>

Solución

Ecuaciones Diferenciales
Segundo Examen Final
Semestre 2013-2

2013 junio 2

> *restart*

1) Resuelva la ecuación diferencial

> $x\cdot\cdot 2\cdot y' = 1 - x\cdot\cdot 2 + y\cdot\cdot 2 - x\cdot\cdot 2\cdot y\cdot\cdot 2$

$$x^2\left(\frac{d}{dx}\,y(x)\right) = 1 - x^2 + y(x)^2 - x^2\,y(x)^2 \quad \textbf{(1)}$$

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Respuesta 1)

> $Ecuacion := x^2\left(\frac{d}{dx}\,y(x)\right) = 1 - x^2 + y(x)^2 - x^2\,y(x)^2$

$$Ecuacion := x^2\left(\frac{d}{dx}\,y(x)\right) = 1 - x^2 + y(x)^2 - x^2\,y(x)^2 \quad \textbf{(2)}$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[\_separable] \quad \textbf{(3)}$$

> $EcuacionSeparable := lhs(Ecuacion) - rhs(Ecuacion) = 0$

$$EcuacionSeparable := x^2\left(\frac{d}{dx}\,y(x)\right) - 1 + x^2 - y(x)^2 + x^2\,y(x)^2 = 0 \quad \textbf{(4)}$$

> $EcuacionSeparada := factor\left(-1 + x^2 - y(x)^2 + x^2\,y(x)^2\right) + x^2\left(\frac{d}{dx}\,y(x)\right) = 0$

$$EcuacionSeparada := (x-1)\,(x+1)\,\left(1 + y(x)^2\right) + x^2\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(5)}$$

> $P := (x-1)\,(x+1);\ Q := 1 + y^2$

$$P := (x-1)\,(x+1)$$
$$Q := 1 + y^2 \quad \textbf{(6)}$$

> $R := x^2;\ S := 1$

$$R := x^2$$
$$S := 1 \quad \textbf{(7)}$$

> $SolucionGeneral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = C_1$

$$SolucionGeneral := x + \frac{1}{x} + \arctan(y) = C_1 \quad \textbf{(8)}$$

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Fin respuesta 1)

> *restart*

2) Resuelva la ecuación diferencial

> $4\cdot y'' + 36\cdot y = \csc(3\,x)$

(9)

$$4\left(\frac{d^2}{dx^2}y(x)\right)+36\,y(x)=\csc(3\,x) \qquad \textbf{(9)}$$

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Respuesta 2)

> $Ecuacion := 4\left(\frac{d^2}{dx^2}y(x)\right)+36\,y(x)=\csc(3\,x)$

$$Ecuacion := 4\left(\frac{d^2}{dx^2}y(x)\right)+36\,y(x)=\csc(3\,x) \qquad \textbf{(10)}$$

> $EcuacionNormal := simplify\left(\frac{lhs(Ecuacion)}{4}\right)=\frac{rhs(Ecuacion)}{4}$

$$EcuacionNormal := \frac{d^2}{dx^2}y(x)+9\,y(x)=\frac{1}{4}\csc(3\,x) \qquad \textbf{(11)}$$

> $EcuacionHom := lhs(EcuacionNormal)=0$

$$EcuacionHom := \frac{d^2}{dx^2}y(x)+9\,y(x)=0 \qquad \textbf{(12)}$$

> $Q := rhs(EcuacionNormal)$

$$Q := \frac{1}{4}\csc(3\,x) \qquad \textbf{(13)}$$

> $EcuacionCaract := m\cdot\cdot 2+9=0$

$$EcuacionCaract := m^2+9=0 \qquad \textbf{(14)}$$

> $Raiz := solve(EcuacionCaract)$

$$Raiz := 3\,I,\ -3\,I \qquad \textbf{(15)}$$

> $SolUno := y(x)=\exp\left(\mathrm{Re}\left(Raiz_1\right)\cdot x\right)\cdot\cos\left(\mathrm{Im}\left(Raiz_1\right)\cdot x\right)$

