> *restart*

SOLUCIÓN

UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE CIENCIAS APLICADAS
DEPARTAMENTO DE ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL

SEMESTRE 2015 -1
DURACIÓN MÁXIMA 2.0 HORAS

TIPO 1
27 DE NOVIEMBRE DE 2014

> *restart*

1. Resolver $xyy' = 3y^2 + x^2$ sujeta a $y(-1) = 2$

>

Respuesta 1)

> $Ecuacion := x \cdot y(x) \cdot y'(x) = 3 \cdot y(x) \cdot \cdot 2 + x \cdot \cdot 2$

$$Ecuacion := x\, y(x) \left(\frac{d}{dx}\, y(x) \right) = 3\, y(x)^2 + x^2 \qquad \textbf{(1)}$$

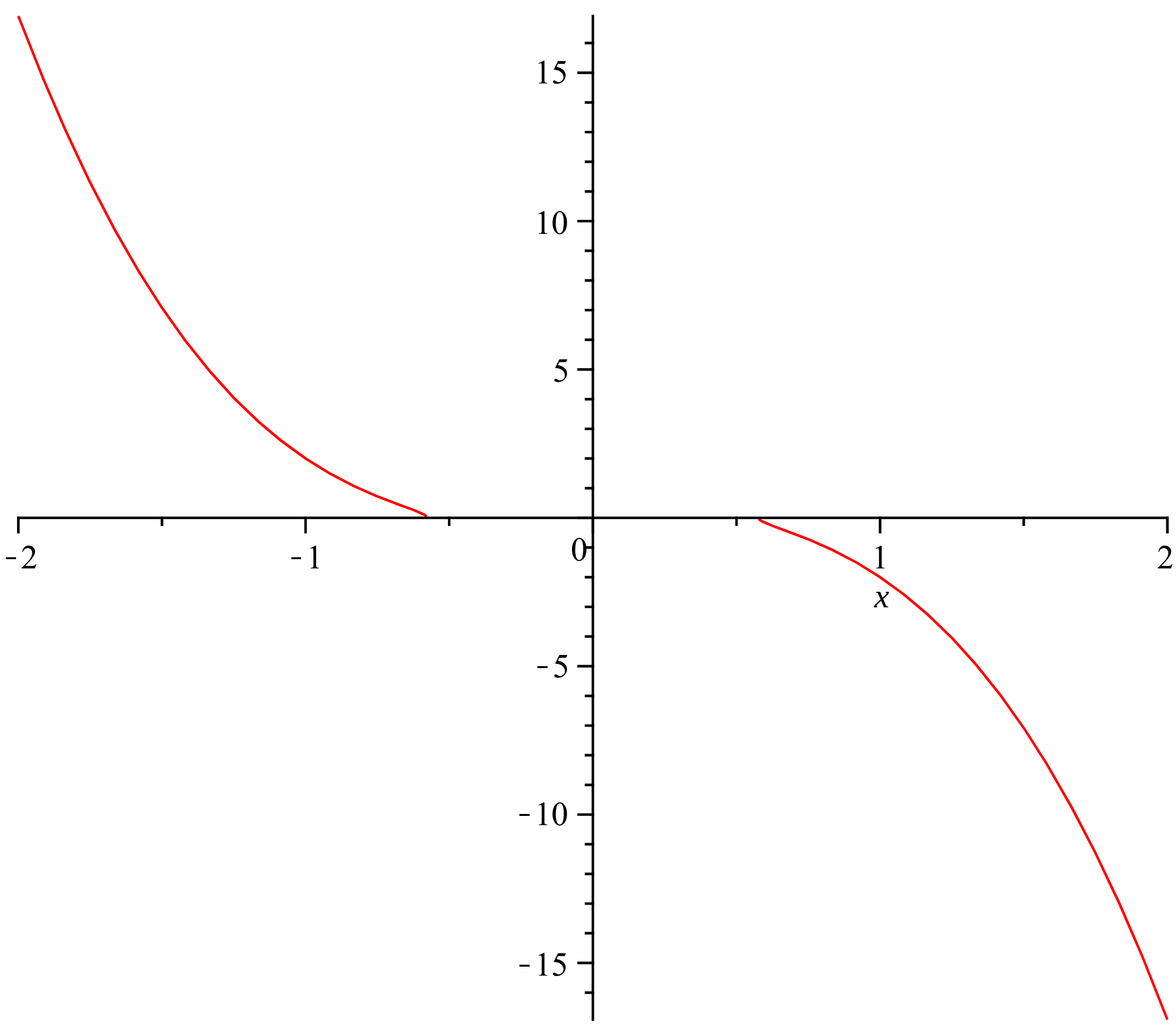
> $Condicion := y(-1) = 2$

$$Condicion := y(-1) = 2 \qquad \textbf{(2)}$$

> $SolucionParticular := dsolve(\{Ecuacion, Condicion\})$

$$SolucionParticular := y(x) = -\frac{1}{2} \sqrt{-2 + 18\, x^4}\; x \qquad \textbf{(3)}$$

