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SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL (TEMAS 4 Y 5)
SEMESTRE 2014-1

2013 NOVIEMBRE 15

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1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE **(sin usar dsolve)**:
a) **(15/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS
b) **(15/100 puntos)** GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA PARA UN INTERVALO DE 0 < t < 3

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RESPUESTA 1a)

\> $Ecuacion := \frac{d^2}{dt^2}\, y(t) + 16\, y(t) = 64\,(t-2)\,\mathrm{Heaviside}(t-2)\cos(3\,t-6)$

$$Ecuacion := \frac{d^2}{dt^2}\, y(t) + 16\, y(t) = 64\,(t-2)\,\mathrm{Heaviside}(t-2)\cos(3\,t-6) \quad \textbf{(1)}$$

\> $Condiciones := y(0) = 1, \mathrm{D}(y)(0) = 0;$

$$Condiciones := y(0) = 1, \mathrm{D}(y)(0) = 0 \quad \textbf{(2)}$$

\> $with(inttrans):$

\> $TransLapEcuacion := simplify(subs(Condiciones, laplace(Ecuacion, t, s)))$

$$TransLapEcuacion := s^2\, laplace(y(t), t, s) - s + 16\, laplace(y(t), t, s) = \frac{64\, e^{-2s}\,(s^2-9)}{(s^2+9)^2} \quad \textbf{(3)}$$

\> $TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))$

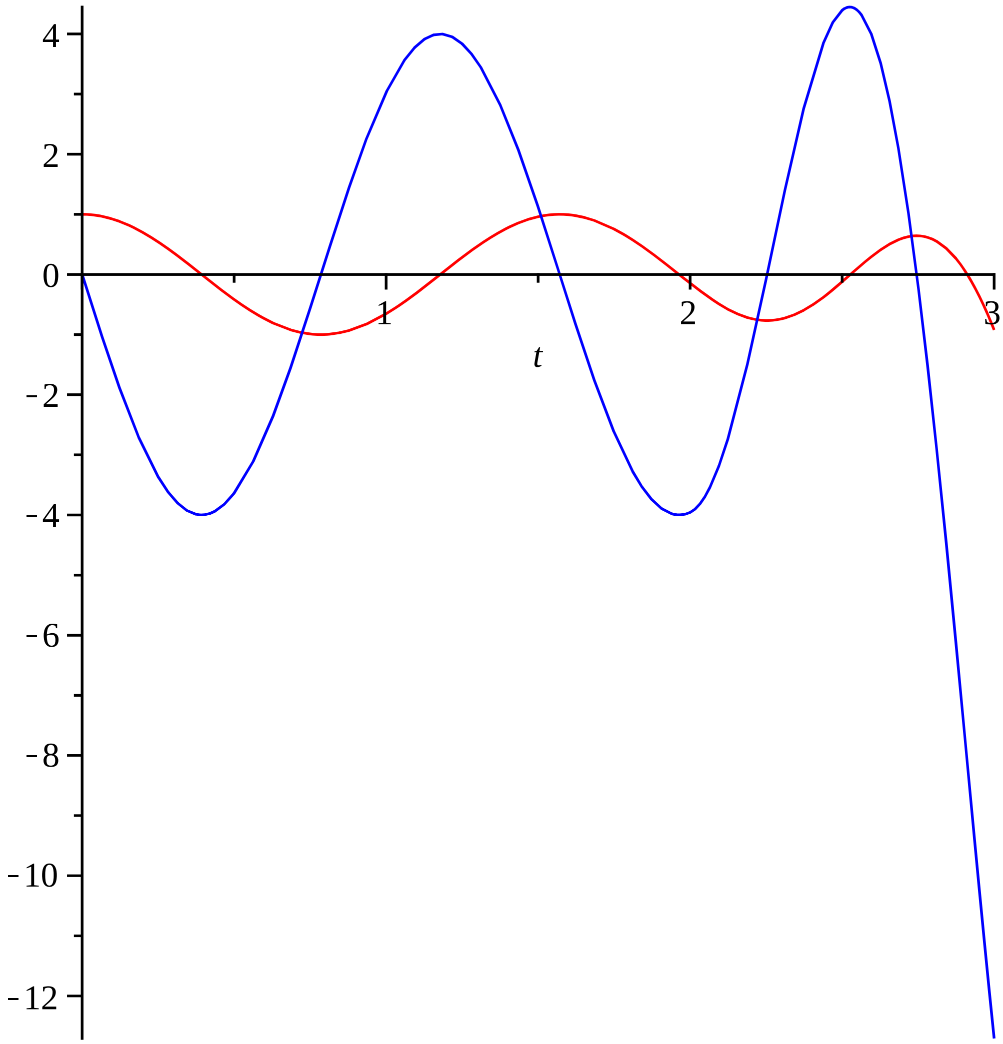
$$TransLapSolucion := laplace(y(t), t, s) = \frac{64\, e^{-2s} s^2 - 576\, e^{-2s} + s^5 + 18\, s^3 + 81\, s}{(s^2+9)^2\,(s^2+16)} \quad \textbf{(4)}$$

