>

SOLUCIÓN

ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL COLEGIADO
semestre 2014-1

3 DICIEMBRE 2013

> *restart*

1) Si la familia de rectas tangentes a una familia de curvas cortan al eje de las absisas en el punto

> $P\left(\frac{x}{2}, 0\right)$

$$P\left(\frac{1}{2}\,x, 0\right) \tag{1}$$

>

RESPUESTA 1)

> $Ecuacion := diff(y(x), x) = subs\left(y_1 = 0, x_1 = \frac{x}{2}, \frac{(y(x) - y_1)}{(x - x_1)}\right)$

$$Ecuacion := \frac{d}{dx}\,y(x) = \frac{2\,y(x)}{x} \tag{2}$$

> $SolucionGeneral := dsolve(Ecuacion)$

$$SolucionGeneral := y(x) = \_C1\,x^2 \tag{3}$$

> $Condiciones := y(1) = 1$

$$Condiciones := y(1) = 1 \tag{4}$$

> $SolucionParticular := dsolve(\{Ecuacion, Condiciones\})$

$$SolucionParticular := y(x) = x^2 \tag{5}$$

FIN RESPUESTA 1)

> *restart*

2) Resuelva

> $Ecuacion := y'' + 2\,y' + 2\,y = \sin(x)$

$$Ecuacion := \frac{d^2}{dx^2}\,y(x) + 2\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = \sin(x) \tag{6}$$

RESPUESTA 2)

> $Solucion := dsolve(Ecuacion)$

$$Solucion := y(x) = e^{-x}\sin(x)\,\_C2 + e^{-x}\cos(x)\,\_C1 - \frac{2}{5}\cos(x) + \frac{1}{5}\sin(x) \tag{7}$$

FIN RESPUESTA 2)

> *restart*

3) Obtenga y(t)

> $Sistema := diff(x(t), t) - diff(y(t), t) - y(t) = -\exp(t), x(t) + diff(y(t), t) - y(t) = 0 :$
> $Sistema_1; Sistema_2;$

$$\frac{d}{dt}\,x(t) - \left(\frac{d}{dt}\,y(t)\right) - y(t) = -e^t$$

$$x(t) + \frac{d}{dt}\,y(t) - y(t) = 0 \tag{8}$$

RESPUESTA 3)

> $Solucion := dsolve(\{Sistema\}) : Solucion_2; Solucion_1$

$$y(t) = \frac{1}{2}\,\_C1\cos(t) - \frac{1}{2}\,\_C2\sin(t) + \frac{1}{2}\,\_C1\sin(t) + \frac{1}{2}\,\_C2\cos(t) + \frac{1}{2}\,e^{t}$$

$$x(t) = \_C1\sin(t) + \_C2\cos(t) \quad \textbf{(9)}$$

> $SolucionUno := x(t) = C_1\sin(t) + C_2\cos(t); SolucionDos := y(t) = \left(\frac{C_1}{2} + \frac{C_2}{2}\right)\cdot\cos(t) + \left(\frac{C_1}{2} - \frac{C_2}{2}\right)\cdot\sin(t) + \frac{\exp(t)}{2}$

$$SolucionUno := x(t) = C_1\sin(t) + C_2\cos(t)$$

$$SolucionDos := y(t) = \left(\frac{1}{2}C_1 + \frac{1}{2}C_2\right)\cos(t) + \left(\frac{1}{2}C_1 - \frac{1}{2}C_2\right)\sin(t) + \frac{1}{2}e^{t} \quad \textbf{(10)}$$

> $Comprobacion_1 := simplify(eval(subs(x(t) = rhs(SolucionUno), y(t) = rhs(SolucionDos), lhs(Sistema_1) - rhs(Sistema_1) = 0)))$

$$Comprobacion_1 := 0 = 0 \quad \textbf{(11)}$$

> $Comprobacion_2 := simplify(eval(subs(x(t) = rhs(SolucionUno), y(t) = rhs(SolucionDos), lhs(Sistema_2) - rhs(Sistema_2) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad \textbf{(12)}$$

