>

SOLUCIÓN

ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL
SEMESTRE 2014-1 (tipo A)

NOVIEMBRE 26 DE 2013

> *restart*

1) Resolver

> $Ecuacion := y(x)\cdot\exp(x+y(x))\cdot diff(y(x),x)+x\cdot\exp(x+y(x))=0$

$$Ecuacion := y(x)\, e^{x+y(x)} \left(\frac{d}{dx} y(x)\right) + x\, e^{x+y(x)} = 0 \quad \textbf{(1)}$$

> $Condicion := y(0)=-1;$

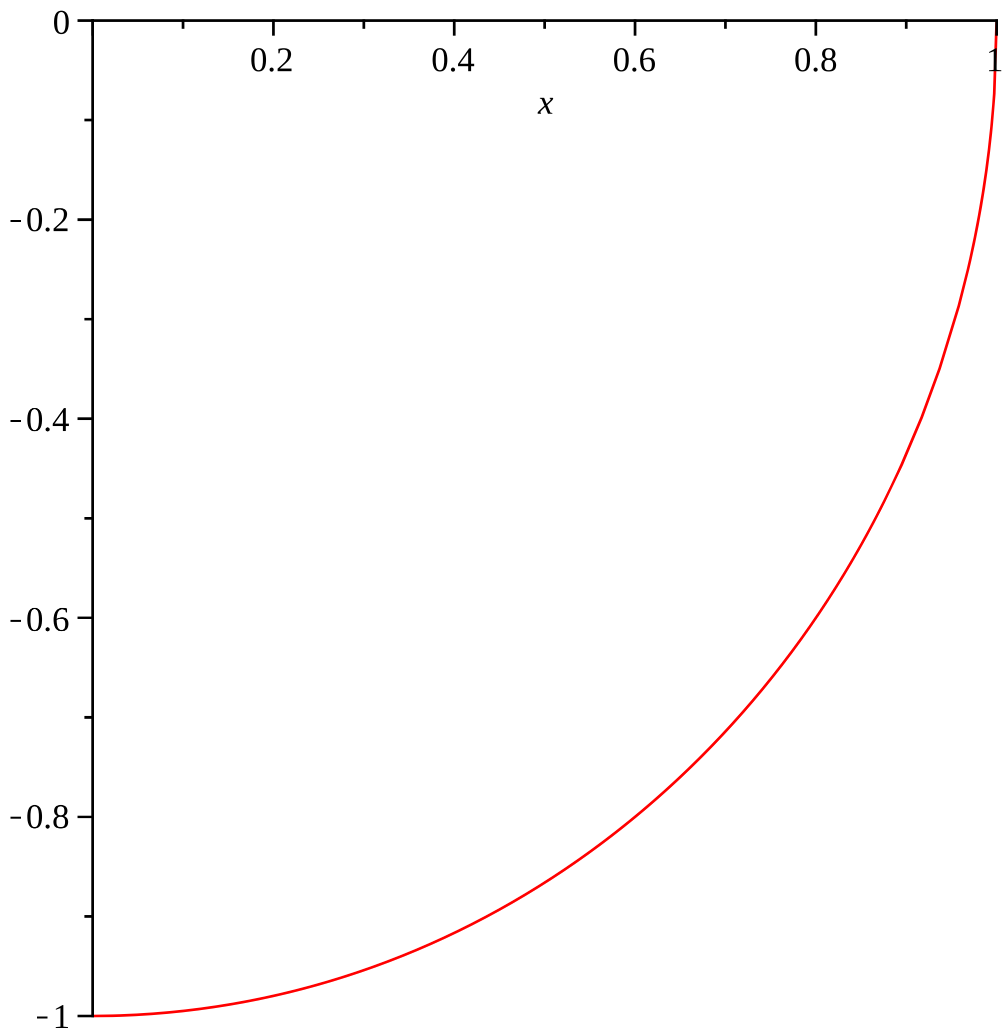
$$Condicion := y(0) = -1 \quad \textbf{(2)}$$

RESPUESTA 1)

> $Solucion := dsolve(\{Ecuacion, Condicion\})$

$$Solucion := y(x) = -\sqrt{-x^2+1} \quad \textbf{(3)}$$

> $plot(rhs(Solucion), x=0..1)$



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MÉTODO DOS

> *Ecuacion*

$$y(x)\,e^{x+y(x)}\left(\frac{d}{dx}\,y(x)\right)+x\,e^{x+y(x)}=0 \quad \textbf{(4)}$$