\> $SolucionParticular := simplify(invlaplace(TransLapSolucion, s, t))$

$$SolucionParticular := y(t) = \cos(4\,t) - \frac{400}{49}\,\mathrm{Heaviside}(t-2)\sin(4\,t-8) + \frac{384}{49}\,\mathrm{Heaviside}(t-2)\sin(3\,t-6) + \frac{64}{7}\,\mathrm{Heaviside}(t-2)\cos(3\,t-6)\, t - \frac{128}{7}\,\mathrm{Heaviside}(t-2)\cos(3\,t-6) \quad \textbf{(5)}$$

RESPUESTA 1b)

\> $plot([rhs(SolucionParticular), rhs(diff(SolucionParticular, t))], t = 0..3, color = [red, blue])$



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FIN RESPUESTA 1)

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2) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:

a) **(15/100 puntos)** OBTENER SU TRANSFORMADA DE LAPLACE.
b) **(25/100 puntos)** OBTENER Y GRAFICAR SU SERIE COSENO DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

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***RESPUESTA 2 a*).**

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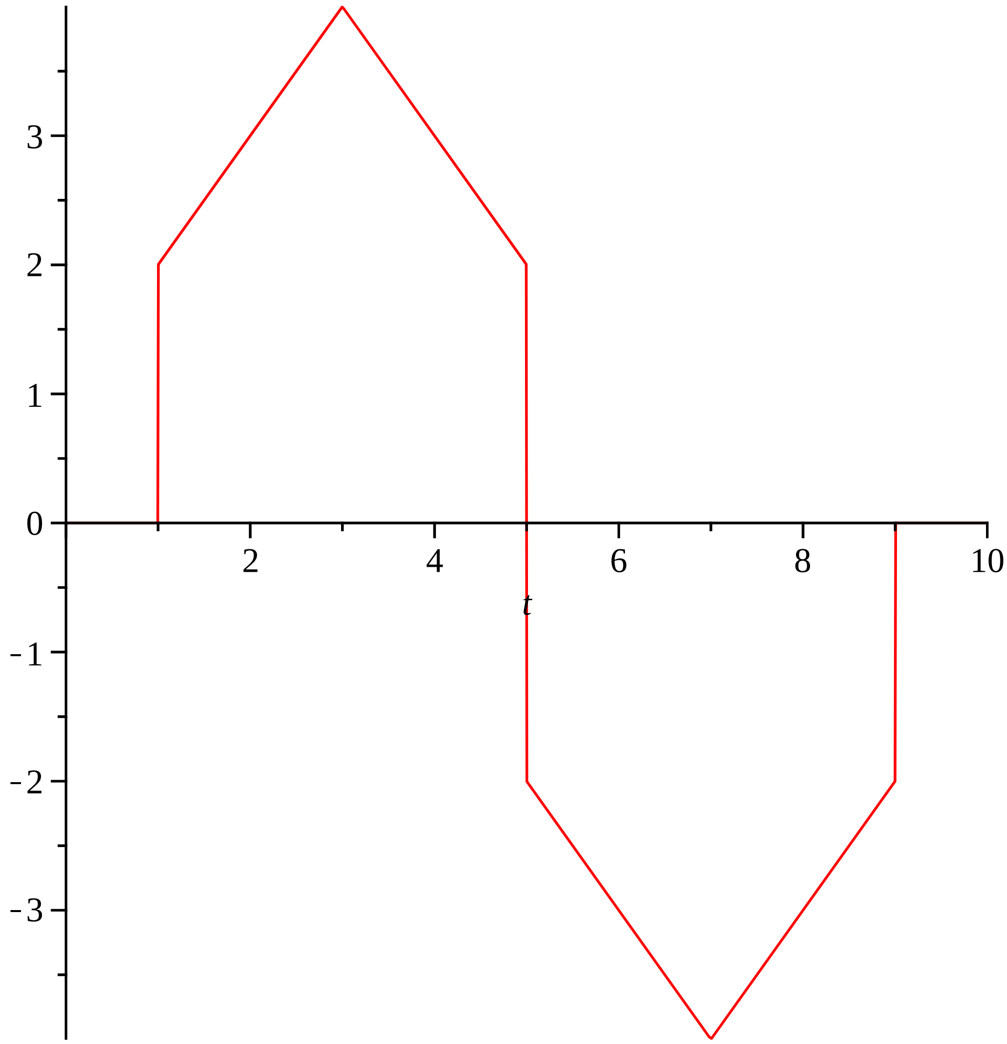
> $p := 2\cdot \text{Heaviside}(t-1) + (t-1)\cdot\text{Heaviside}(t-1) - 2\cdot(t-3)\cdot\text{Heaviside}(t-3) + (t-5)\cdot\text{Heaviside}(t-5) - 4\cdot\text{Heaviside}(t-5) - (t-5)\cdot\text{Heaviside}(t-5) + 2\cdot(t-7)\cdot\text{Heaviside}(t-7) - (t-9)\cdot\text{Heaviside}(t-9) + 2\cdot\text{Heaviside}(t-9) : plot(p, t = 0..10)$



