> 

SOLUCIÓN

ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL
SEMESTRE 2014-1 (tipo B)

NOVIEMBRE 26 DE 2013

> *restart*

1) Resolver

> $Ecuacion := x \cdot \exp(x + y(x)) \cdot diff(y(x), x) + y(x) \cdot \exp(x + y(x)) = 0$

$$Ecuacion := x\, e^{x + y(x)} \left( \frac{d}{dx}\, y(x) \right) + y(x)\, e^{x + y(x)} = 0 \qquad \textbf{(1)}$$
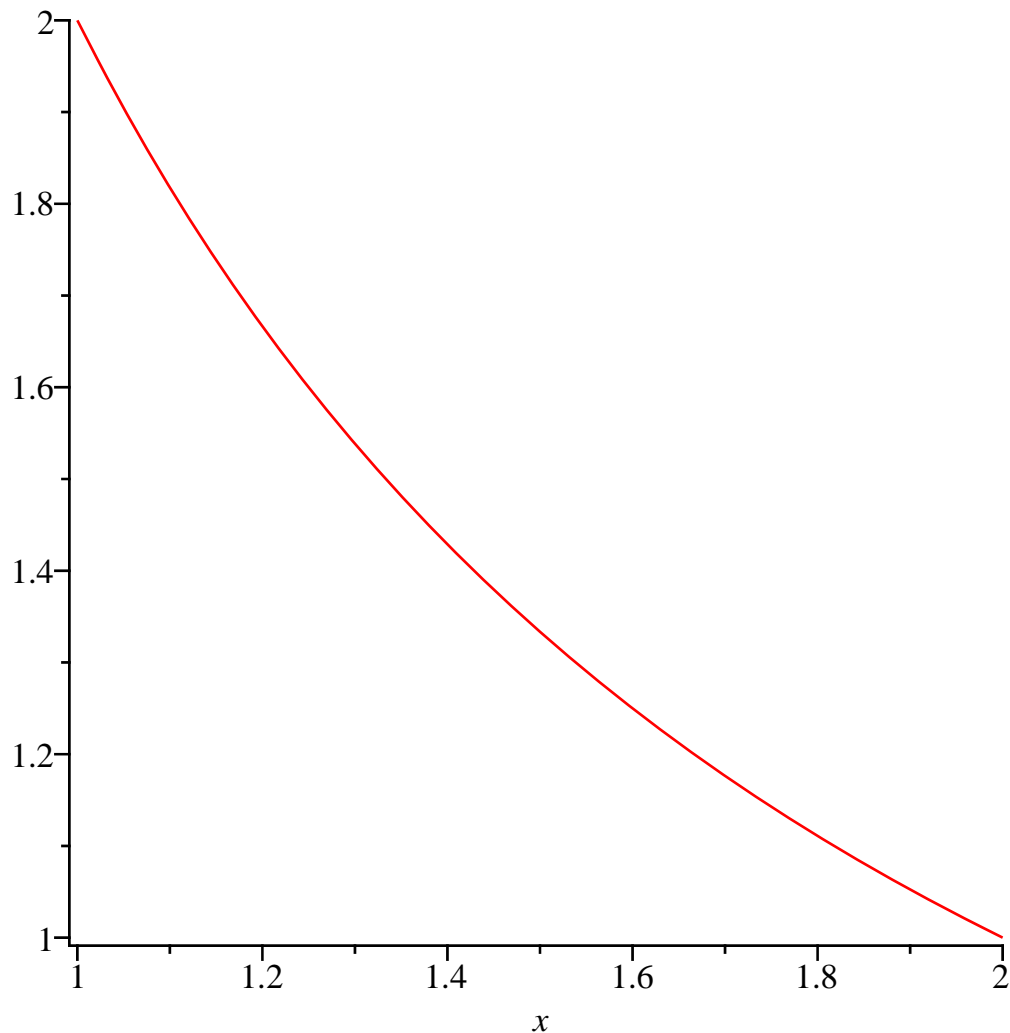
> $Condicion := y(1) = 2;$

$$Condicion := y(1) = 2 \qquad \textbf{(2)}$$

**RESPUESTA 1)**

> $Solucion := dsolve(\{Ecuacion, Condicion\})$

$$Solucion := y(x) = \frac{2}{x} \qquad \textbf{(3)}$$

> $plot(rhs(Solucion), x = 1..2)$



>

MÉTODO DOS

> *Ecuacion*

$$x\, e^{x+y(x)} \left( \frac{d}{dx}\, y(x) \right) + y(x)\, e^{x+y(x)} = 0 \tag{4}$$