> with(DEtools) :

> odeadvisor(Ecuacion)

$$[\_separable] \quad \textbf{(5)}$$

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Por variables separables

> $M := x\exp(x)\cdot\exp(y);\ N := y\cdot\exp(x)\cdot\exp(y)$

$$M := e^{x}\,e^{y}\,x$$

$$N := e^{x}\,e^{y}\,y \quad \textbf{(6)}$$

> $P := x\cdot\exp(x);\ Q := \exp(y);\ R := \exp(x);\ S := y\cdot\exp(y)$

$$P := x\,e^{x}$$

$$Q := e^{y}$$

$$R := e^{x}$$

$$S := y\,e^{y} \quad \textbf{(7)}$$

> $Solucion := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = C_1$

$$Solucion := \frac{1}{2}\,x^{2}+\frac{1}{2}\,y^{2}=C_{1} \quad \textbf{(8)}$$

> $SolucionGeneral := lhs(Solucion)\cdot 2 = C_1$

$$SolucionGeneral := x^{2}+y^{2}=C_{1} \quad \textbf{(9)}$$

> $Parametro := subs(x=0, y=-1, SolucionGeneral)$

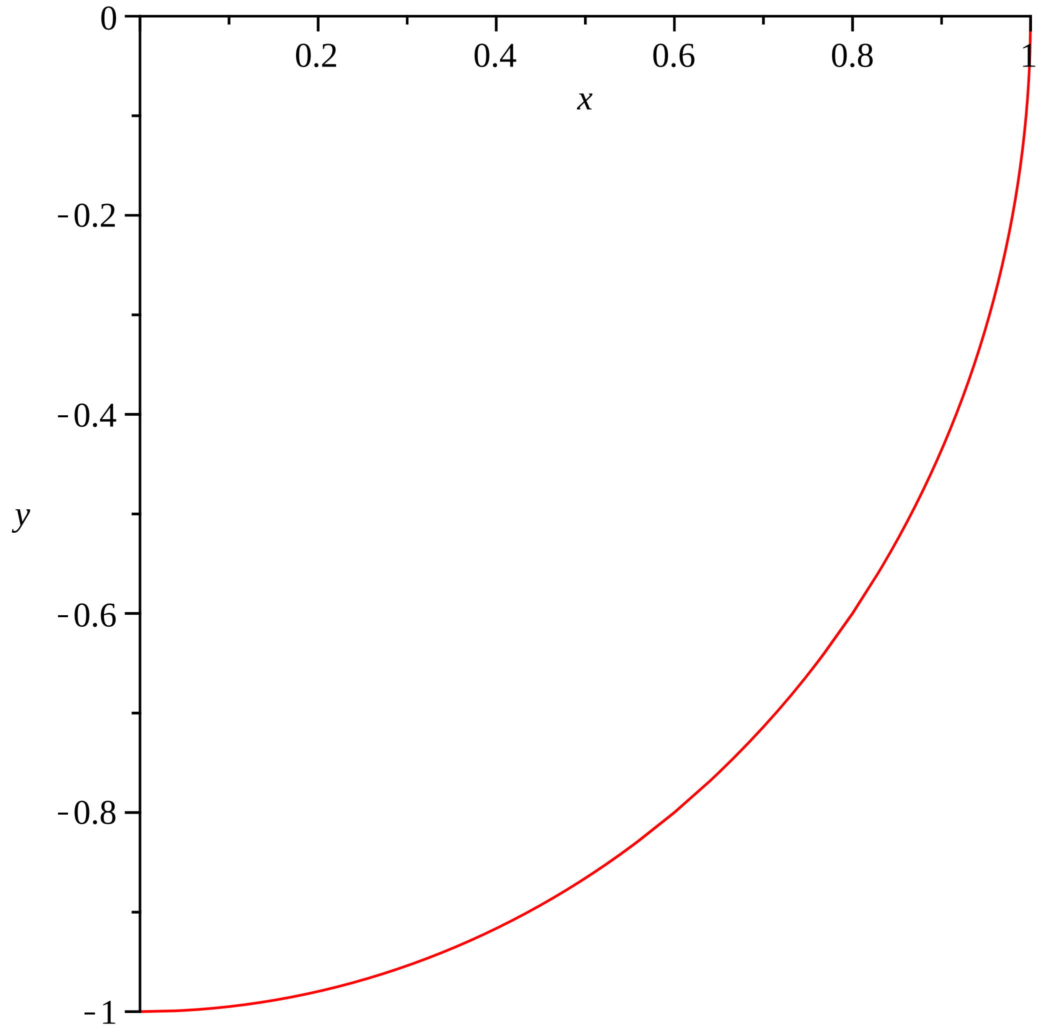
$$Parametro := 1 = C_{1} \quad \textbf{(10)}$$