FIN RESPUESTA 3)

> *restart*

4)

> $Ecuacion := diff(y(t), t\$2) + y(t) = \mathrm{Dirac}(t - \mathrm{Pi})$

$$Ecuacion := \frac{d^2}{dt^2}\,y(t) + y(t) = \mathrm{Dirac}(t - \pi) \quad \textbf{(13)}$$

> $Condiciones := y(0) = 1, y'(0) = 0$

$$Condiciones := y(0) = 1, \mathrm{D}(y)(0) = 0 \quad \textbf{(14)}$$

RESPUESTA 4)

> $with(inttrans) :$

> $TransLapEcua := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcua := s^2\,laplace(y(t), t, s) - s + laplace(y(t), t, s) = e^{-s\pi} \quad \textbf{(15)}$$

> $TransLapSol := isolate(TransLapEcua, laplace(y(t), t, s))$

$$TransLapSol := laplace(y(t), t, s) = \frac{e^{-s\pi} + s}{s^2 + 1} \quad \textbf{(16)}$$

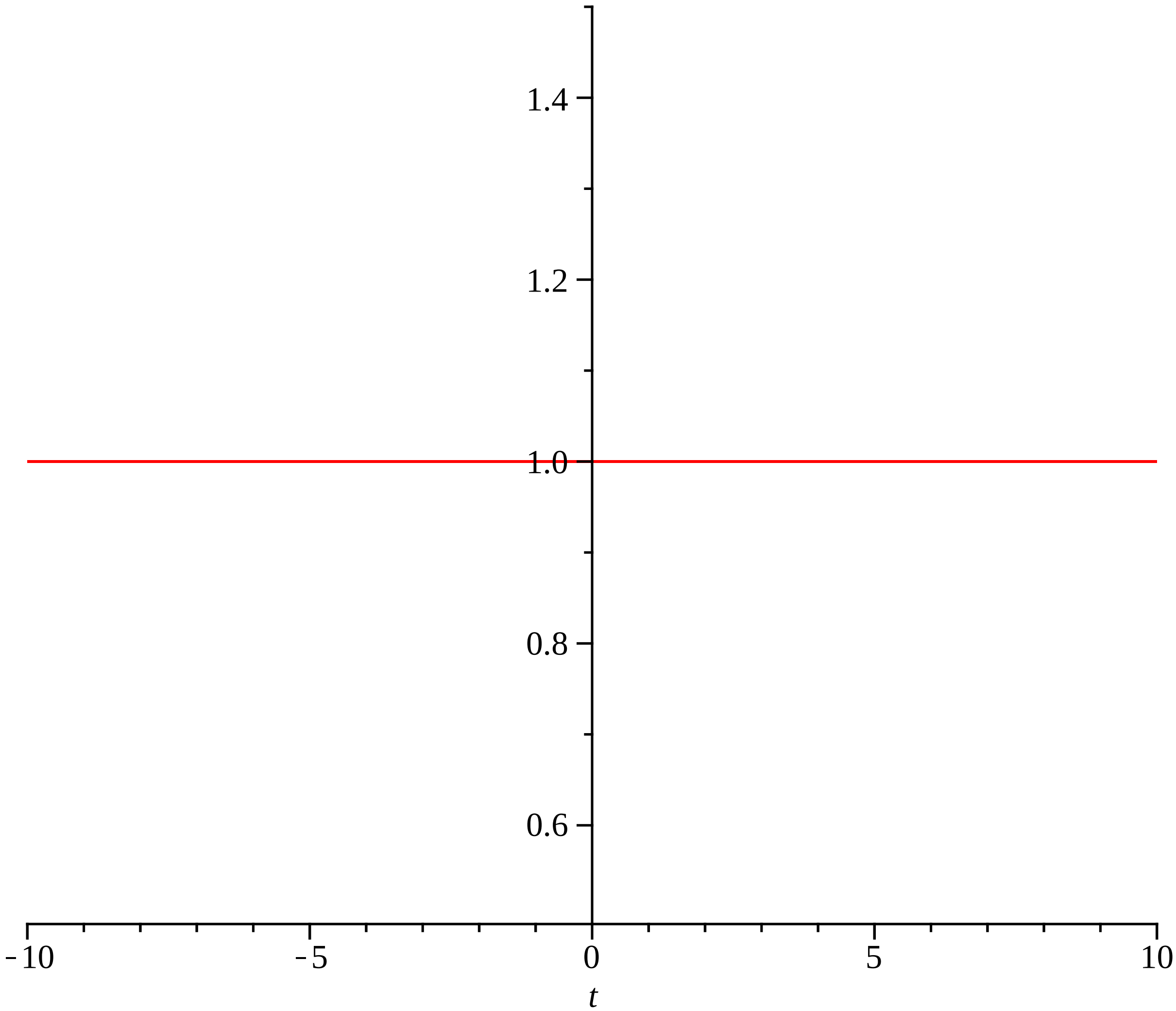
> $Solucion := invlaplace(TransLapSol, s, t)$