> $plot(rhs(SolucionParticular), x = -2 ..2)$



> *Ecuacion*

$$x\,y(x)\left(\frac{d}{dx}\,y(x)\right) = 3\,y(x)^2 + x^2 \quad (4)$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

[[_homogeneous, class A], _rational, _Bernoulli] **(5)**

> *FactInt* := *intfactor*(*Ecuacion*)

$$FactInt := \frac{1}{x^7} \quad (6)$$

>

Por Factor Integrante

> $M := -\left(3\,y^2 + x^2\right)$

$$M := -3\,y^2 - x^2 \quad (7)$$

> $N := x\,y$

$$N := x\,y \quad (8)$$

> $comprobacion_1 := simplify(diff(M, y) - diff(N, x) = 0)$

$$comprobacion_1 := -7\,y = 0 \quad (9)$$

> $MM := expand(M \cdot FactInt)$

$$MM := -\frac{3y^2}{x^7} - \frac{1}{x^5} \quad \textbf{(10)}$$

> $NN := expand(N \cdot FactInt)$

$$NN := \frac{y}{x^6} \quad \textbf{(11)}$$

> $comprobacion_2 := simplify(diff(MM, y) - diff(NN, x) = 0)$

$$comprobacion_2 := 0 = 0 \quad \textbf{(12)}$$

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{1}{2}\frac{y^2}{x^6} + \frac{1}{4x^4} \quad \textbf{(13)}$$

> $SolucionGeneral_2 := 2 \cdot (IntMMx + int((NN - diff(IntMMx, y)), y)) = C_1$

$$SolucionGeneral_2 := \frac{y^2}{x^6} + \frac{1}{2x^4} = C_1 \quad \textbf{(14)}$$

> $Parametro := subs(x = -1, y = rhs(Condicion), SolucionGeneral_2)$

$$Parametro := \frac{9}{2} = C_1 \quad \textbf{(15)}$$

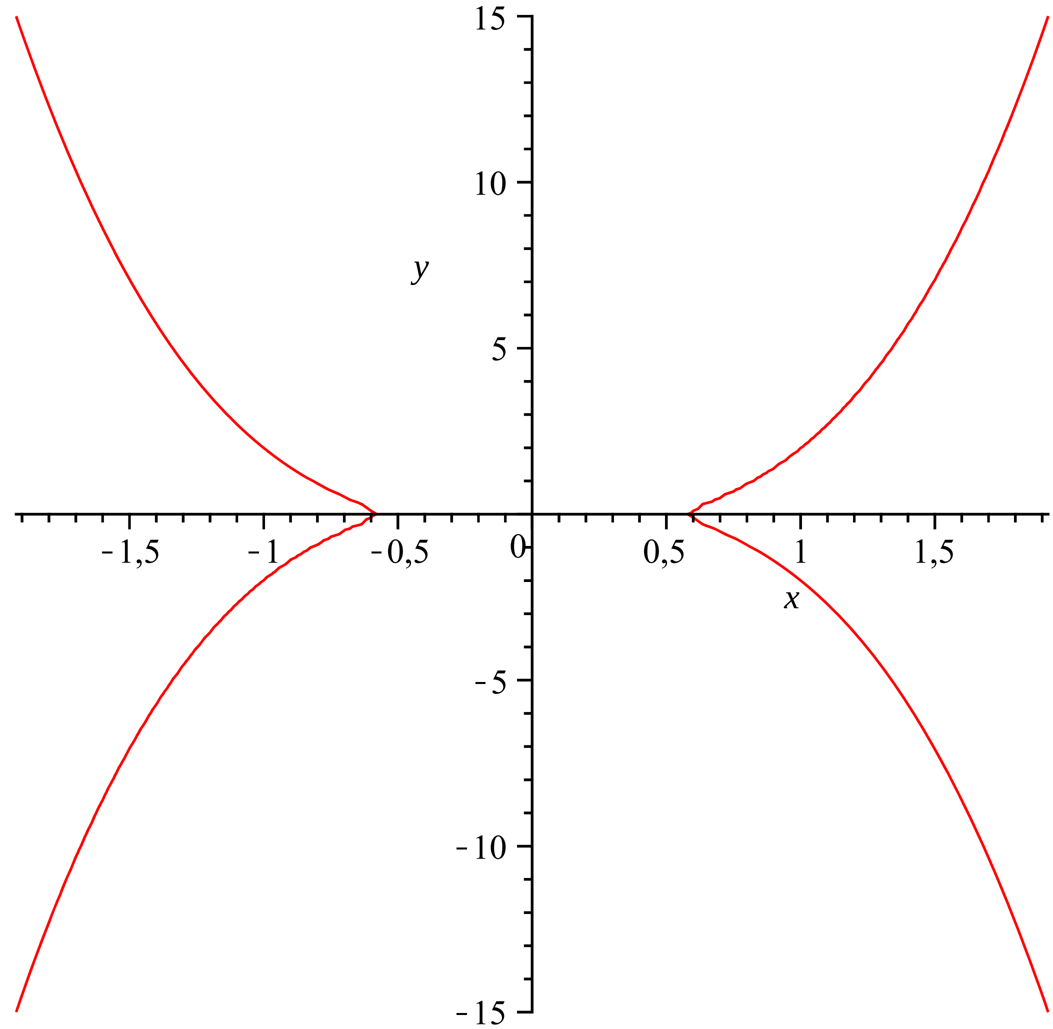
> $SolucionParticular_2 := subs(C_1 = lhs(Parametro), SolucionGeneral_2)$