> $with(inttrans):$

> $P := laplace(p, t, s)$

$$P := \frac{e^{-s} - e^{-9s} + 2\,e^{-7s} - 2\,e^{-3s}}{s^2} + \frac{2\,(e^{-s} + e^{-9s} - 2\,e^{-5s})}{s} \tag{6}$$

***RESPUESTA* 2 *b*).**

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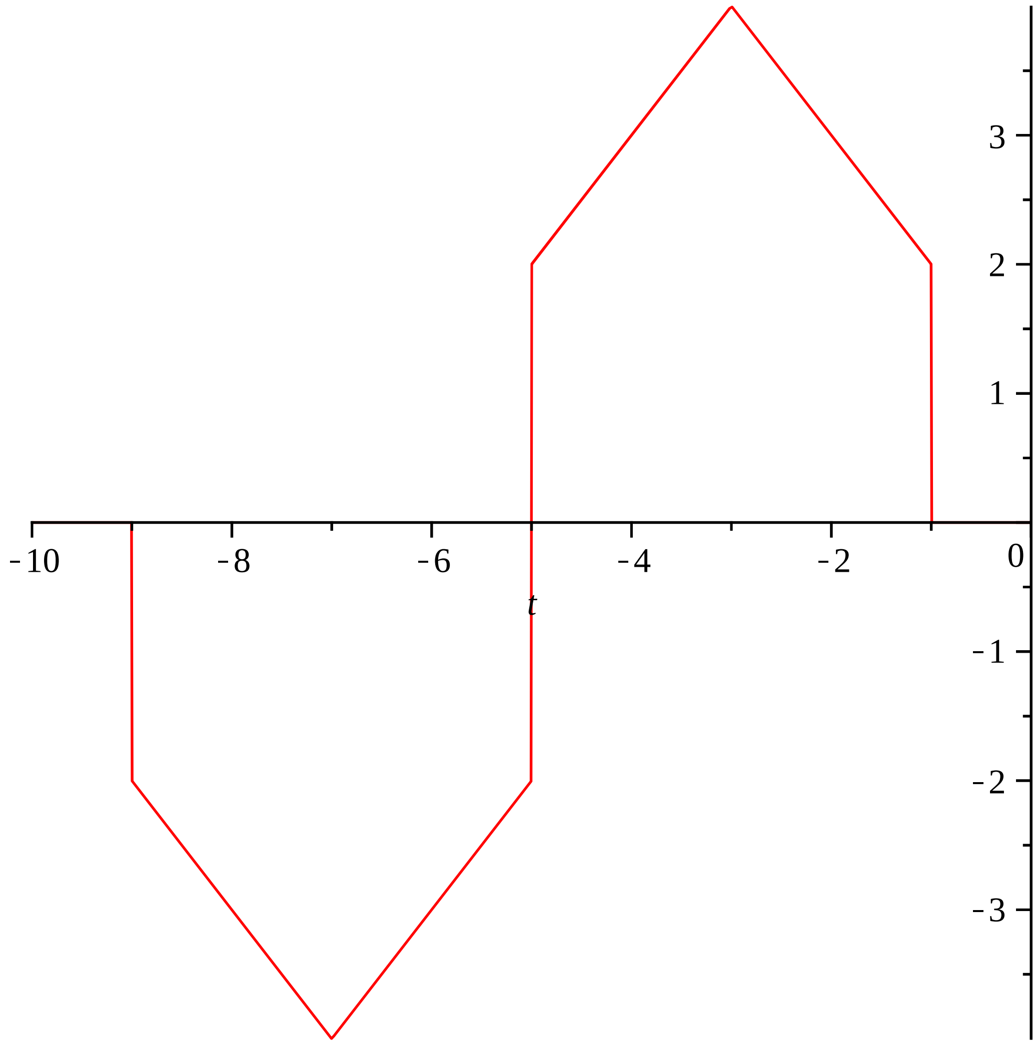
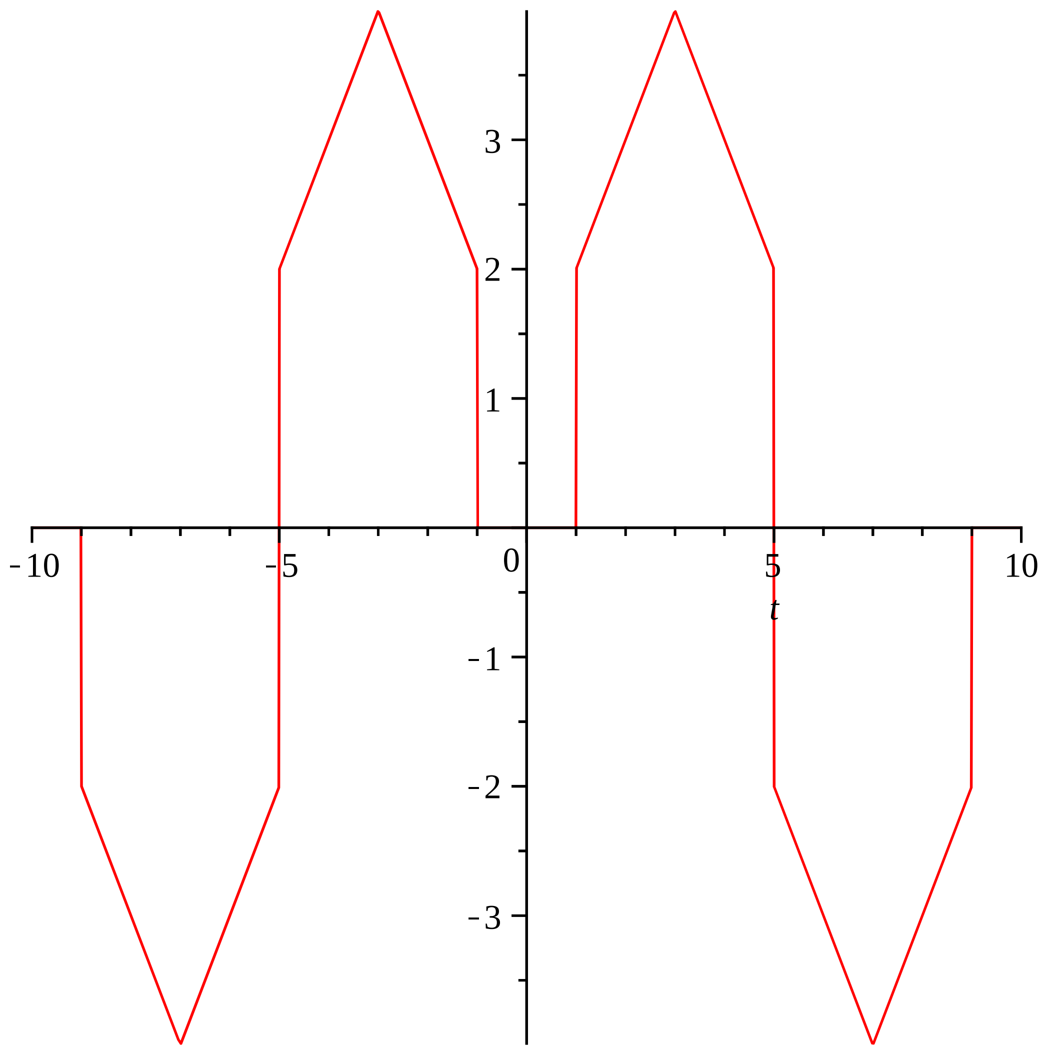
> $r := -2\cdot\text{Heaviside}(t+9) - (t+9)\cdot\text{Heaviside}(t+9) + 2\cdot(t+7)\cdot\text{Heaviside}(t+7) - (t+5)\cdot\text{Heaviside}(t+5) + 4\cdot\text{Heaviside}(t+5) + (t+5)\cdot\text{Heaviside}(t+5) - 2\cdot(t+3)\cdot\text{Heaviside}(t+3) + (t+1)\cdot\text{Heaviside}(t+1) - 2\cdot\text{Heaviside}(t+1) : plot(r, t=-10..0)$