$$Solucion := y(t) = -\mathrm{Heaviside}(t - \pi)\sin(t) + \cos(t) \quad \textbf{(17)}$$

> $funcion := simplify(subs(t = 2\cdot\mathrm{Pi}, Solucion))$

$$funcion := y(2\pi) = 1 \quad \textbf{(18)}$$

> $plot(rhs(funcion), t = -10..10)$



FIN RESPUESTA 4)

> *restart*

5) Obtenga la transformada inversa de Laplace

> $F := -\dfrac{\exp(-\mathrm{Pi}\cdot s)}{s\cdot\cdot 2+1} + \dfrac{\exp\left(-\dfrac{\mathrm{Pi}\cdot s}{2}\right)}{s} - \dfrac{\exp(-\mathrm{Pi}\cdot s)}{s}$

$$F := -\frac{e^{-\pi s}}{s^2+1} + \frac{e^{-\frac{1}{2}\pi s}}{s} - \frac{e^{-\pi s}}{s} \qquad \textbf{(19)}$$

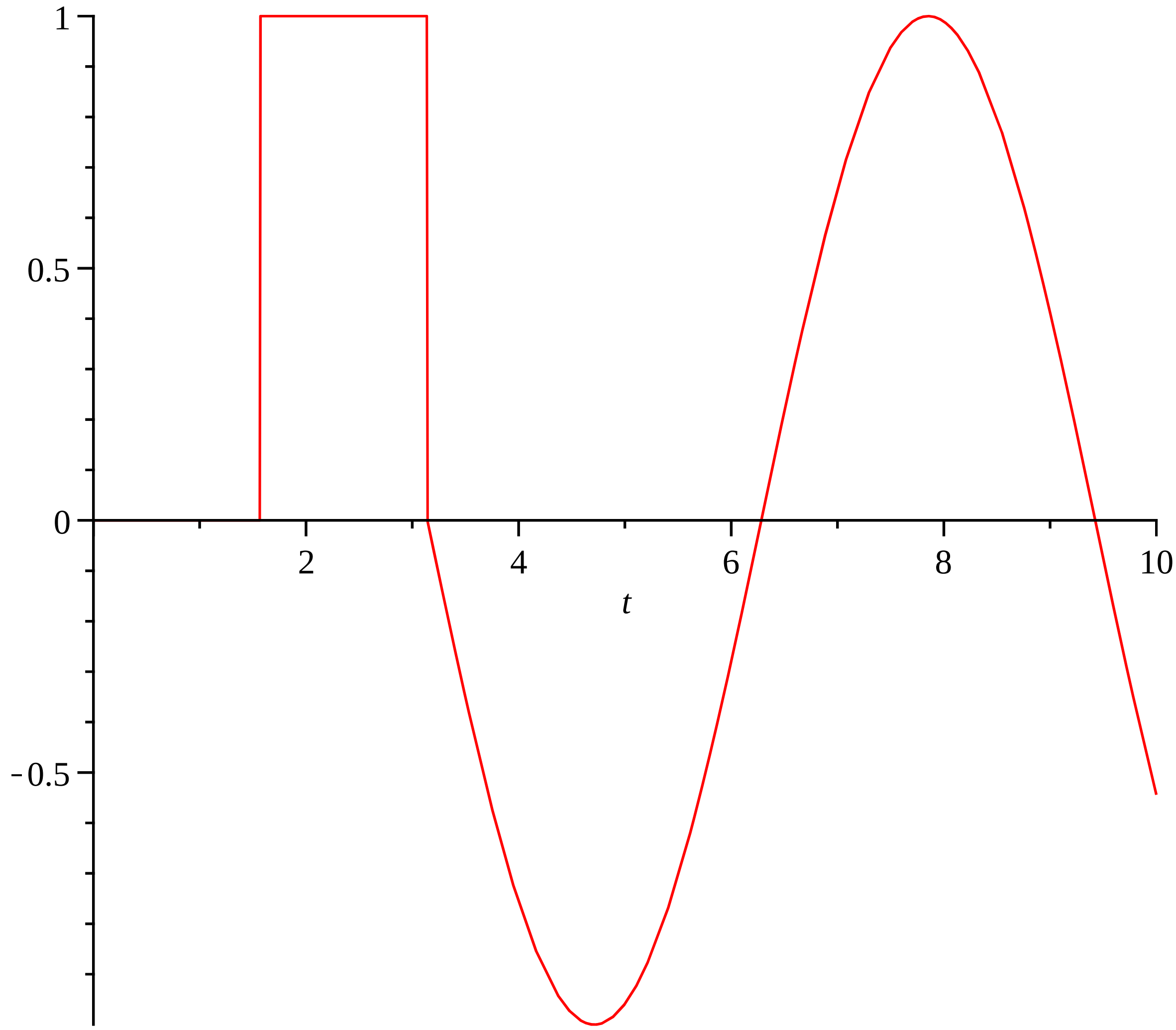
RESPUESTA 5)

> *with*(*inttrans*) :

> $f := invlaplace(F, s, t)$

$$f := \mathrm{Heaviside}\left(t - \frac{1}{2}\pi\right) + \mathrm{Heaviside}(t-\pi)\,(\sin(t) - 1) \qquad \textbf{(20)}$$