> $SolucionParticular := subs(C_1 = lhs(Parametro), SolucionGeneral)$

$$SolucionParticular := x^{2}+y^{2}=1 \quad \textbf{(11)}$$

> with(plots) :

> implicitplot(SolucionParticular, x = 0 ..1, y = −1 ..0)



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Por factor integrante

> $intfactor(Ecuacion)$

$$\frac{1}{e^{x+y(x)}}, \frac{1}{\left(x^2+y(x)^2\right) e^{x+y(x)}} \quad \textbf{(12)}$$

> $FactInt := \frac{1}{e^{x+y}}$

$$FactInt := \frac{1}{e^{x+y}} \quad \textbf{(13)}$$

> $M; N;$

$$e^x e^y x$$
$$e^x e^y y \quad \textbf{(14)}$$

> $MM := simplify(FactInt \cdot M); NN := simplify(FactInt \cdot N)$

$$MM := x$$
$$NN := y \quad \textbf{(15)}$$

> $SolucionDos := int(MM, x) + int((NN - diff(int(MM, x), y)), y) = C_1$

(16)

$$SolucionDos := \frac{1}{2}x^2 + \frac{1}{2}y^2 = C_1 \quad \textbf{(16)}$$

> $SolucionGralDos := lhs(SolucionDos) \cdot 2 = C_1$

$$SolucionGralDos := x^2 + y^2 = C_1 \quad \textbf{(17)}$$

> $ParametroDos := subs(x = 0, y = -1, SolucionGralDos)$

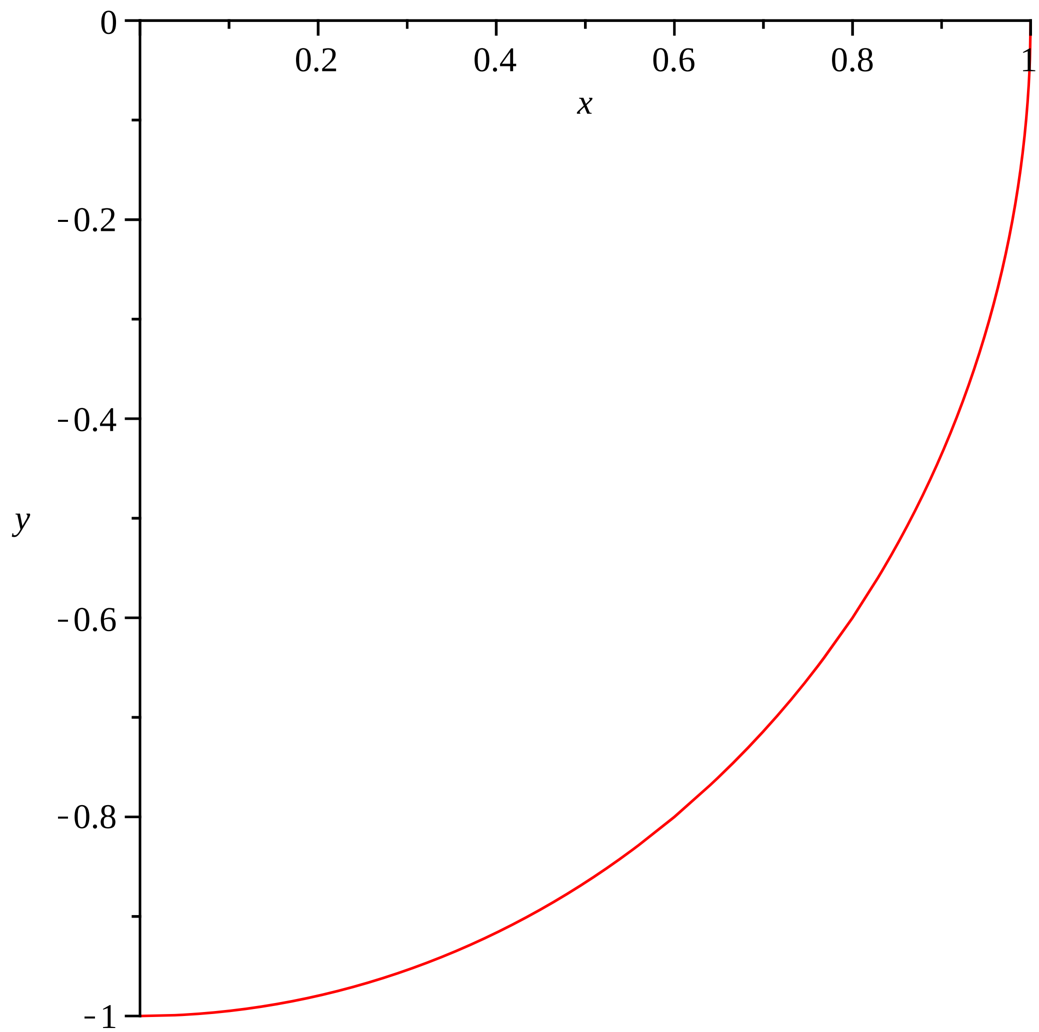
$$ParametroDos := 1 = C_1 \quad \textbf{(18)}$$