$$SolUno := y(x)=\cos(3\,x) \qquad \textbf{(16)}$$

> $SolDos := y(x)=\exp\left(\mathrm{Re}\left(Raiz_1\right)\cdot x\right)\cdot\sin\left(\mathrm{Im}\left(Raiz_1\right)\cdot x\right)$

$$SolDos := y(x)=\sin(3\,x) \qquad \textbf{(17)}$$

> $SolucionHom := y(x)=C_1\cdot rhs(SolUno)+C_2\cdot rhs(SolDos)$

$$SolucionHom := y(x)=C_1\cos(3\,x)+C_2\sin(3\,x) \qquad \textbf{(18)}$$

> $with(linalg):$

> $WW := wronskian([rhs(SolUno), rhs(SolDos)], x)$

$$WW := \begin{bmatrix}\cos(3\,x) & \sin(3\,x)\\ -3\sin(3\,x) & 3\cos(3\,x)\end{bmatrix} \qquad \textbf{(19)}$$

> $AA := array([0, Q])$

$$AA := \begin{bmatrix}0 & \frac{1}{4}\csc(3\,x)\end{bmatrix} \qquad \textbf{(20)}$$

> $SOL := simplify(linsolve(WW, AA))$

$$SOL := \begin{bmatrix}-\frac{1}{12} & \frac{1}{12}\,\frac{\cos(3\,x)}{\sin(3\,x)}\end{bmatrix} \qquad \textbf{(21)}$$

> $Aprima := SOL_1;\ Bprima := SOL_2$

$$Aprima := -\frac{1}{12}$$

$$Bprima := \frac{1}{12}\frac{\cos(3x)}{\sin(3x)} \quad \textbf{(22)}$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2$

$$A := -\frac{1}{12}x + C_1$$

$$B := \frac{1}{36}\ln(\sin(3x)) + C_2 \quad \textbf{(23)}$$

> $SolucionNoHom := y(x) = simplify(A \cdot rhs(SolUno) + B \cdot rhs(SolDos))$

$$SolucionNoHom := y(x) = -\frac{1}{12}\cos(3x)\,x + C_1\cos(3x) + \frac{1}{36}\sin(3x)\ln(\sin(3x)) + C_2\sin(3x) \quad \textbf{(24)}$$

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Fin respuesta 2)

> *restart*

3) Obtenga la solución del sitema de ecuaciones diferenciales

> $diff(x(t), t\$2) - 2 \cdot diff(y(t), t) + 3 \cdot x(t) = 0; diff(y(t), t\$2) + 2 \cdot diff(x(t), t) + 3 \cdot y(t) = 0$

$$\frac{d^2}{dt^2}x(t) - 2\left(\frac{d}{dt}y(t)\right) + 3x(t) = 0$$

$$\frac{d^2}{dt^2}y(t) + 2\left(\frac{d}{dt}x(t)\right) + 3y(t) = 0 \quad \textbf{(25)}$$

sujeta a las condiciones

> $x(0) = 4, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0$

$$x(0) = 4, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0 \quad \textbf{(26)}$$

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Respuesta 3)

> $Sistema := diff(x(t), t\$2) - 2 \cdot diff(y(t), t) + 3 \cdot x(t) = 0, diff(y(t), t\$2) + 2 \cdot diff(x(t), t) + 3 \cdot y(t) = 0 : Sistema_1; Sistema_2$

$$\frac{d^2}{dt^2}x(t) - 2\left(\frac{d}{dt}y(t)\right) + 3x(t) = 0$$

$$\frac{d^2}{dt^2}y(t) + 2\left(\frac{d}{dt}x(t)\right) + 3y(t) = 0 \quad \textbf{(27)}$$

> $Condiciones := x(0) = 4, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0$