> *with*(*DEtools*) :
> *odeadvisor*(*Ecuacion*)

$$[\_separable] \tag{5}$$

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Por variables separables

> $M := y \cdot \exp(x) \cdot \exp(y); N := x \cdot \exp(x) \cdot \exp(y)$

$$M := e^x\, e^y\, y$$

$$N := e^x\, e^y\, x \tag{6}$$

> $P := \exp(x); Q := y \cdot \exp(y); R := x \cdot \exp(x); S := \exp(y)$

$$P := e^x$$

$$Q := y\, e^y$$

$$R := x\, e^x$$

$$S := e^y \tag{7}$$

> $Solucion := int\left( \dfrac{P}{R}, x \right) + int\left( \dfrac{S}{Q}, y \right) = C_1$

$$Solucion := \ln(x) + \ln(y) = C_1 \tag{8}$$

> $SolucionGeneral := simplify(\exp(lhs(Solucion))) = C_1$

$$SolucionGeneral := x\, y = C_1 \tag{9}$$

> $Parametro := subs(x = 1, y = 2, SolucionGeneral)$

$$Parametro := 2 = C_1 \tag{10}$$
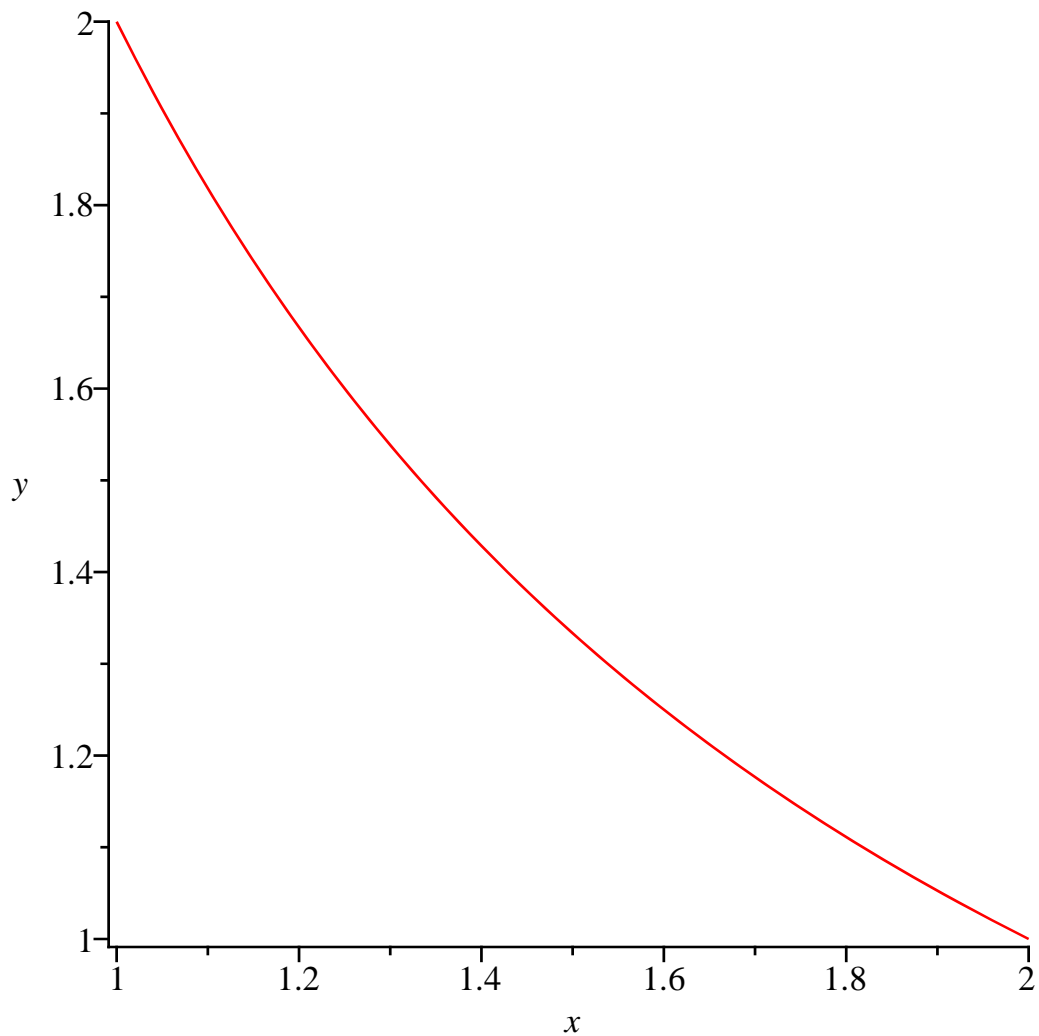
> $SolucionParticular := subs\left( C_1 = lhs(Parametro), SolucionGeneral \right)$

$$SolucionParticular := x\, y = 2 \tag{11}$$

> *with*(*plots*) :
> *implicitplot*(*SolucionParticular*, $x = 1 ..2, y = 1 ..2$)