> $SolucionParticularDos := subs(C_1 = lhs(ParametroDos), SolucionGralDos)$

$$SolucionParticularDos := x^2 + y^2 = 1 \quad \textbf{(19)}$$

> $with(plots):$

> $implicitplot(SolucionParticularDos, x = 0..1, y = -1..0)$



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FIN RESPUESTA 1)

> *restart*

2) las funciones

> $SolUno := y(x) = x \cdot \left(-\frac{1}{2}\right) \cdot \cos(x); SolDos := y(x) = x \cdot \left(-\frac{1}{2}\right) \cdot \sin(x)$

$$SolUno := y(x) = \frac{\cos(x)}{\sqrt{x}}$$

$$SolDos := y(x) = \frac{\sin(x)}{\sqrt{x}} \quad \textbf{(20)}$$

> $EcuacionHom := x\cdot\cdot 2\cdot y'' + x\cdot y' + \left(x\cdot\cdot 2 - \frac{1}{4}\right)\cdot y = 0$

$$EcuacionHom := x^2 \left(\frac{d^2}{dx^2} y(x)\right) + x \left(\frac{d}{dx} y(x)\right) + \left(x^2 - \frac{1}{4}\right) y(x) = 0 \quad \textbf{(21)}$$

> $EcuacionNoHom := lhs(EcuacionHom) = x\cdot\cdot\left(\frac{3}{2}\right)$

$$EcuacionNoHom := x^2 \left(\frac{d^2}{dx^2} y(x)\right) + x \left(\frac{d}{dx} y(x)\right) + \left(x^2 - \frac{1}{4}\right) y(x) = x^{3/2} \quad \textbf{(22)}$$

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RESPUESTA 2)

> $SolucionHomogenea := y(x) = C_1\cdot rhs(SolUno) + C_2\cdot rhs(SolDos)$

$$SolucionHomogenea := y(x) = \frac{C_1 \cos(x)}{\sqrt{x}} + \frac{C_2 \sin(x)}{\sqrt{x}} \quad \textbf{(23)}$$

> $Comprobacion_1 := simplify(eval(subs(y(x) = rhs(SolucionHomogenea), EcuacionHom)))$

$$Comprobacion_1 := 0 = 0 \quad \textbf{(24)}$$

> $EcuacionHomNormalizada := expand\left(\frac{lhs(EcuacionHom)}{x\cdot\cdot 2}\right) = \frac{rhs(EcuacionHom)}{x\cdot\cdot 2}$

$$EcuacionHomNormalizada := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} + y(x) - \frac{1}{4} \frac{y(x)}{x^2} = 0 \quad \textbf{(25)}$$

> $EcuacionNoHomNormalizada := expand\left(\frac{lhs(EcuacionNoHom)}{x\cdot\cdot 2}\right) = \frac{rhs(EcuacionNoHom)}{x\cdot\cdot 2}$

$$EcuacionNoHomNormalizada := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} + y(x) - \frac{1}{4} \frac{y(x)}{x^2} = \frac{1}{\sqrt{x}} \quad \textbf{(26)}$$