> *plot*(*f*, *t* = 0 ..10)



FIN RESPUESTA 5)

> *restart*

6) Resuelva para constante de separación positiva

> $Ecuacion := diff(u(x,t),x) = 2\cdot diff(u(x,t),t) + u(x,t)$

$$Ecuacion := \frac{\partial}{\partial x}\, u(x,t) = 2\left(\frac{\partial}{\partial t}\, u(x,t)\right) + u(x,t) \qquad \textbf{(21)}$$

RESPUESTA 6)

> $EcuacionSep := eval(subs(u(x,t) = F(x)\cdot G(t), Ecuacion))$

$$EcuacionSep := \left(\frac{d}{dx}\, F(x)\right) G(t) = 2\, F(x)\left(\frac{d}{dt}\, G(t)\right) + F(x)\, G(t) \qquad \textbf{(22)}$$

> $EcuacionUno := simplify\left(\frac{lhs(EcuacionSep)}{F(x)\cdot G(t)}\right) = simplify\left(\frac{rhs(EcuacionSep)}{F(x)\cdot G(t)}\right)$

$$EcuacionUno := \frac{\frac{d}{dx}\, F(x)}{F(x)} = \frac{2\left(\frac{d}{dt}\, G(t)\right) + G(t)}{G(t)} \qquad \textbf{(23)}$$