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Por factor integrante

> $intfactor(Ecuacion)$

$$\frac{1}{e^{x+y(x)}}, \frac{1}{y(x)\, e^{x+y(x)}\, x} \tag{12}$$

> $FactInt := \dfrac{1}{e^{x+y}}$

$$FactInt := \frac{1}{e^{x+y}} \tag{13}$$

> $M; N;$

$$e^x\, e^y\, y$$
$$e^x\, e^y\, x \tag{14}$$

> $MM := simplify(FactInt \cdot M); NN := simplify(FactInt \cdot N)$
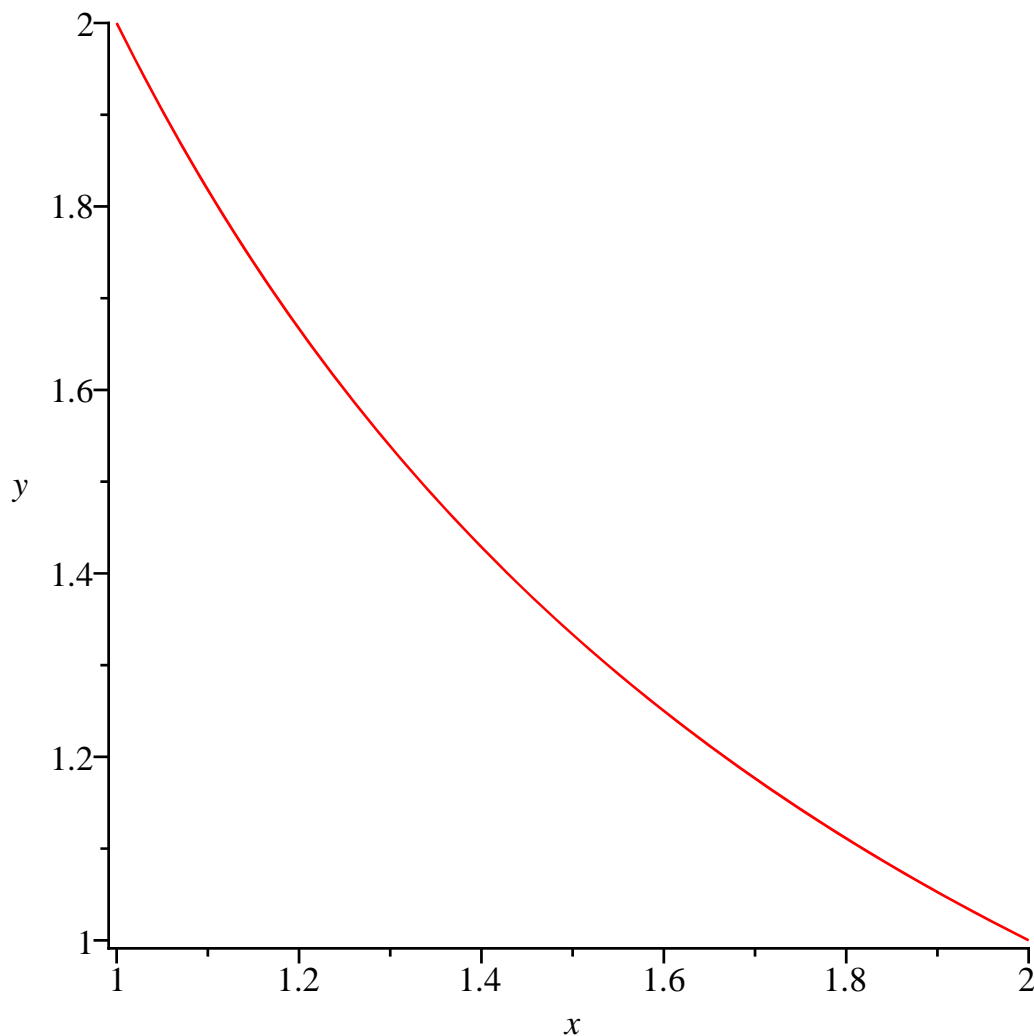
$$MM := y$$
$$NN := x \tag{15}$$