> $with(linalg):$

> $WW := wronskian([rhs(SolUno), rhs(SolDos)], x)$

$$WW := \begin{bmatrix} \frac{\cos(x)}{\sqrt{x}} & \frac{\sin(x)}{\sqrt{x}} \\ -\frac{1}{2}\frac{\cos(x)}{x^{3/2}} - \frac{\sin(x)}{\sqrt{x}} & -\frac{1}{2}\frac{\sin(x)}{x^{3/2}} + \frac{\cos(x)}{\sqrt{x}} \end{bmatrix} \quad \textbf{(27)}$$

> $BB := array([0, rhs(EcuacionNoHomNormalizada)])$

$$BB := \begin{bmatrix} 0 & \frac{1}{\sqrt{x}} \end{bmatrix} \quad \textbf{(28)}$$

> $Parametro := simplify(linsolve(WW, BB))$

$$Parametro := \begin{bmatrix} -\sin(x) & \cos(x) \end{bmatrix} \quad (29)$$

> $Aprima := Parametro_1; Bprima := Parametro_2$

$$Aprima := -\sin(x)$$
$$Bprima := \cos(x) \quad (30)$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2$

$$A := \cos(x) + C_1$$
$$B := \sin(x) + C_2 \quad (31)$$

> $SolucionFinal := y(x) = simplify(expand(A \cdot rhs(SolUno) + B \cdot rhs(SolDos)))$

$$SolucionFinal := y(x) = \frac{1 + C_1 \cos(x) + C_2 \sin(x)}{\sqrt{x}} \quad (32)$$

> $EcuacionNoHom$

$$x^2 \left(\frac{d^2}{dx^2} y(x)\right) + x \left(\frac{d}{dx} y(x)\right) + \left(x^2 - \frac{1}{4}\right) y(x) = x^{3/2} \quad (33)$$

> $Comprobacion_2 := simplify(eval(subs(y(x) = rhs(SolucionFinal), lhs(EcuacionNoHom) - rhs(EcuacionNoHom) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad (34)$$

> $SolucionComprobatoria := dsolve(EcuacionNoHom)$

$$SolucionComprobatoria := y(x) = \frac{\sin(x)\ \_C2}{\sqrt{x}} + \frac{\cos(x)\ \_C1}{\sqrt{x}} + \frac{1}{\sqrt{x}} \quad (35)$$

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FIN RESPUESTA 2)

> *restart*

3) Resolver

> $Ecuacion := y'' + y' - 2\, y = x + 1$

$$Ecuacion := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) - 2\, y(x) = x + 1 \quad (36)$$

> $Solucion := dsolve(Ecuacion)$

$$Solucion := y(x) = e^{x}\ \_C2 + e^{-2x}\ \_C1 - \frac{3}{4} - \frac{1}{2} x \quad (37)$$

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FIN RESPUESTA 3)

> *restart*

4) Determinar y(t) para

> $Sistema := diff(x(t), t) - x(t) + y(t) = -\sin(t), diff(y(t), t) + x(t) - y(t) = \cos(t) :$
> $Sistema_1; Sistema_2$

$$\frac{d}{dt} x(t) - x(t) + y(t) = -\sin(t)$$
$$\frac{d}{dt} y(t) + x(t) - y(t) = \cos(t) \quad (38)$$

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RESPUESTA 4)

> $Solucion := dsolve(\{Sistema\}) : Solucion_2$

$$y(t) = \frac{1}{5}\cos(t) + \frac{2}{5}\sin(t) - \frac{1}{2}e^{2t}\_C1 + \_C2 \quad (39)$$

> $Comprobacion_1 := simplify(eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2), lhs(Sistema_1) - rhs(Sistema_1) = 0)))$

$$Comprobacion_1 := 0 = 0 \quad (40)$$

> $Comprobacion_2 := simplify(eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2), lhs(Sistema_2) - rhs(Sistema_2) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad (41)$$

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FIN RESPUESTA 4)

> *restart*

5) Resolver

> $Ecuacion := diff(y(t), t\$2) + 2\cdot diff(y(t), t) + y(t) = \mathrm{Dirac}(t)$

$$Ecuacion := \frac{d^2}{dt^2}y(t) + 2\left(\frac{d}{dt}y(t)\right) + y(t) = \mathrm{Dirac}(t) \quad (42)$$