$$SolucionParticular_2 := \frac{y^2}{x^6} + \frac{1}{2x^4} = \frac{9}{2} \quad \textbf{(16)}$$

> $with(plots):$

>

> $implicitplot(SolucionParticular_2, x = -2..2, y = -15..15, gridrefine = 2)$



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Por homogenea

> *Ecuacion*

$$x\,y(x)\left(\frac{d}{dx}\,y(x)\right)=3\,y(x)^2+x^2 \quad \textbf{(17)}$$

> *EcuacionDos* := *simplify*(*isolate*(*eval*(*subs*(*y*(*x*) = *x*·*u*(*x*), *Ecuacion*)), *diff*(*u*(*x*), *x*)))

$$EcuacionDos := \frac{d}{dx}\,u(x)=\frac{2\,u(x)^2+1}{x\,u(x)} \quad \textbf{(18)}$$

> $P := \frac{2\,u^2+1}{u}$

$$P := \frac{2\,u^2+1}{u} \quad \textbf{(19)}$$

> $SolucionUno := int\left(\frac{1}{P}, u\right) - int\left(\frac{1}{x}, x\right) = C_2$

$$SolucionUno := \frac{1}{4}\,\ln\left(2\,u^2+1\right)-\ln(x)=C_2 \quad \textbf{(20)}$$

> $SolucionDos := simplify(\exp(lhs(SolucionUno))\cdot\cdot 4) = C_2$

$$SolucionDos := \frac{2\,u^2+1}{x^4} = C_2 \tag{21}$$

> $SolucionTres := expand\left(subs\left(u = \frac{y}{x}, SolucionDos\right)\right)$