> $EcuacionUnoX := lhs(EcuacionUno) = \mathrm{beta}\cdot\cdot 2;\ EcuacionUnoT := rhs(EcuacionUno) = \mathrm{beta}\cdot\cdot 2$

$$EcuacionUnoX := \frac{\frac{d}{dx} F(x)}{F(x)} = \beta^2$$

$$EcuacionUnoT := \frac{2 \left(\frac{d}{dt} G(t)\right) + G(t)}{G(t)} = \beta^2 \tag{24}$$

> $SolucionUnoX := dsolve(EcuacionUnoX);\ SolucionUnoT := dsolve(EcuacionUnoT)$

$$SolucionUnoX := F(x) = \_C1\, e^{\beta^2 x}$$

$$SolucionUnoT := G(t) = \_C1\, e^{\frac{1}{2} (\beta - 1) (\beta + 1) t} \tag{25}$$

> $SolucionGeneralUno := u(x, t) = rhs(SolucionUnoX) \cdot subs(\_C1 = 1, rhs(SolucionUnoT))$

$$SolucionGeneralUno := u(x, t) = \_C1\, e^{\beta^2 x}\, e^{\frac{1}{2} (\beta - 1) (\beta + 1) t} \tag{26}$$

> $Parametro := simplify(subs(t = 0, rhs(SolucionGeneralUno))) = 6 \cdot \exp(3\, x)$

$$Parametro := \_C1\, e^{\beta^2 x} = 6\, e^{3 x} \tag{27}$$

> $SolucionParticular := simplify(subs(\_C1 = 6, \mathrm{beta} = \mathrm{sqrt}(3), SolucionGeneralUno))$

$$SolucionParticular := u(x, t) = 6\, e^{3 x + t} \tag{28}$$

>

Método alternativo

> $EcuacionDos := simplify\left(\frac{(lhs(EcuacionSep) - F(x) \cdot G(t))}{F(x) \cdot G(t)}\right) = simplify\left(\frac{(rhs(EcuacionSep) - F(x) \cdot G(t))}{F(x) \cdot G(t)}\right)$

$$EcuacionDos := \frac{\frac{d}{dx} F(x) - F(x)}{F(x)} = \frac{2 \left(\frac{d}{dt} G(t)\right)}{G(t)} \tag{29}$$

> $EcuacionDosX := lhs(EcuacionDos) = \mathrm{beta} \cdot \cdot 2;\ EcuacionDosT := rhs(EcuacionDos) = \mathrm{beta} \cdot \cdot 2$

$$EcuacionDosX := \frac{\frac{d}{dx} F(x) - F(x)}{F(x)} = \beta^2$$

$$EcuacionDosT := \frac{2 \left(\frac{d}{dt} G(t)\right)}{G(t)} = \beta^2 \tag{30}$$

> $SolucionDosX := dsolve(EcuacionDosX);\ SolucionDosT := dsolve(EcuacionDosT)$

$$SolucionDosX := F(x) = \_C1\, e^{(\beta^2 + 1) x}$$

$$SolucionDosT := G(t) = \_C1\, e^{\frac{1}{2} \beta^2 t} \tag{31}$$

> $SolucionGeneralDos := u(x, t) = rhs(SolucionDosX) \cdot subs(\_C1 = 1, rhs(SolucionDosT))$

$$SolucionGeneralDos := u(x, t) = \_C1\, e^{(\beta^2 + 1) x}\, e^{\frac{1}{2} \beta^2 t} \tag{32}$$

> $ParametroDos := simplify(subs(t = 0, rhs(SolucionGeneralDos))) = 6 \cdot \exp(3\, x)$

$$ParametroDos := \_C1\, e^{(\beta^2 + 1) x} = 6\, e^{3 x} \tag{33}$$

> $SolucionParticular := simplify(subs(\_C1 = 6, \mathrm{beta} = \mathrm{sqrt}(3 - 1), SolucionGeneralDos))$

$$SolucionParticular := u(x, t) = 6\,\mathrm{e}^{3x + t} \tag{34}$$

FIN RESPUESTA 6)

> *restart*

FIN EXAMEN

> *restart*

>

>

>