> $Condiciones := y(0) = 0, \mathrm{D}(y)(0) = 0$

$$Condiciones := y(0) = 0, \mathrm{D}(y)(0) = 0 \quad (43)$$

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RESPUESTA 5)

> $with(inttrans) :$

> $TransLapEcuacion := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s^2\, laplace(y(t), t, s) + 2\, s\, laplace(y(t), t, s) + laplace(y(t), t, s) = 1 \quad (44)$$

> $TransLapSolucion := isolate(TransLapEcuacion, laplace(y(t), t, s))$

$$TransLapSolucion := laplace(y(t), t, s) = \frac{1}{s^2 + 2\, s + 1} \quad (45)$$

> $SolucionParticular := invlaplace(TransLapSolucion, s, t)$

$$SolucionParticular := y(t) = t\, e^{-t} \quad (46)$$

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FIN RESPUESTA 5)

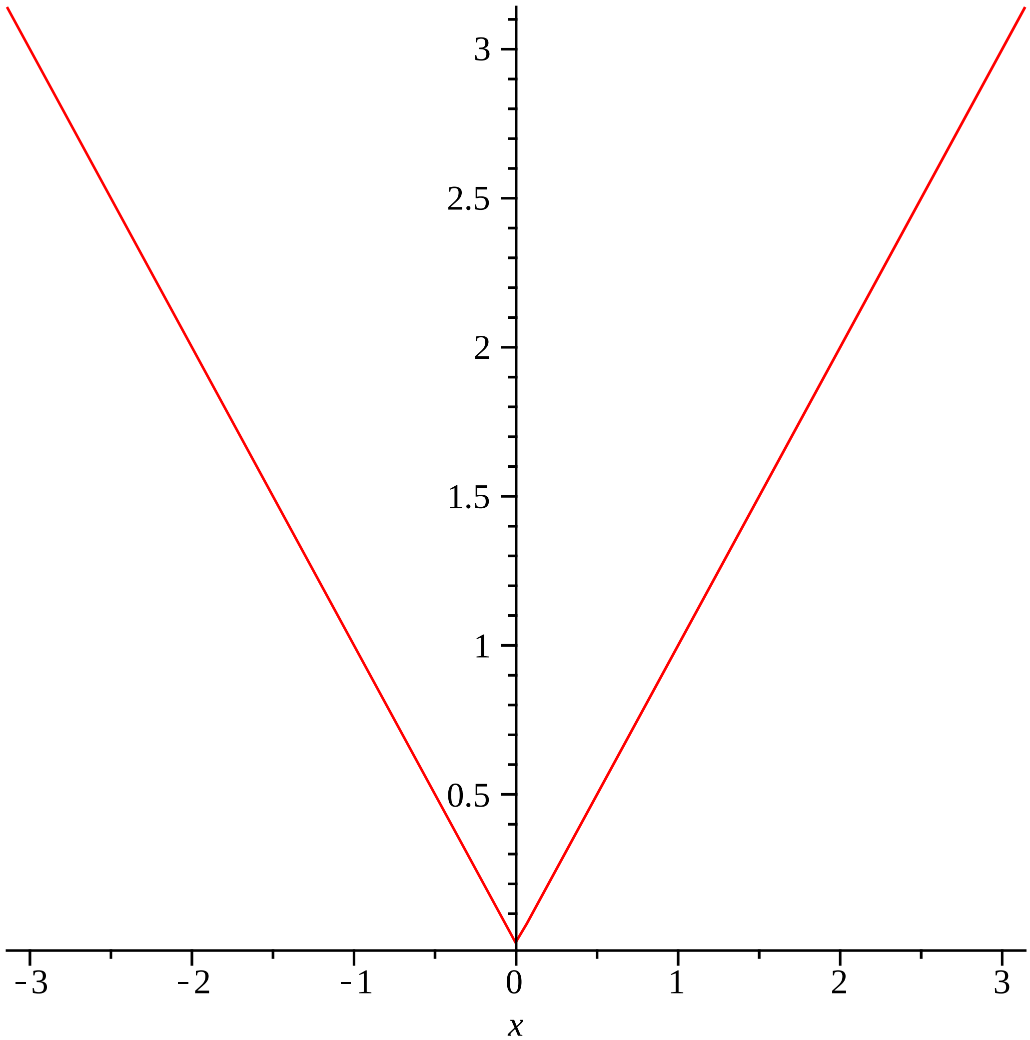
> *restart*

6) Desarrollar en serie trigonométrica de Fourier

> $f := \mathrm{abs}(x)$

$$f := |x| \quad (47)$$

> $plot(f, x = -\mathrm{Pi}..\mathrm{Pi})$



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RESPUESTA 6)