> $q := p + r; plot(q, t = -10 .. 10)$

$$q := 2\,\text{Heaviside}(t-1) + (t-1)\,\text{Heaviside}(t-1) - 2\,(t-3)\,\text{Heaviside}(t-3) - 4\,\text{Heaviside}(t-5) + 2\,(t-7)\,\text{Heaviside}(t-7) - (t-9)\,\text{Heaviside}(t-9) + 2\,\text{Heaviside}(t-9) - 2\,\text{Heaviside}(t+9) - (t+9)\,\text{Heaviside}(t+9) + 2\,(t+7)\,\text{Heaviside}(t+7) + 4\,\text{Heaviside}(t+5) - 2\,(t+3)\,\text{Heaviside}(t+3) + (t+1)\,\text{Heaviside}(t+1) - 2\,\text{Heaviside}(t+1)$$



> $L := 10$

$$L := 10 \qquad (7)$$

> $a_0 := \left(\frac{1}{10}\right) \cdot int(q, t = -10..10)$

$$a_0 := 0 \qquad (8)$$

> $a_n := simplify\left(\left(\frac{1}{10}\right) \cdot int\left(q \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -10..10\right)\right)$

$$a_n := -\frac{1}{n^2 \pi^2}\left(4\left(n \pi \sin\left(\frac{1}{10} n \pi\right) + 5 \cos\left(\frac{1}{10} n \pi\right) - 10 \cos\left(\frac{3}{10} n \pi\right) + 10 \cos\left(\frac{7}{10} n \pi\right) - 5 \cos\left(\frac{9}{10} n \pi\right) + n \pi \sin\left(\frac{9}{10} n \pi\right) - 2 \sin\left(\frac{1}{2} n \pi\right) n \pi\right)\right) \qquad (9)$$

> $b_n := simplify\left(\left(\frac{1}{10}\right) \cdot int\left(q \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -10..10\right)\right)$