$$SolucionTres := \frac{2\,y^2}{x^6} + \frac{1}{x^4} = C_2 \tag{22}$$

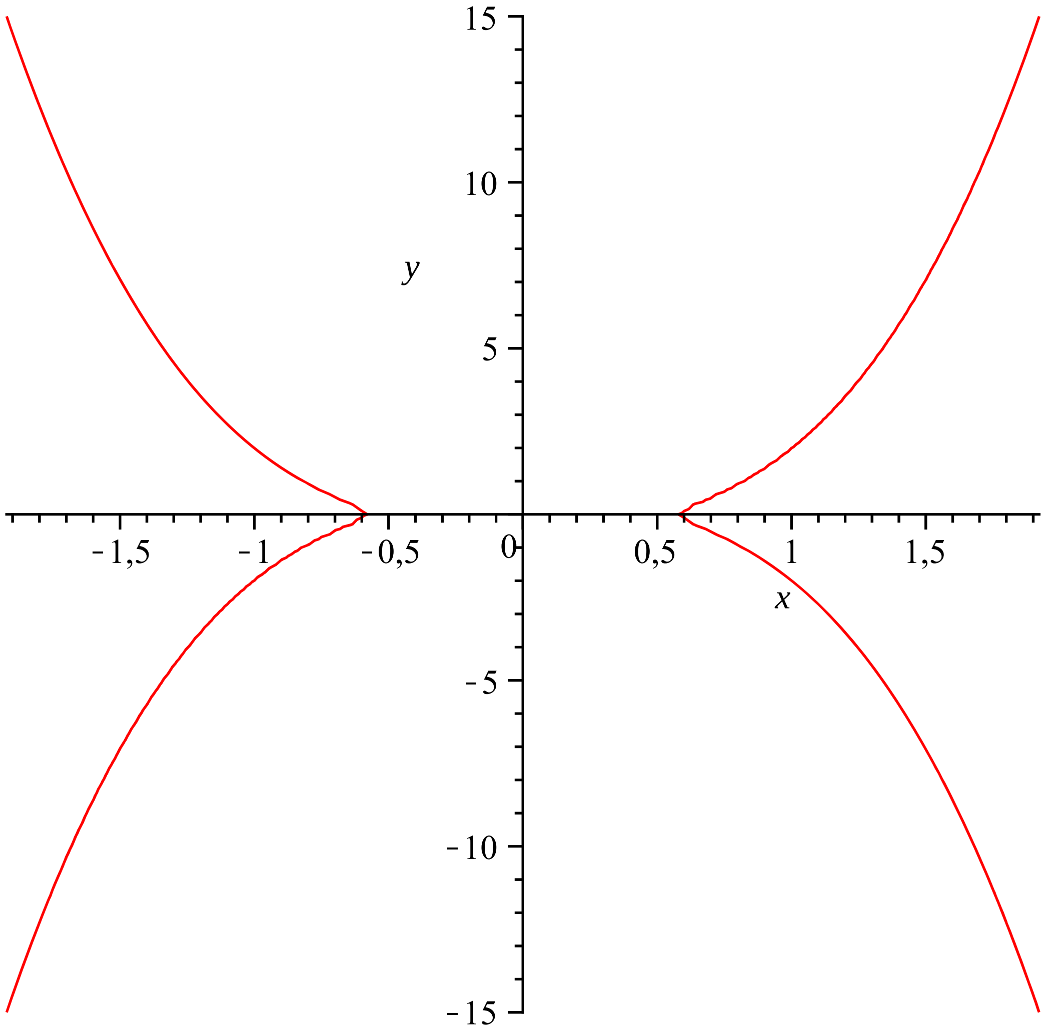
> $ParametroDos := subs(x = -1, y = rhs(Condicion), SolucionTres)$

$$ParametroDos := 9 = C_2 \tag{23}$$

> $SolucionParticular_3 := subs(C_2 = lhs(ParametroDos), SolucionTres)$

$$SolucionParticular_3 := \frac{2\,y^2}{x^6} + \frac{1}{x^4} = 9 \tag{24}$$

> $implicitplot(SolucionParticular_3, x = -2\,..2, y = -15\,..15, gridrefine = 2)$



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fin respuesta 1)

> *restart*

2. Determinar la solución para condiciones iniciales de la ecuación diferencial $\frac{d^2s}{dt^2}+4s=2\cos(2t)$

tal que $s=0$ y $\frac{ds}{dt}=2$ cuando $t=0$

2 PUNTOS

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Respuesta 2)