> $L := \text{Pi}$

$$L := \pi \quad \textbf{(48)}$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(f, x = -L..L)$

$$a_0 := \pi \quad \textbf{(49)}$$

> $C := \frac{a_0}{2}$

$$C := \frac{1}{2}\pi \quad \textbf{(50)}$$

> $a_n := subs\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^{\cdot \cdot} n, \left(\frac{1}{L}\right) \cdot int\left(f \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

$$a_n := \frac{2\left((-1)^n - 1\right)}{\pi n^2} \quad \textbf{(51)}$$

> $b_n := subs\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^{\cdot \cdot} n, \left(\frac{1}{L}\right) \cdot int\left(f \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

(52)

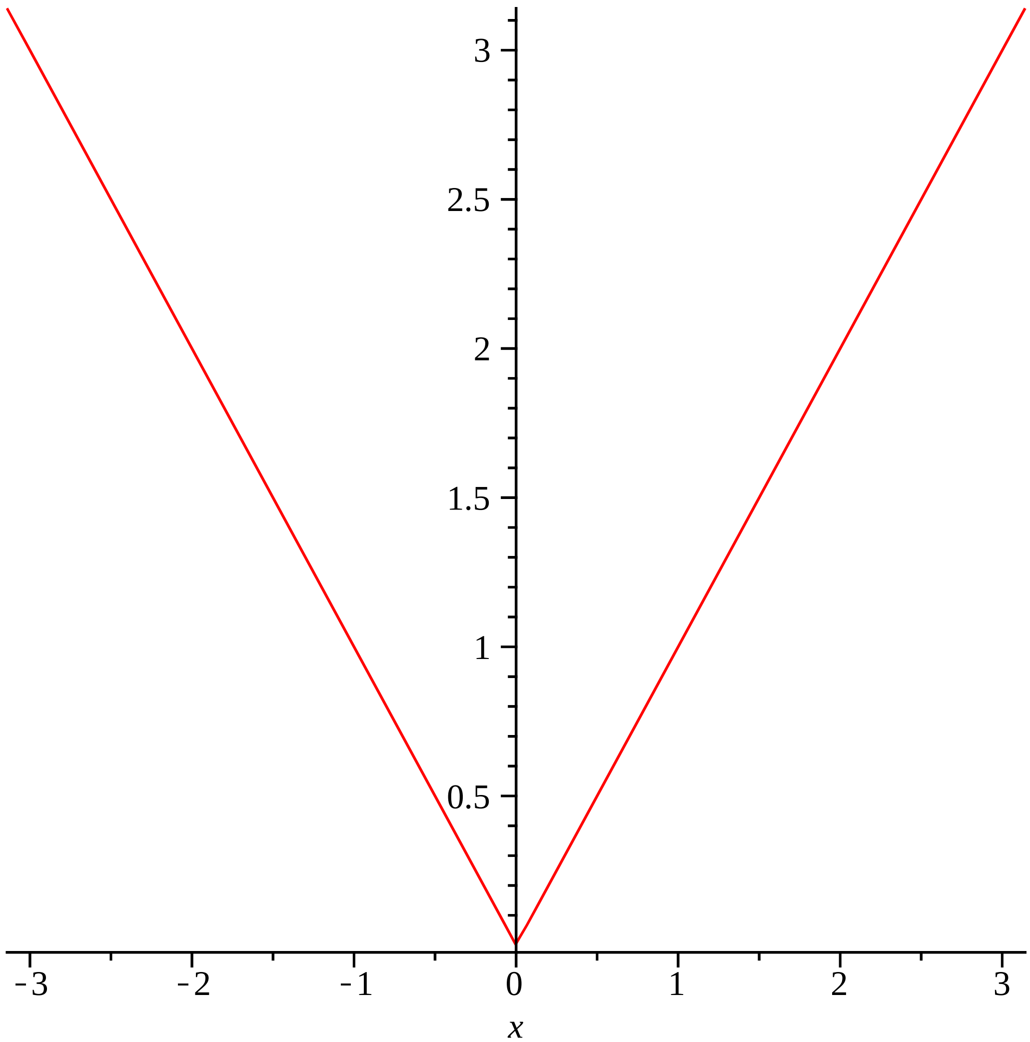
$$b_n := 0 \quad \textbf{(52)}$$

> $STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1..infinity\right)$

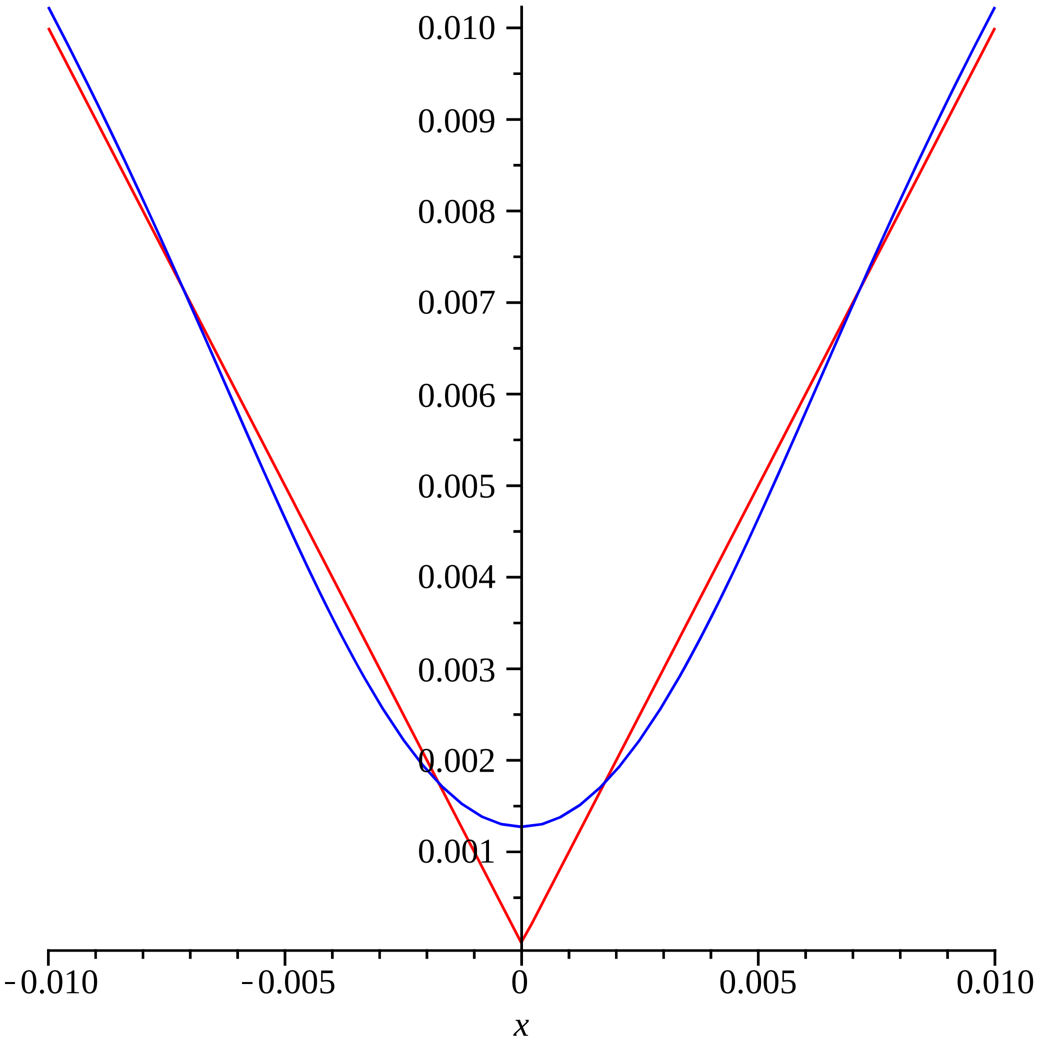
$$STF := \frac{1}{2}\pi + \sum_{n=1}^{\infty} \frac{2\left((-1)^n - 1\right)\cos(n\,x)}{\pi\, n^2} \quad \textbf{(53)}$$

> $STF_{500} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1..500\right) :$

> $plot(STF_{500}, x = -\text{Pi}..\text{Pi})$



> $plot([f, STF_{500}], x = -0.01..0.01, color = [red, blue])$



FIN RESPUESTA 6)

> *restart*

FIN EXAMEN

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