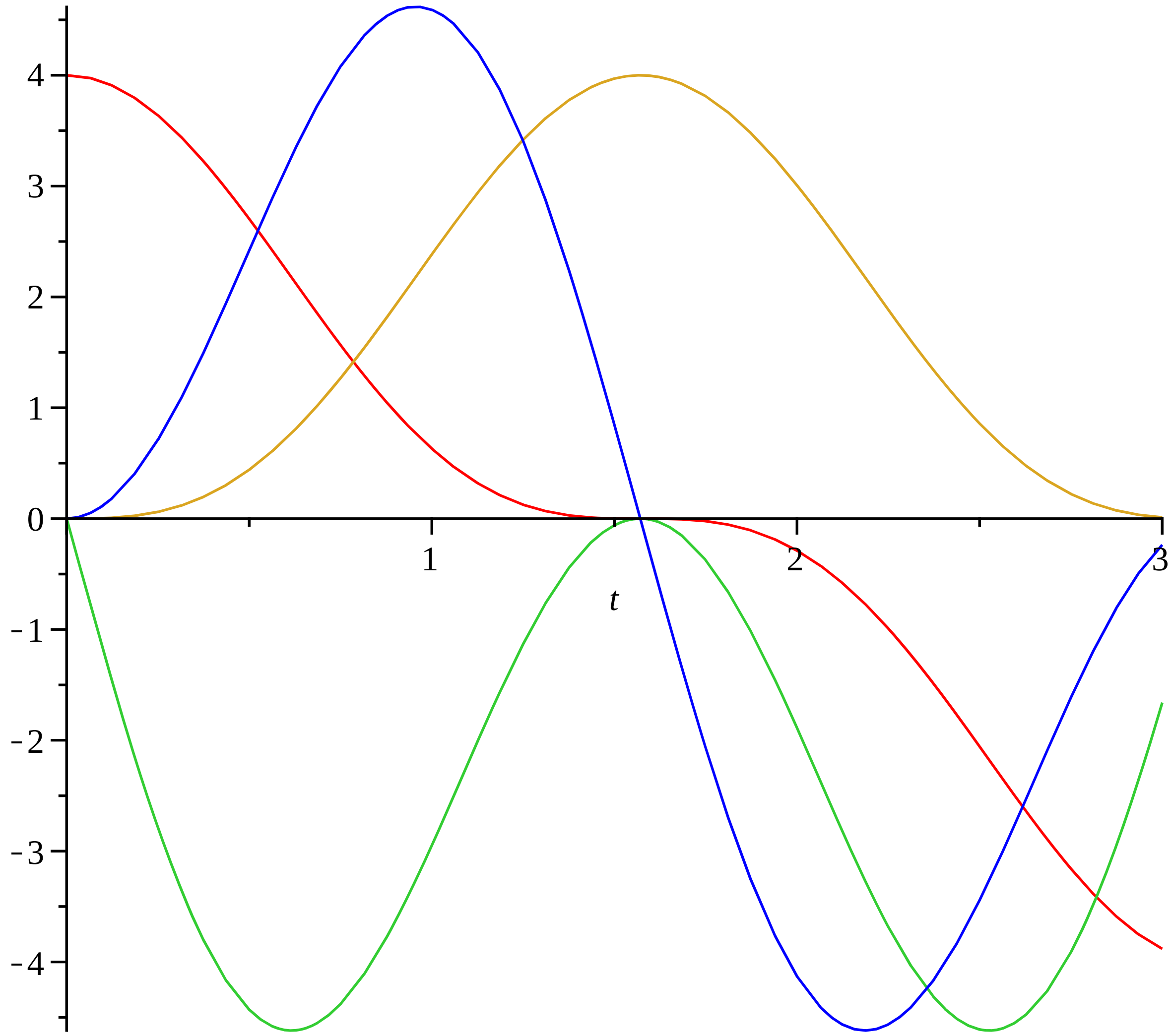
$$Condiciones := x(0) = 4, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0 \quad \textbf{(28)}$$