> $Ecuacion := s''(t) + 4\cdot s(t) = 2\cdot\cos(2\,t)$

$$Ecuacion := D^{(2)}(s)(t) + 4\,s(t) = 2\cos(2\,t) \quad \textbf{(25)}$$

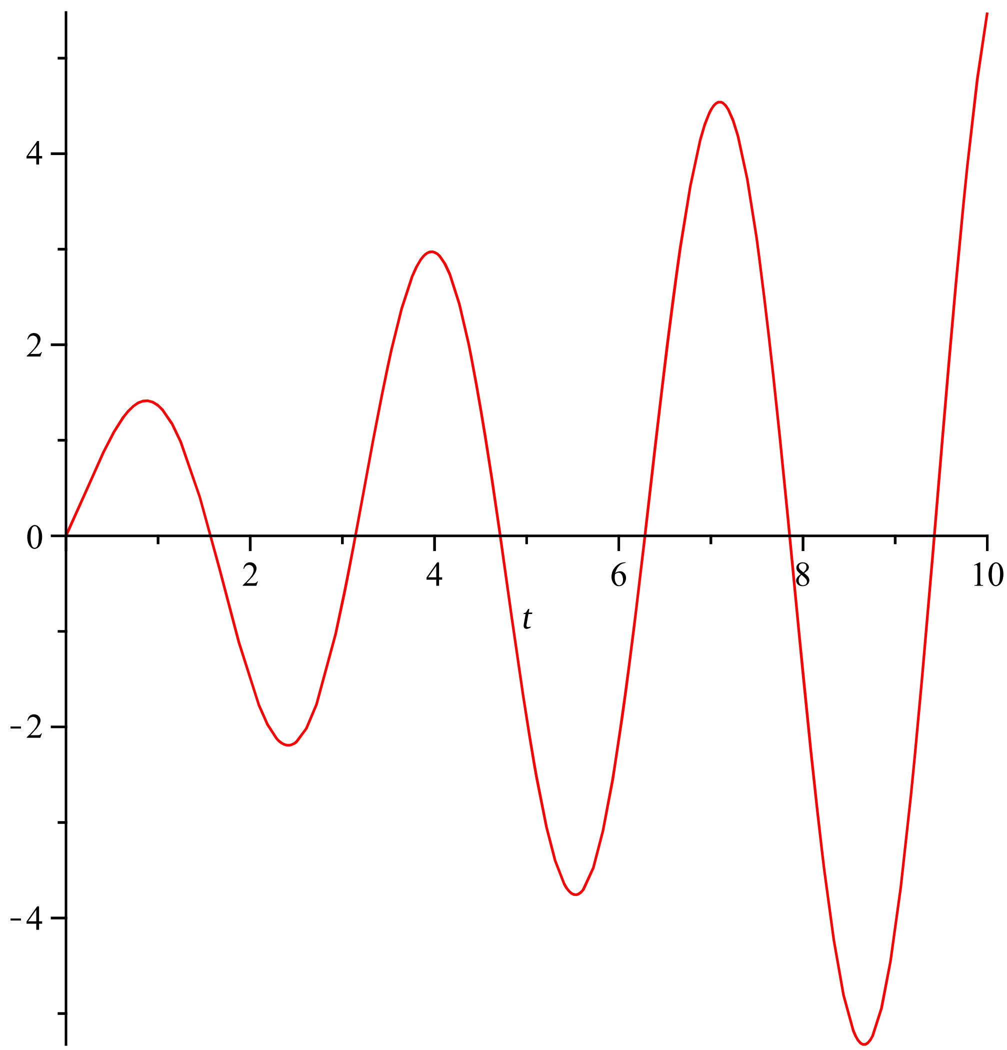
> $Condiciones := s(0) = 0, D(s)(0) = 2$

$$Condiciones := s(0) = 0, D(s)(0) = 2 \quad \textbf{(26)}$$

> $SolucionParticular := dsolve(\{Ecuacion, Condiciones\})$

$$SolucionParticular := s(t) = \sin(2\,t) + \frac{1}{2}\sin(2\,t)\,t \quad \textbf{(27)}$$

> $plot(rhs(SolucionParticular), t = 0..10)$



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Por parámetros variables

> *Ecuacion*

$$D^{(2)}(s)(t) + 4\, s(t) = 2\cos(2\,t) \quad \textbf{(28)}$$

> *EcuacionHom* := *lhs*(*Ecuacion*) = 0

$$EcuacionHom := D^{(2)}(s)(t) + 4\, s(t) = 0 \quad \textbf{(29)}$$

> *Q* := *rhs*(*Ecuacion*)

$$Q := 2\cos(2\,t) \quad \textbf{(30)}$$

> *EcuacionCarac* := *m*··2 + 4 = 0

$$EcuacionCarac := m^2 + 4 = 0 \quad \textbf{(31)}$$

> *Raiz* := *solve*(*EcuacionCarac*)

$$Raiz := 2\,I, -2\,I \quad \textbf{(32)}$$

> *SolUno* := *s*(*t*) = exp(Re(*Raiz*₁)·*t*)·cos(Im(*Raiz*₁)·*t*); *SolDos* := *s*(*t*) = exp(Re(*Raiz*₁)·*t*)·sin(Im(*Raiz*₁)·*t*)

$$SolUno := s(t) = \cos(2\,t)$$
$$SolDos := s(t) = \sin(2\,t) \quad \textbf{(33)}$$

> *with*(*linalg*) :