> $SolucionGralDos := int(MM, x) + int((NN - diff(int(MM, x), y)), y) = C_1$

$$SolucionGralDos := x\, y = C_1 \tag{16}$$

> 
> $ParametroDos := subs(x = 1, y = 2, SolucionGralDos)$
$$ParametroDos := 2 = C_1 \tag{17}$$

> $SolucionParticularDos := subs\big(C_1 = lhs(ParametroDos), SolucionGralDos\big)$
$$SolucionParticularDos := x\, y = 2 \tag{18}$$

> $with(plots):$
> $implicitplot(SolucionParticularDos, x = 1..2, y = 1..2)$



> 
**FIN RESPUESTA 1)**
> $restart$
2) las funciones
> $SolUno := y(x) = x \cdot \left( -\dfrac{1}{2} \right) \cdot \cos(x); \ SolDos := y(x) = x \cdot \left( -\dfrac{1}{2} \right) \cdot \sin(x)$
$$SolUno := y(x) = \frac{\cos(x)}{\sqrt{x}}$$
$$SolDos := y(x) = \frac{\sin(x)}{\sqrt{x}} \tag{19}$$

> $EcuacionHom := x^{..}2 \cdot y'' + x \cdot y' + \left( x^{..}2 - \dfrac{1}{4} \right) \cdot y = 0$

$$EcuacionHom := x^2 \left( \frac{d^2}{dx^2}\, y(x) \right) + x \left( \frac{d}{dx}\, y(x) \right) + \left( x^2 - \frac{1}{4} \right) y(x) = 0 \tag{20}$$

> $EcuacionNoHom := lhs(EcuacionHom) = x^{..}\left( \dfrac{3}{2} \right)$

$$EcuacionNoHom := x^2 \left( \frac{d^2}{dx^2}\, y(x) \right) + x \left( \frac{d}{dx}\, y(x) \right) + \left( x^2 - \frac{1}{4} \right) y(x) = x^{3/2} \tag{21}$$

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**RESPUESTA 2)**

> $SolucionHomogenea := y(x) = C_1 \cdot rhs(SolUno) + C_2 \cdot rhs(SolDos)$

$$SolucionHomogenea := y(x) = \frac{C_1 \cos(x)}{\sqrt{x}} + \frac{C_2 \sin(x)}{\sqrt{x}} \tag{22}$$

> $Comprobacion_1 := simplify(eval(subs(y(x) = rhs(SolucionHomogenea), EcuacionHom)))$

$$Comprobacion_1 := 0 = 0 \tag{23}$$

> $EcuacionHomNormalizada := expand\left( \dfrac{lhs(EcuacionHom)}{x^{..}2} \right) = \dfrac{rhs(EcuacionHom)}{x^{..}2}$

$$EcuacionHomNormalizada := \frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} + y(x) - \frac{1}{4}\, \frac{y(x)}{x^2} = 0 \tag{24}$$

> $EcuacionNoHomNormalizada := expand\left( \dfrac{lhs(EcuacionNoHom)}{x^{..}2} \right) = \dfrac{rhs(EcuacionNoHom)}{x^{..}2}$

$$EcuacionNoHomNormalizada := \frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} + y(x) - \frac{1}{4}\, \frac{y(x)}{x^2} = \frac{1}{\sqrt{x}} \tag{25}$$

> $with(linalg) :$
> $WW := wronskian([rhs(SolUno), rhs(SolDos)], x)$

$$WW := \begin{bmatrix} \dfrac{\cos(x)}{\sqrt{x}} & \dfrac{\sin(x)}{\sqrt{x}} \\ -\dfrac{1}{2}\, \dfrac{\cos(x)}{x^{3/2}} - \dfrac{\sin(x)}{\sqrt{x}} & -\dfrac{1}{2}\, \dfrac{\sin(x)}{x^{3/2}} + \dfrac{\cos(x)}{\sqrt{x}} \end{bmatrix} \tag{26}$$