> $Solucion := dsolve(\{Sistema, Condiciones\}) : Solucion_1; Solucion_2$

$$x(t) = 3\cos(t) + \cos(3t)$$

$$y(t) = 3\sin(t) - \sin(3t) \quad \textbf{(29)}$$

> $plot([rhs(Solucion_1), rhs(diff(Solucion_1, t)), rhs(Solucion_2), rhs(diff(Solucion_2, t))], t = 0..3)$



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Fin respuesta 3)

> *restart*

4) Resuelva la ecuacion diferencial

> *diff*(y(t), t$2) + y(t) = f; y(0) = 0; D(y)(0) = 0

$$\frac{d^2}{dt^2} y(t) + y(t) = f$$

$$y(0) = 0$$

$$D(y)(0) = 0 \qquad \textbf{(30)}$$

donde

> $f := 2 \cdot \text{Heaviside}(t-1);$

$$f := 2\ \text{Heaviside}(t-1) \qquad \textbf{(31)}$$

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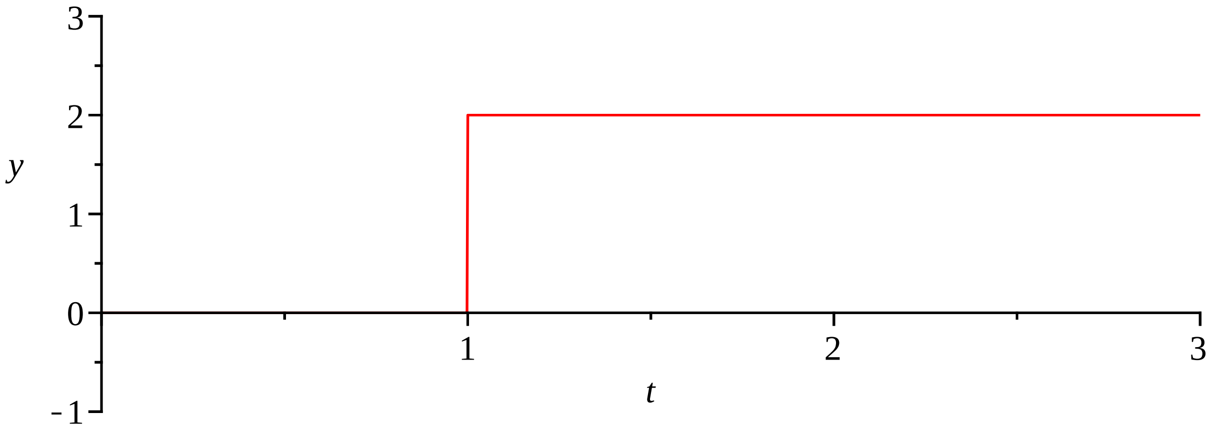
Respuesta 4)

> *plot*(f, t = 0 ..3, y =−1 ..3)



> $Ecuacion := diff(y(t), t\$2) + y(t) = f$

$$Ecuacion := \frac{d^2}{dt^2} y(t) + y(t) = 2 \text{ Heaviside}(t-1) \quad \textbf{(32)}$$

> $Condiciones := y(0) = 0, D(y)(0) = 0$

$$Condiciones := y(0) = 0, D(y)(0) = 0 \quad \textbf{(33)}$$

> $with(inttrans):$

> $TransLapEcuacion := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s^2\, laplace(y(t), t, s) + laplace(y(t), t, s) = \frac{2\, e^{-s}}{s} \quad \textbf{(34)}$$