> *WW* := *wronskian*([*rhs*(*SolUno*), *rhs*(*SolDos*)], *t*)

$$WW := \begin{bmatrix} \cos(2\,t) & \sin(2\,t) \\ -2\sin(2\,t) & 2\cos(2\,t) \end{bmatrix} \quad \textbf{(34)}$$

> *BB* := *array*([0, *Q*])

$$BB := \begin{bmatrix} 0 & 2\cos(2\,t) \end{bmatrix} \quad \textbf{(35)}$$

> *SOL* := *simplify*(*linsolve*(*WW*, *BB*)) :

> *Aprima* := *SOL*₁; *Bprima* := *SOL*₂

$$Aprima := -\cos(2\,t)\sin(2\,t)$$
$$Bprima := \cos(2\,t)^2 \quad \textbf{(36)}$$

> *A* := *int*(*Aprima*, *t*) + *C*₁

$$A := \frac{1}{4}\cos(2\,t)^2 + C_1 \quad \textbf{(37)}$$

> *B* := *int*(*Bprima*, *t*) + *C*₂

$$B := \frac{1}{4}\cos(2\,t)\sin(2\,t) + \frac{1}{2}\,t + C_2 \quad \textbf{(38)}$$

> *SolucionGeneral*₂ := *s*(*t*) = *simplify*(*A*·*rhs*(*SolUno*) + *B*·*rhs*(*SolDos*))

$$SolucionGeneral_2 := s(t) = \frac{1}{2}\sin(2\,t)\,t + \sin(2\,t)\,C_2 + \cos(2\,t)\,C_1 + \frac{1}{4}\cos(2\,t) \quad \textbf{(39)}$$

> *Condiciones*

$$s(0) = 0, D(s)(0) = 2 \quad \textbf{(40)}$$

> *Sistemita* := *subs*(*t* = 0, *rhs*(*SolucionGeneral*₂) = *rhs*(*Condiciones*₁)), *subs*(*t* = 0, *rhs*(*diff*(*SolucionGeneral*₂, *t*)) = *rhs*(*Condiciones*₂)) : *Sistemita*₁; *Sistemita*₂

$$\frac{1}{4} + C_1 = 0$$
$$2\,C_2 = 2 \quad \textbf{(41)}$$

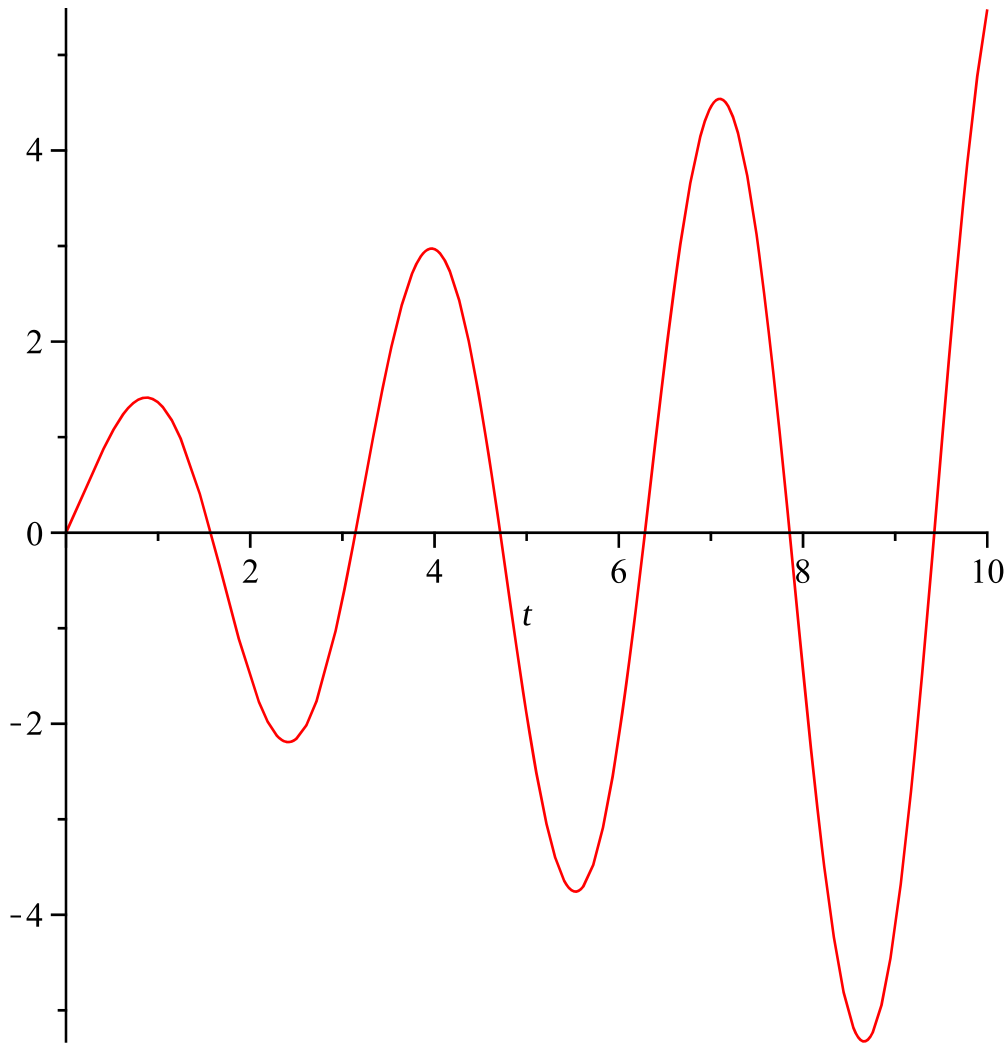
> $Parametros := solve(\{Sistemita\}, \{C_1, C_2\})$