$$b_n := 0 \qquad (10)$$

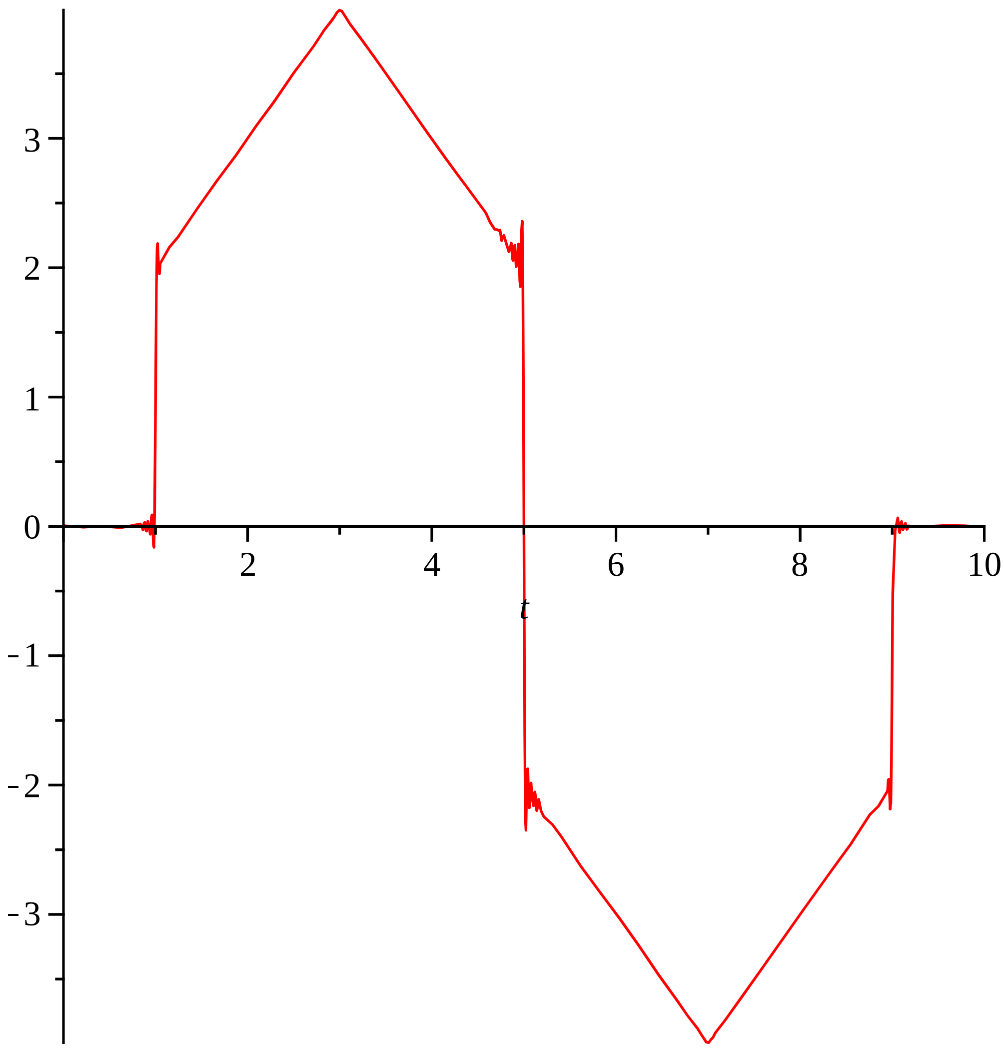
> $STF := Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..infinity\right);$

(11)

$$STF := \sum_{n=1}^{\infty} \left(-\frac{1}{n^2 \pi^2} \left(4 \left(n \pi \sin\left(\frac{1}{10} n \pi\right) + 5 \cos\left(\frac{1}{10} n \pi\right) - 10 \cos\left(\frac{3}{10} n \pi\right) + 10 \cos\left(\frac{7}{10} n \pi\right) - 5 \cos\left(\frac{9}{10} n \pi\right) + n \pi \sin\left(\frac{9}{10} n \pi\right) - 2 \sin\left(\frac{1}{2} n \pi\right) n \pi \right) \cos\left(\frac{1}{10} n \pi t\right) \right) \right) \quad (11)$$

> $STF_{500} := sum\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..500\right) :$

> $plot(STF_{500}, t = 0..10)$



FIN RESPUESTA 2)

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3) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA **CONSTANTE DE SEPARACIÓN POSITIVA**:

$$\frac{\partial^2}{\partial x^2} z(x, t) + t^2 \left(\frac{\partial}{\partial t} z(x, t) \right) = \frac{\partial}{\partial x} z(x, t) \quad (12)$$

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RESPUESTA 3)

> $Ecuacion := \frac{\partial^2}{\partial x^2} z(x,t) + t^2 \left(\frac{\partial}{\partial t} z(x,t)\right) = \frac{\partial}{\partial x} z(x,t)$

$$Ecuacion := \frac{\partial^2}{\partial x^2} z(x,t) + t^2 \left(\frac{\partial}{\partial t} z(x,t)\right) = \frac{\partial}{\partial x} z(x,t) \qquad (13)$$