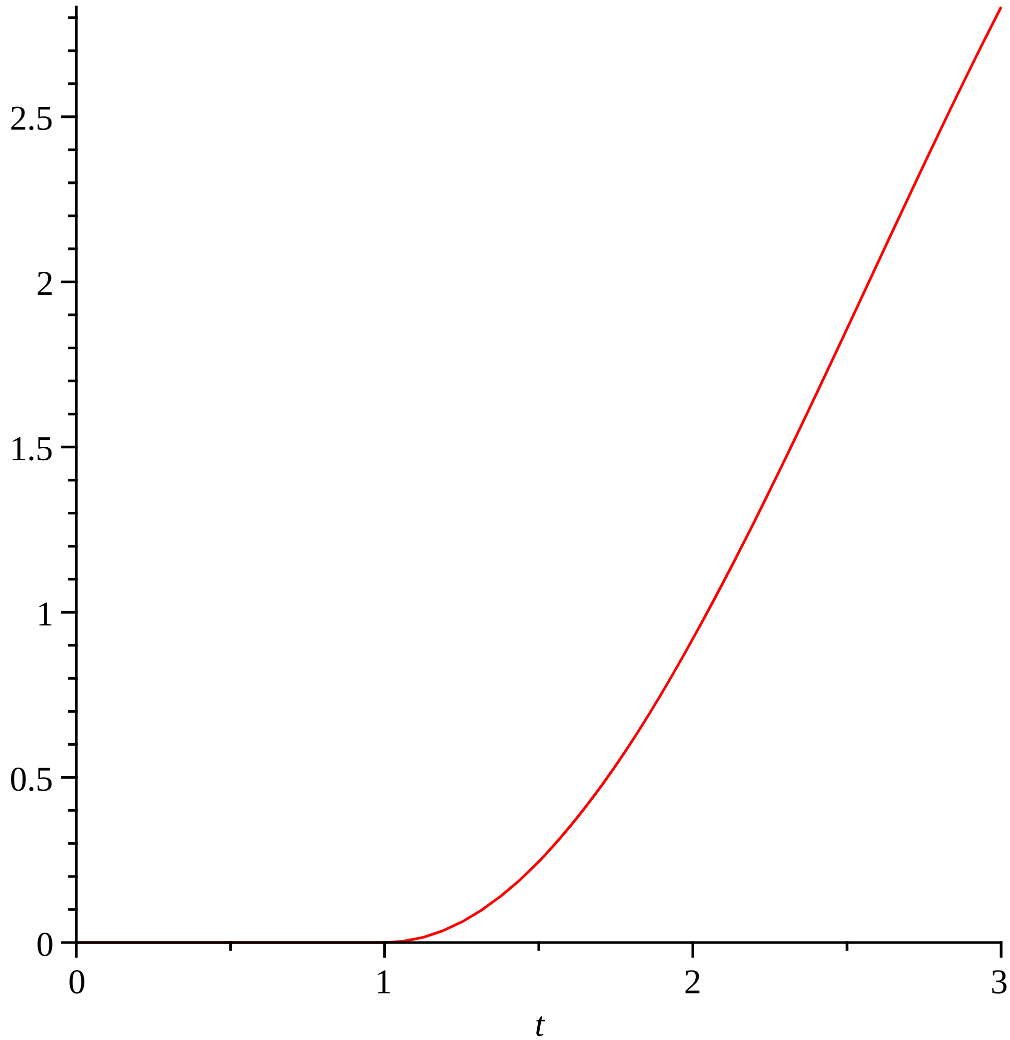
> $TransLapSolucion := isolate(TransLapEcuacion, laplace(y(t), t, s))$

$$TransLapSolucion := laplace(y(t), t, s) = \frac{2\, e^{-s}}{s\,(s^2+1)} \quad \textbf{(35)}$$

> $SolucionUno := simplify(invlaplace(TransLapSolucion, s, t))$

$$SolucionUno := y(t) = -4 \text{ Heaviside}(t-1) \left(-1 + \cos\left(\frac{1}{2}\, t - \frac{1}{2}\right)^2\right) \quad \textbf{(36)}$$

> $plot(rhs(SolucionUno), t = 0..3)$



> $SolucionDos := y(t) = 2\cdot(1-\cos(t-1))\cdot\text{Heaviside}(t-1)$

$$SolucionDos := y(t) = 2\ (1-\cos(t-1))\ \text{Heaviside}(t-1) \tag{37}$$

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Fin respuesta 4)

