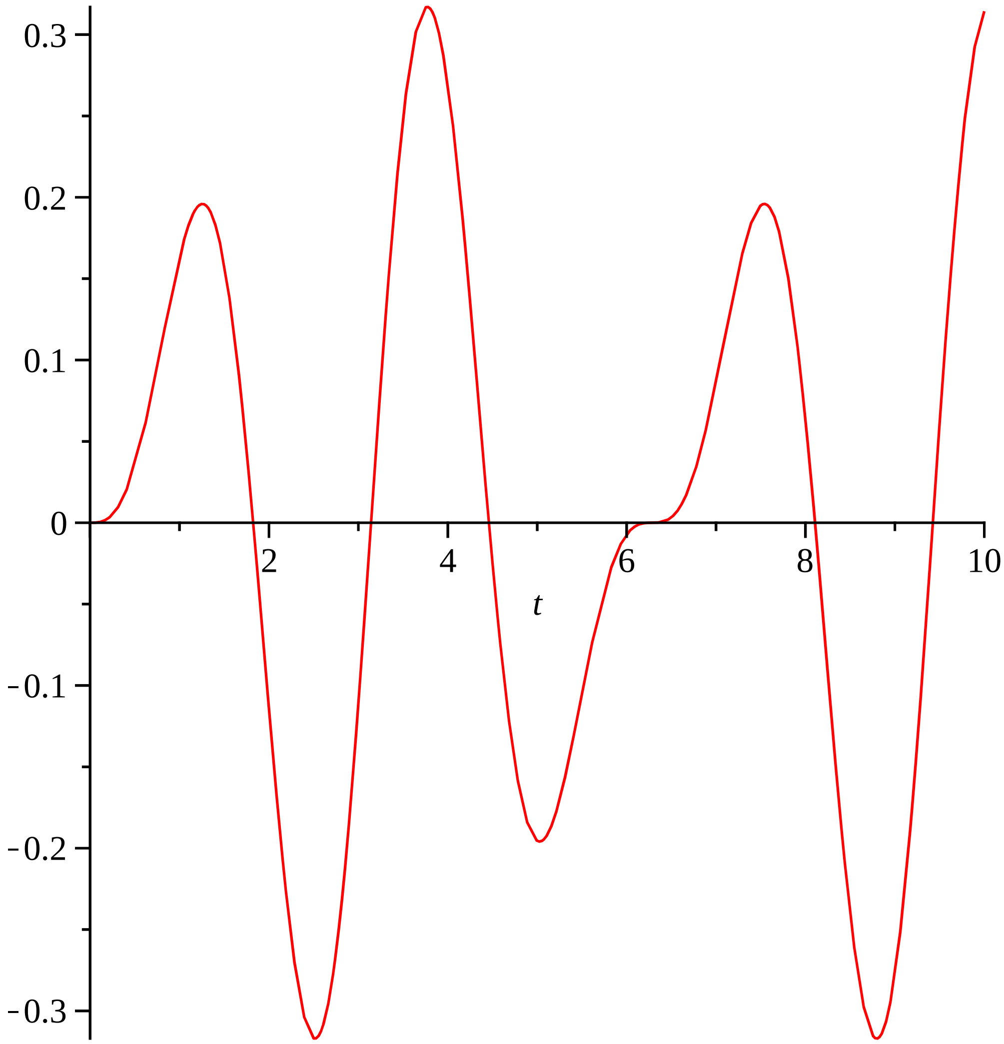
> $TransLapSol := isolate(TransLapEcua, laplace(x(t), t, s))$

$$TransLapSol := laplace(x(t), t, s) = \frac{2}{(s^2 + 4)\,(s^2 + 9)} \quad \textbf{(45)}$$

> $SolucionParticular := simplify(invlaplace(TransLapSol, s, t))$

$$SolucionParticular := x(t) = \frac{1}{5} \sin(2\, t) - \frac{2}{15} \sin(3\, t) \quad \textbf{(46)}$$