$$Parametros := \left\{C_1 = -\frac{1}{4}, C_2 = 1\right\} \tag{42}$$

> $SolucionParticular_2 := subs(C_1 = rhs(Parametros_1), C_2 = rhs(Parametros_2), SolucionGeneral_2)$

$$SolucionParticular_2 := s(t) = \frac{1}{2}\,\sin(2\,t)\,t + \sin(2\,t) \tag{43}$$

> $plot(rhs(SolucionParticular_2), t = 0\,..10)$



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Fin respuesta 2)

> *restart*

3. Resolver el sistema para $y(t)$

$$x' - 4x + y'' = t$$
$$x' + x + y' = 0$$

2 PUNTOS

> $Sistema := x'(t) - 4\cdot x(t) + y''(t) = t, x'(t) + x(t) + y'(t) = 0 : Sistema_1; Sistema_2$

$$D(x)(t) - 4\,x(t) + D^{(2)}(y)(t) = t$$
$$D(x)(t) + x(t) + D(y)(t) = 0 \qquad \textbf{(44)}$$

> $Solucion := dsolve(\{Sistema\}) : Solucion_1; Solucion_2$

$$x(t) = -\frac{1}{5}\sin(2\,t)\,\_C2 - \frac{1}{5}\cos(2\,t)\,\_C1 - \frac{1}{4}\,t + \frac{2}{5}\cos(2\,t)\,\_C2 - \frac{2}{5}\sin(2\,t)\,\_C1$$
$$y(t) = -\frac{1}{2}\cos(2\,t)\,\_C2 + \frac{1}{2}\sin(2\,t)\,\_C1 + \frac{1}{8}\,t^2 + \frac{1}{4}\,t + \_C3 \qquad \textbf{(45)}$$

>

Por Matriz Exponencial

> $SistemaDos := x'(t) = -(x(t) + z(t)), y'(t) = z(t), +z'(t) = -(-x(t) - z(t) - 4\cdot x(t)) + t :$
$SistemaDos_1; SistemaDos_2; SistemaDos_3;$

$$D(x)(t) = -x(t) - z(t)$$
$$D(y)(t) = z(t)$$
$$D(z)(t) = 5\,x(t) + z(t) + t \qquad \textbf{(46)}$$

> $AA := array([[-1, 0, -1], [0, 0, 1], [5, 0, 1]])$

$$AA := \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 5 & 0 & 1 \end{bmatrix} \qquad \textbf{(47)}$$

> $BB := array([0, 0, t])$

$$BB := \begin{bmatrix} 0 & 0 & t \end{bmatrix} \qquad \textbf{(48)}$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} -\frac{1}{2}\sin(2\,t) + \cos(2\,t) & 0 & -\frac{1}{2}\sin(2\,t) \\ -\frac{5}{4}\cos(2\,t) + \frac{5}{4} & 1 & \frac{1}{2}\sin(2\,t) - \frac{1}{4}\cos(2\,t) + \frac{1}{4} \\ \frac{5}{2}\sin(2\,t) & 0 & \frac{1}{2}\sin(2\,t) + \cos(2\,t) \end{bmatrix} \qquad \textbf{(49)}$$