> $BB := array([0, rhs(EcuacionNoHomNormalizada)])$

$$BB := \begin{bmatrix} 0 & \dfrac{1}{\sqrt{x}} \end{bmatrix} \tag{27}$$

> $Parametro := simplify(linsolve(WW, BB))$

$$Parametro := \begin{bmatrix} -\sin(x) & \cos(x) \end{bmatrix} \tag{28}$$

> $Aprima := Parametro_1; \quad Bprima := Parametro_2$

$$Aprima := -\sin(x)$$
$$Bprima := \cos(x) \tag{29}$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2$

$$A := \cos(x) + C_1$$

$$B := \sin(x) + C_2 \tag{30}$$

> $SolucionFinal := y(x) = simplify(expand(A \cdot rhs(SolUno) + B \cdot rhs(SolDos)))$

$$SolucionFinal := y(x) = \frac{1 + C_1 \cos(x) + C_2 \sin(x)}{\sqrt{x}} \tag{31}$$

> $EcuacionNoHom$

$$x^2 \left( \frac{d^2}{dx^2} y(x) \right) + x \left( \frac{d}{dx} y(x) \right) + \left( x^2 - \frac{1}{4} \right) y(x) = x^{3/2} \tag{32}$$

> $Comprobacion_2 := simplify(eval(subs(y(x) = rhs(SolucionFinal), lhs(EcuacionNoHom)$
$\quad - rhs(EcuacionNoHom) = 0)))$

$$Comprobacion_2 := 0 = 0 \tag{33}$$

> $SolucionComprobatoria := dsolve(EcuacionNoHom)$

$$SolucionComprobatoria := y(x) = \frac{\sin(x)\_C2}{\sqrt{x}} + \frac{\cos(x)\_C1}{\sqrt{x}} + \frac{1}{\sqrt{x}} \tag{34}$$

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**FIN RESPUESTA 2)**

> $restart$

3) Resolver

> $Ecuacion := y'' - y' - 2\, y = x - 1$

$$Ecuacion := \frac{d^2}{dx^2} y(x) - \left( \frac{d}{dx} y(x) \right) - 2\, y(x) = x - 1 \tag{35}$$

> $Solucion := dsolve(Ecuacion)$

$$Solucion := y(x) = e^{2x}\_C2 + e^{-x}\_C1 + \frac{3}{4} - \frac{1}{2}\, x \tag{36}$$

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FIN RESPUESTA 3)

> $restart$

4) Determinar y(t) para

> $Sistema := diff(x(t), t) - x(t) + y(t) = -\sin(t), diff(y(t), t) + x(t) - y(t) = \cos(t) :$
$\quad Sistema_1; Sistema_2$

$$\frac{d}{dt} x(t) - x(t) + y(t) = -\sin(t)$$

$$\frac{d}{dt} y(t) + x(t) - y(t) = \cos(t) \tag{37}$$

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**RESPUESTA 4)**

> $Solucion := dsolve(\{Sistema\}) : Solucion_1$

$$x(t) = \frac{3}{5} \sin(t) + \frac{4}{5} \cos(t) + \frac{1}{2}\, e^{2t}\_C1 + \_C2 \tag{38}$$

> $Comprobacion_1 := simplify(eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2),$

$$lhs\left(Sistema_1\right) - rhs\left(Sistema_1\right) = 0\,\bigr)\,\bigr)\,\bigr)$$

$$Comprobacion_1 := 0 = 0 \tag{39}$$

> $Comprobacion_2 := simplify\bigl(eval\bigl(subs\bigl(x(t) = rhs\left(Solucion_1\right), y(t) = rhs\left(Solucion_2\right),$
  $lhs\left(Sistema_2\right) - rhs\left(Sistema_2\right) = 0\,\bigr)\,\bigr)\,\bigr)$

$$Comprobacion_2 := 0 = 0 \tag{40}$$

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**FIN RESPUESTA 4)**

> *restart*

5) Resolver

> $Ecuacion := diff(y(t), t\$2) - 2 \cdot diff(y(t), t) + y(t) = \mathrm{Dirac}(t)$

$$Ecuacion := \frac{d^2}{dt^2}\,y(t) - 2\left(\frac{d}{dt}\,y(t)\right) + y(t) = \mathrm{Dirac}(t) \tag{41}$$

> $Condiciones := y(0) = 0, \mathrm{D}(y)(0) = 0$

$$Condiciones := y(0) = 0, \mathrm{D}(y)(0) = 0 \tag{42}$$

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**RESPUESTA 5)**

> $with(inttrans):$

> $TransLapEcuacion := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s^2\,laplace(y(t), t, s) - 2\,s\,laplace(y(t), t, s) + laplace(y(t), t, s) = 1 \tag{43}$$