> $plot(rhs(SolucionParticular), t = 0 ..10)$



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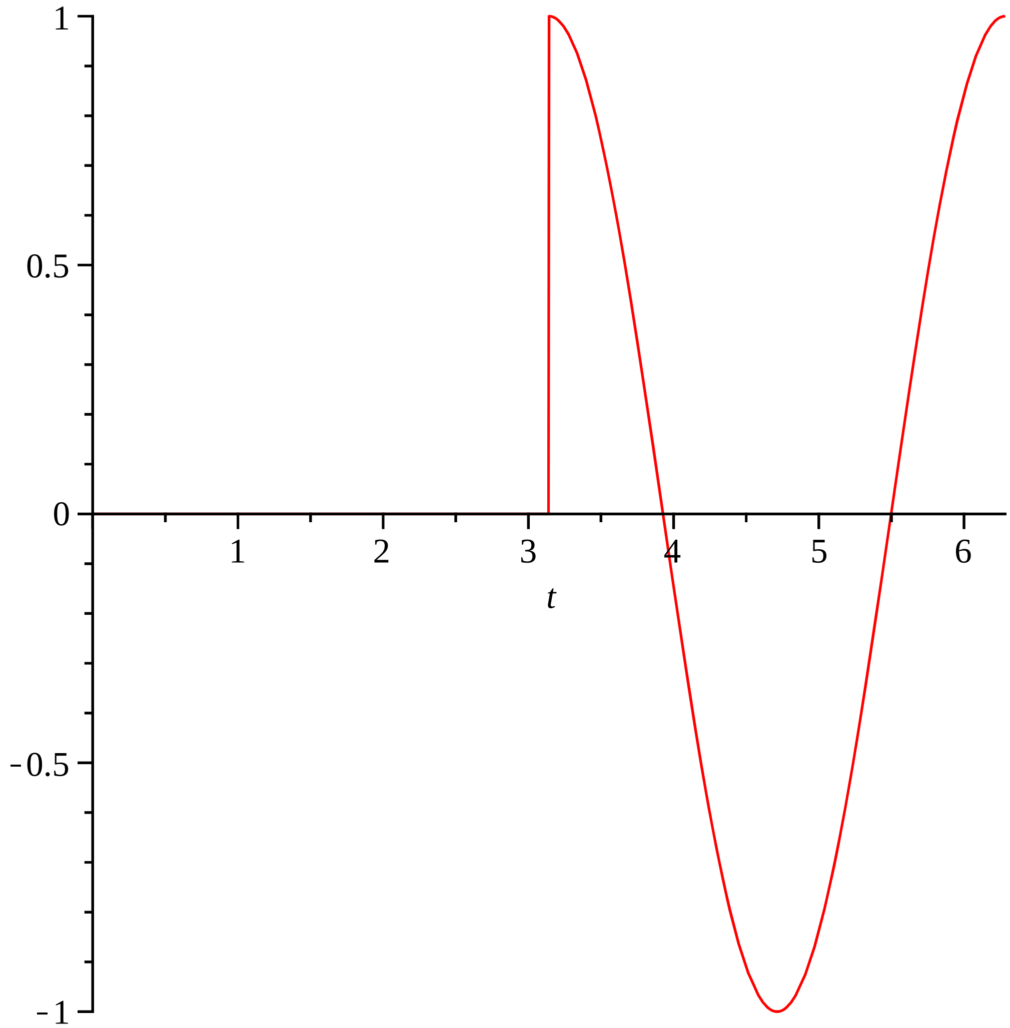
FIN RESPUESTA 4)

\> *restart*

5) calcule la Transformada de Laplace de la función graficada

\> $f := \text{Heaviside}(t - \text{Pi}) \cdot \cos(2 \cdot t) : plot(f, t = 0..2 \cdot \text{Pi})$



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RESPUESTA 5)

> *eval*(*f*);

$$\mathrm{Heaviside}(t-\pi)\cos(2\,t) \qquad \textbf{(47)}$$

> *with*(*inttrans*) :

> *F* := *laplace*(*f*, *t*, *s*)

$$F := \frac{e^{-s\,\pi}\,s}{s^2+4} \qquad \textbf{(48)}$$

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FIN RESPUESTA 5)