> *restart*

5) Obtenga la solución de la ecuación integral

> $f(t) = 2\cdot t - 4\cdot int(\sin(\text{tau})\cdot f(\text{t}-\text{tau}), \text{tau} = 0\,..t)$

$$f(t) = 2\,t - 4\left(\int_0^t \sin(\tau)\, f(t-\tau)\,\mathrm{d}\tau\right) \tag{38}$$

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Respuesta 5)

> $Ecuacion := f(t) = 2\cdot t - 4\cdot int(\sin(\text{tau})\cdot f(t-\text{tau}), \text{tau} = 0\,..t)$

$$Ecuacion := f(t) = 2\,t - 4\left(\int_0^t \sin(\tau)\, f(t-\tau)\,\mathrm{d}\tau\right) \tag{39}$$

> *with*(*inttrans*) :

> *TransLapEcuacion* := *expand*(*laplace*(*Ecuacion*, *t*, *s*))

(40)

$$TransLapEcuacion := laplace(f(t), t, s) = \frac{2}{s^2} - \frac{4\, laplace(f(t), t, s)}{s^2+1} \tag{40}$$

> TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(f(t), t, s)))

$$TransLapSolucion := laplace(f(t), t, s) = \frac{2\,(s^2+1)}{s^2\,(s^2+5)} \tag{41}$$

> Solucion := invlaplace(TransLapSolucion, s, t)

$$Solucion := f(t) = \frac{2}{5}\,t + \frac{8}{25}\,\sqrt{5}\,\sin\left(\sqrt{5}\,t\right) \tag{42}$$

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Fin respuesta 5)

> restart

6) Utilice el método de separación de variables para resolver la ecuación en derivadas parciales

> s·diff (u(s, t), s) = t·diff (u(s, t), t)

$$s\left(\frac{\partial}{\partial s}\,u(s,t)\right) = t\left(\frac{\partial}{\partial t}\,u(s,t)\right) \tag{43}$$

con una constante de separación negativa

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Respuesta 6)

> Ecuacion := s·diff (u(s, t), s) = t·diff (u(s, t), t)

$$Ecuacion := s\left(\frac{\partial}{\partial s}\,u(s,t)\right) = t\left(\frac{\partial}{\partial t}\,u(s,t)\right) \tag{44}$$

> EcuacionSeparable := simplify(eval(subs(u(s, t) = F(s)·G(t), Ecuacion)))

$$EcuacionSeparable := s\left(\frac{d}{ds}\,F(s)\right)G(t) = t\,F(s)\left(\frac{d}{dt}\,G(t)\right) \tag{45}$$

> $EcuacionSeparada := \frac{lhs(EcuacionSeparable)}{F(s)\cdot G(t)} = \frac{rhs(EcuacionSeparable)}{F(s)\cdot G(t)}$

$$EcuacionSeparada := \frac{s\left(\frac{d}{ds}\,F(s)\right)}{F(s)} = \frac{t\left(\frac{d}{dt}\,G(t)\right)}{G(t)} \tag{46}$$

> EcuacionS := lhs(EcuacionSeparada) =-beta· ·2; EcuacionT := rhs(EcuacionSeparada) = -beta· ·2

$$EcuacionS := \frac{s\left(\frac{d}{ds}\,F(s)\right)}{F(s)} = -\beta^2$$

$$EcuacionT := \frac{t\left(\frac{d}{dt}\,G(t)\right)}{G(t)} = -\beta^2 \tag{47}$$

> SolucionS := dsolve(EcuacionS); SolucionT := dsolve(EcuacionT)

$$SolucionS := F(s) = \_C1\; s^{-\beta^2}$$

$$SolucionT := G(t) = \_C1\; t^{-\beta^2} \tag{48}$$

> SolucionGeneral := u(s, t) = rhs(SolucionS)·subs(_C1 = 1, rhs(SolucionT))

$$SolucionGeneral := u(s,t) = \_C1\; s^{-\beta^2}\, t^{-\beta^2} \tag{49}$$

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Fin respuesta 6)

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Fin examen

> *restart*

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