> $TransLapSolucion := isolate(TransLapEcuacion, laplace(y(t), t, s))$

$$TransLapSolucion := laplace(y(t), t, s) = \frac{1}{s^2 - 2\,s + 1} \tag{44}$$

> $SolucionParticular := invlaplace(TransLapSolucion, s, t)$

$$SolucionParticular := y(t) = t\,e^{t} \tag{45}$$

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**FIN RESPUESTA 5)**

> *restart*

6) Desarrollar en serie trigonométrica de Fourier

> $f := abs(x)$

$$f := |x| \tag{46}$$

> $plot(f, x = -\mathrm{Pi}\,..\,\mathrm{Pi})$

>

**RESPUESTA 6)**

> $L := \mathrm{Pi}$
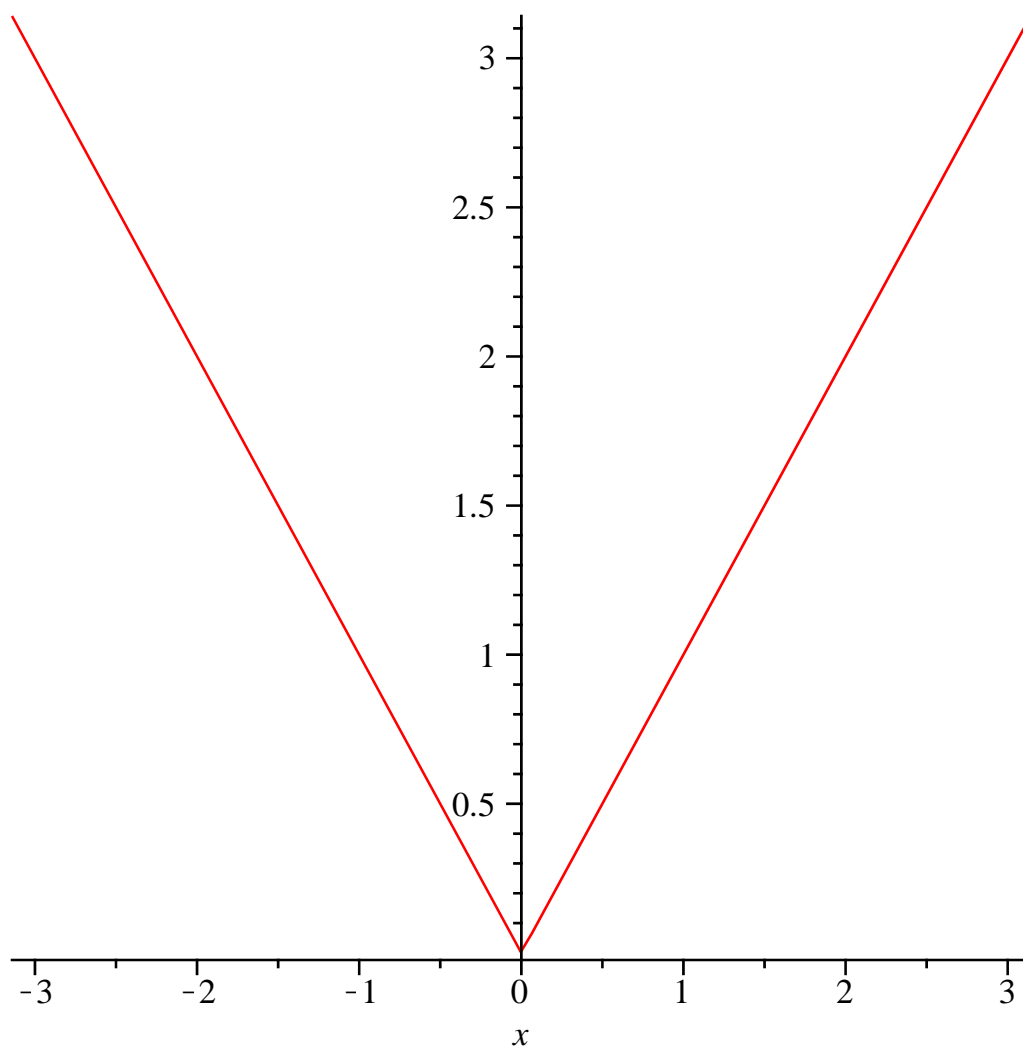
$$L := \pi \tag{47}$$