> $EcuacionSeparable := simplify(eval(subs(z(x,t) = F(x) \cdot G(t), Ecuacion)))$

$$EcuacionSeparable := \left(\frac{d^2}{dx^2} F(x)\right) G(t) + t^2 F(x) \left(\frac{d}{dt} G(t)\right) = \left(\frac{d}{dx} F(x)\right) G(t) \qquad (14)$$

> $EcuacionSeparada := simplify\left(\frac{\left(lhs(EcuacionSeparable) - t^2 F(x)\left(\frac{d}{dt} G(t)\right) - \left(\frac{d}{dx} F(x)\right) G(t)\right)}{F(x) \cdot G(t)} = \frac{\left(rhs(EcuacionSeparable) - t^2 F(x)\left(\frac{d}{dt} G(t)\right) - \left(\frac{d}{dx} F(x)\right) G(t)\right)}{F(x) \cdot G(t)}\right)$

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} \qquad (15)$$

> $EcuacionX := lhs(EcuacionSeparada) = \text{alpha}; EcuacionT := rhs(EcuacionSeparada) = \text{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = \alpha$$

$$EcuacionT := -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} = \alpha \qquad (16)$$

> $SolucionXpos := dsolve(subs(\text{alpha} = \text{beta} \cdot \cdot 2, EcuacionX))$

$$SolucionXpos := F(x) = \_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1+4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1+4\beta^2}\right)x} \qquad (17)$$

> $SolucionTpos := dsolve(subs(\text{alpha} = \text{beta} \cdot \cdot 2, EcuacionT))$

$$SolucionTpos := G(t) = \_C1\, e^{\frac{\beta^2}{t}} \qquad (18)$$

> $SoluicionGeneral := z(x,t) = (rhs(SolucionXpos) \cdot subs(\_C1 = 1, rhs(SolucionTpos)))$

$$SoluicionGeneral := z(x,t) = \left(\_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1+4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1+4\beta^2}\right)x}\right) e^{\frac{\beta^2}{t}} \qquad (19)$$

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OPCIÓN DOS

> $EcuacionSeparableDos := simplify(eval(subs(z(x,t) = F(x) + G(t), Ecuacion)))$

$$EcuacionSeparableDos := \frac{d^2}{dx^2} F(x) + t^2 \left(\frac{d}{dt} G(t)\right) = \frac{d}{dx} F(x) \qquad (20)$$

> $EcuacionSeparadaDos := lhs(EcuacionSeparableDos) - \frac{d}{dx} F(x) - t^2 \left(\frac{d}{dt} G(t)\right)$

$$= rhs(EcuacionSeparableDos) - \frac{d}{dx} F(x) - t^2 \left(\frac{d}{dt} G(t)\right)$$

$$EcuacionSeparadaDos := \frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right) = -t^2 \left(\frac{d}{dt} G(t)\right) \quad \textbf{(21)}$$

> $EcuacionXdos := lhs(EcuacionSeparadaDos) = \text{alpha}; EcuacionTdos := rhs(EcuacionSeparadaDos) = \text{alpha}$

$$EcuacionXdos := \frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right) = \alpha$$

$$EcuacionTdos := -t^2 \left(\frac{d}{dt} G(t)\right) = \alpha \quad \textbf{(22)}$$

> $SolucionXposDos := dsolve(subs(\text{alpha} = \text{beta}\cdot\cdot 2, EcuacionXdos))$

$$SolucionXposDos := F(x) = e^x \_C1 - \beta^2 x + \_C2 \quad \textbf{(23)}$$

> $SolucionTposDos := dsolve(subs(\text{alpha} = \text{beta}\cdot\cdot 2, EcuacionTdos))$

$$SolucionTposDos := G(t) = \frac{\beta^2}{t} + \_C1 \quad \textbf{(24)}$$

> $SoluicionGeneralDos := z(x, t) = (rhs(SolucionXposDos) \cdot subs(\_C1 = 1, rhs(SolucionTposDos)))$

$$SoluicionGeneralDos := z(x, t) = \left(e^x \_C1 - \beta^2 x + \_C2\right) \left(\frac{\beta^2}{t} + 1\right) \quad \textbf{(25)}$$

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FIN DEL EXAMEN

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