> *restart*

6) Resuelva la ecuación diferencial en derivadas parciales

> $diff\,(u(x,t),x) = \frac{5}{x}\cdot diff\,(u(x,t),t)$

$$\frac{\partial}{\partial x}\,u(x,t) = \frac{5\left(\frac{\partial}{\partial t}\,u(x,t)\right)}{x} \qquad \textbf{(49)}$$

suponga una constante de separación igual a 3

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RESPUESTA 6)

> $Ecuacion := \frac{\partial}{\partial x} u(x,t) = \frac{5 \left(\frac{\partial}{\partial t} u(x,t)\right)}{x}$

$$Ecuacion := \frac{\partial}{\partial x} u(x,t) = \frac{5 \left(\frac{\partial}{\partial t} u(x,t)\right)}{x} \quad \textbf{(50)}$$

> $EcuacionUno := eval(subs(u(x,t) = F(x) \cdot G(t), Ecuacion))$

$$EcuacionUno := \left(\frac{d}{dx} F(x)\right) G(t) = \frac{5\, F(x) \left(\frac{d}{dt} G(t)\right)}{x} \quad \textbf{(51)}$$

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Respuesta primera _C

> $EcuacionDos := \frac{lhs(EcuacionUno) \cdot x}{F(x) \cdot G(t)} = \frac{rhs(EcuacionUno) \cdot x}{F(x) \cdot G(t)}$

$$EcuacionDos := \frac{\left(\frac{d}{dx} F(x)\right) x}{F(x)} = \frac{5 \left(\frac{d}{dt} G(t)\right)}{G(t)} \quad \textbf{(52)}$$

> $EcuacionX := lhs(EcuacionDos) = \text{alpha}; EcuacionT := rhs(EcuacionDos) = \text{alpha}$

$$EcuacionX := \frac{\left(\frac{d}{dx} F(x)\right) x}{F(x)} = \alpha$$

$$EcuacionT := \frac{5 \left(\frac{d}{dt} G(t)\right)}{G(t)} = \alpha \quad \textbf{(53)}$$

> $SolucionX := dsolve(subs(\text{alpha} = 3, EcuacionX)); SolucionT := dsolve(subs(\text{alpha} = 3, EcuacionT))$

$$SolucionX := F(x) = \_C1\, x^3$$

$$SolucionT := G(t) = \_C1\, e^{\frac{3}{5} t} \quad \textbf{(54)}$$

> $SolucionGeneral := u(x,t) = subs(\_C1 = 1, rhs(SolucionX)) \cdot rhs(SolucionT)$

$$SolucionGeneral := u(x,t) = x^3\, \_C1\, e^{\frac{3}{5} t} \quad \textbf{(55)}$$

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Respuesta segunda

> $EcuacionTres := \frac{lhs(EcuacionUno) \cdot x}{5 \cdot F(x) \cdot G(t)} = \frac{rhs(EcuacionUno) \cdot x}{5 \cdot F(x) \cdot G(t)}$

$$EcuacionTres := \frac{1}{5} \frac{\left(\frac{d}{dx} F(x)\right) x}{F(x)} = \frac{\frac{d}{dt} G(t)}{G(t)} \quad \textbf{(56)}$$

> $EcuacionXX := lhs(EcuacionTres) = \text{alpha}; EcuacionTT := rhs(EcuacionTres) = \text{alpha}$

$$EcuacionXX := \frac{1}{5} \frac{\left(\frac{d}{dx} F(x)\right) x}{F(x)} = \alpha$$

$$EcuacionTT := \frac{\frac{d}{dt} G(t)}{G(t)} = \alpha \quad \textbf{(57)}$$

> $SolucionXX := dsolve(subs(\mathrm{alpha} = 3, EcuacionXX)); SolucionTT := dsolve(subs(\mathrm{alpha} = 3, EcuacionTT))$

$$SolucionXX := F(x) = \_C1\ x^{15}$$

$$SolucionTT := G(t) = \_C1\ e^{3t} \quad \textbf{(58)}$$

> $SolucionGeneralDos := u(x, t) = subs(\_C1 = 1, rhs(SolucionXX)) \cdot rhs(SolucionTT)$

$$SolucionGeneralDos := u(x, t) = x^{15}\ \_C1\ e^{3t} \quad \textbf{(59)}$$

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FIN RESPUESTA 6)

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FIN EXAMEN

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