> $a_0 := \left(\dfrac{1}{L}\right) \cdot int(f, x = -L..L)$

$$a_0 := \pi \tag{48}$$

> $C := \dfrac{a_0}{2}$

$$C := \dfrac{1}{2}\,\pi \tag{49}$$

> $a_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0,\ \cos(n \cdot \mathrm{Pi}) = (-1) \cdot\cdot n,\ \left(\dfrac{1}{L}\right) \cdot int\left(f \cdot \cos\left(\dfrac{n \cdot \mathrm{Pi} \cdot x}{L}\right),\ x = -L..L\right)\right)$

$$a_n := \dfrac{2\left((-1)^n - 1\right)}{\pi\,n^2} \tag{50}$$

> $b_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0,\ \cos(n \cdot \mathrm{Pi}) = (-1) \cdot\cdot n,\ \left(\dfrac{1}{L}\right) \cdot int\left(f \cdot \sin\left(\dfrac{n \cdot \mathrm{Pi} \cdot x}{L}\right),\ x = -L..L\right)\right)$
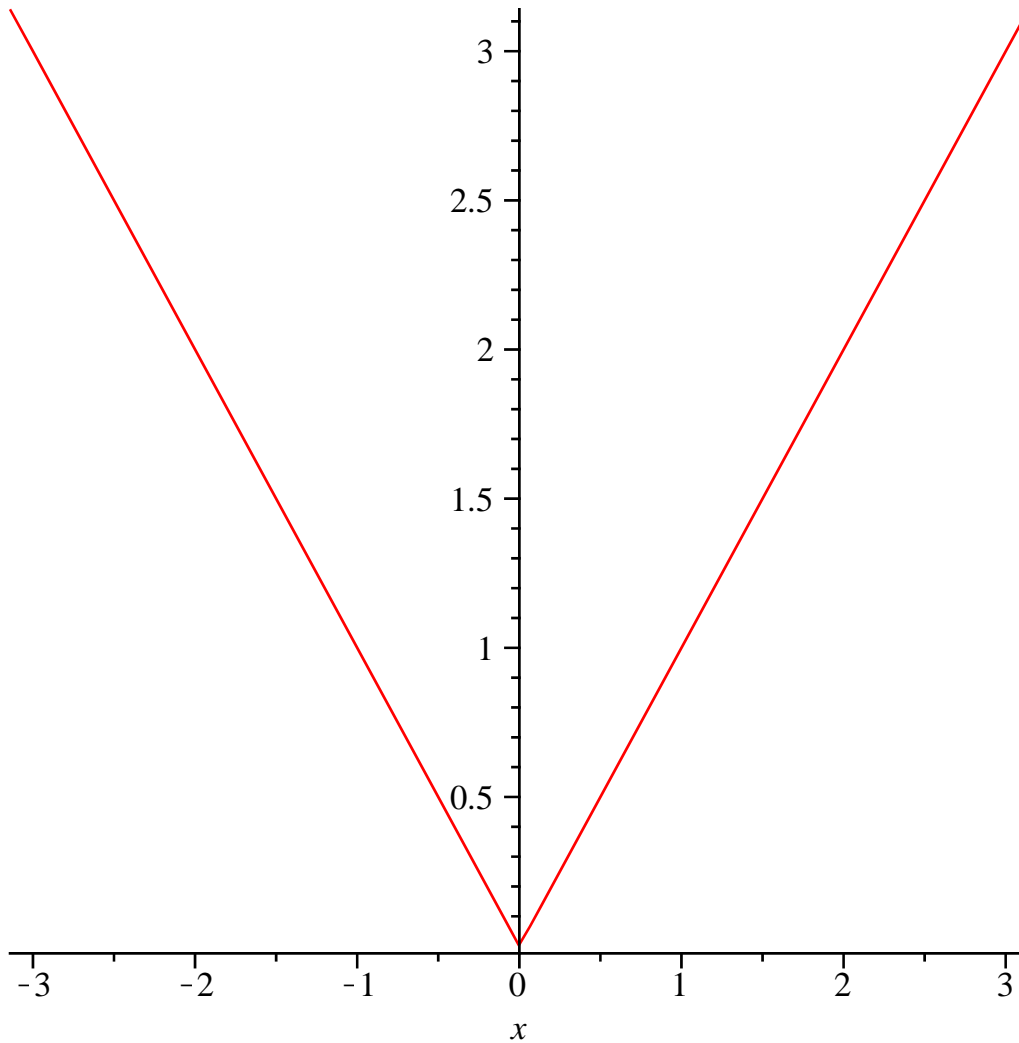
(51)

$$b_n := 0 \tag{51}$$

> $STF := C + Sum\left(a_n \cdot \cos\left(\dfrac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1 \,..\, infinity\right)$

$$STF := \frac{1}{2}\,\pi + \sum_{n=1}^{\infty} \frac{2\left((-1)^n - 1\right)\cos(n\,x)}{\pi\,n^2} \tag{52}$$

> $STF_{500} := C + sum\left(a_n \cdot \cos\left(\dfrac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1 \,..\, 500\right) :$

> $plot\left(STF_{500}, x = -\mathrm{Pi}\,..\,\mathrm{Pi}\right)$



> $plot\left(\left[f, STF_{500}\right], x = -0.01\,..\,0.01, color = [red, blue]\right)$
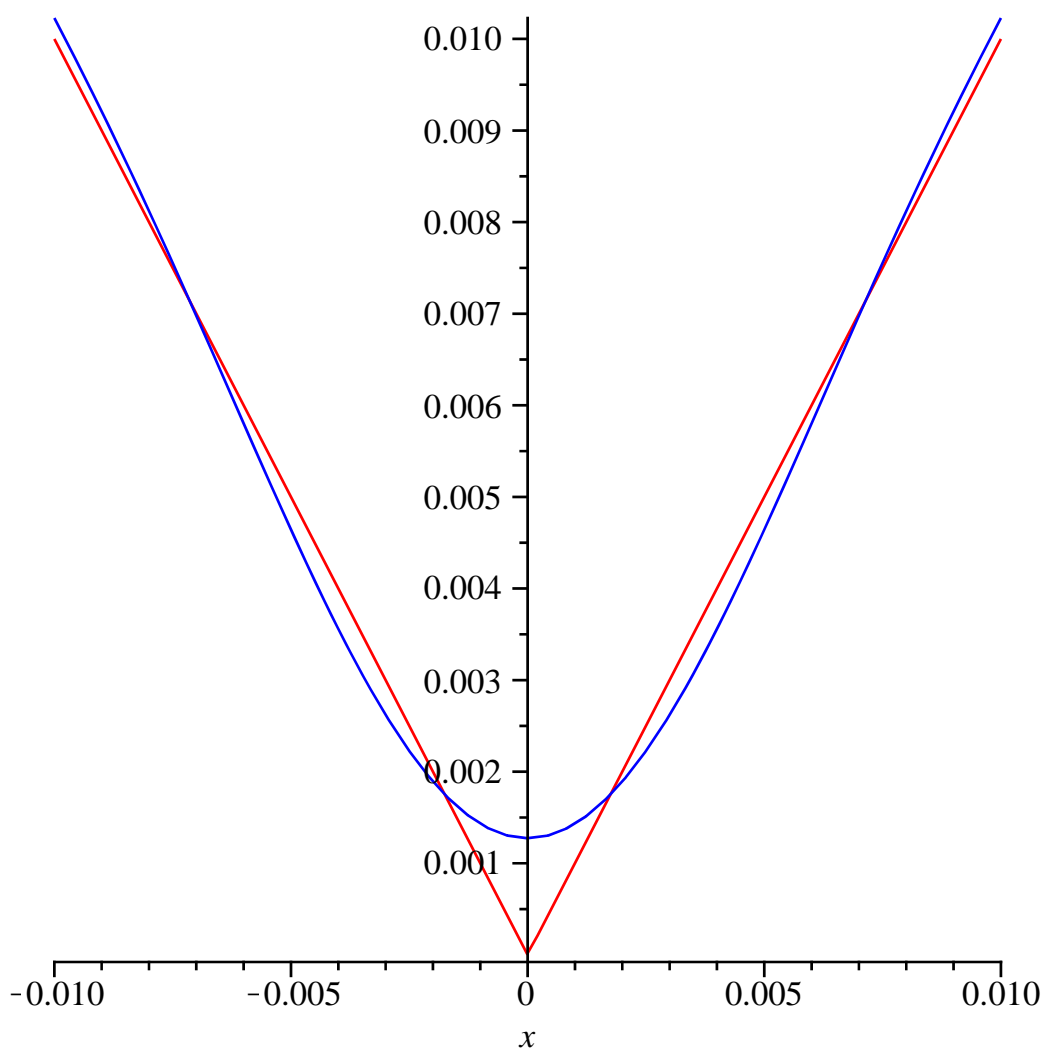
**FIN RESPUESTA 6)**

> *restart*

**FIN EXAMEN**

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