> $Xcero := array([C_1, C_2, C_3])$

$$Xcero := \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix} \qquad \textbf{(50)}$$

> $MatExpTau := map(rcurry(eval, t = 't - \mathrm{tau}'), MatExp)$

$MatExpTau :=$ **(51)**

$$\left[\begin{array}{ccc} -\frac{1}{2}\sin(2t-2\tau)+\cos(2t-2\tau) & 0 & -\frac{1}{2}\sin(2t-2\tau) \\ -\frac{5}{4}\cos(2t-2\tau)+\frac{5}{4} & 1 & \frac{1}{2}\sin(2t-2\tau)-\frac{1}{4}\cos(2t-2\tau)+\frac{1}{4} \\ \frac{5}{2}\sin(2t-2\tau) & 0 & \frac{1}{2}\sin(2t-2\tau)+\cos(2t-2\tau) \end{array}\right]$$

> $BBtau := map(rcurry(eval, t='tau'), BB)$

$$BBtau := \left[\begin{array}{ccc} 0 & 0 & \tau \end{array}\right] \quad (52)$$

> $ProdTau := evalm(\ MatExpTau \&* BBtau)$

$$ProdTau := \left[-\frac{1}{2}\sin(2t-2\tau)\,\tau, \left(\frac{1}{2}\sin(2t-2\tau)-\frac{1}{4}\cos(2t-2\tau)+\frac{1}{4}\right)\tau, \left(\frac{1}{2}\sin(2t-2\tau)+\cos(2t-2\tau)\right)\tau\right] \quad (53)$$

> $IntProTau := simplify(map(int, ProdTau, \mathrm{tau}=0..t))$

$$IntProTau := \left[\frac{1}{8}\sin(2t)-\frac{1}{4}t, -\frac{1}{8}\sin(2t)+\frac{1}{16}\cos(2t)+\frac{1}{8}t^2-\frac{1}{16}+\frac{1}{4}t, -\frac{1}{8}\sin(2t)-\frac{1}{4}\cos(2t)+\frac{1}{4}+\frac{1}{4}t\right] \quad (54)$$

> $SOLSOL := evalm(evalm(MatExp \&* Xcero) + IntProTau) : x(t) = simplify(SOLSOL_1);$
$Solucion_1; y(t) = simplify(SOLSOL_2); Solucion_2; z(t) = simplify(SOLSOL_3)$

$$x(t) = -\frac{1}{2}\sin(2t)\,C_1 + C_1\cos(2t) - \frac{1}{2}\sin(2t)\,C_3 + \frac{1}{8}\sin(2t) - \frac{1}{4}t$$

$$x(t) = -\frac{1}{5}\sin(2t)\,\_C2 - \frac{1}{5}\cos(2t)\,\_C1 - \frac{1}{4}t + \frac{2}{5}\cos(2t)\,\_C2 - \frac{2}{5}\sin(2t)\,\_C1$$

$$y(t) = -\frac{5}{4}C_1\cos(2t) + \frac{5}{4}C_1 + C_2 + \frac{1}{2}\sin(2t)\,C_3 - \frac{1}{4}C_3\cos(2t) + \frac{1}{4}C_3 - \frac{1}{8}\sin(2t) + \frac{1}{16}\cos(2t) + \frac{1}{8}t^2 - \frac{1}{16} + \frac{1}{4}t$$

$$y(t) = -\frac{1}{2}\cos(2t)\,\_C2 + \frac{1}{2}\sin(2t)\,\_C1 + \frac{1}{8}t^2 + \frac{1}{4}t + \_C3$$

$$z(t) = \frac{5}{2}\sin(2t)\,C_1 + \frac{1}{2}\sin(2t)\,C_3 + C_3\cos(2t) - \frac{1}{8}\sin(2t) - \frac{1}{4}\cos(2t) + \frac{1}{4} + \frac{1}{4}t \quad (55)$$

>

Fin respuesta 3)

> *restart*

4. Resolver la ecuación diferencial $y''' - y'' = \delta(t-4)$ sujeta a $y(0) = y'(0) = y''(0) = 0$

2 PUNTOS

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Respuesta 4)

> $Ecuacion := y'''(t) - y''(t) = \mathrm{Dirac}(t-4)$

$$Ecuacion := \mathrm{D}^{(3)}(y)(t) - \mathrm{D}^{(2)}(y)(t) = \mathrm{Dirac}(t-4) \quad \textbf{(56)}$$

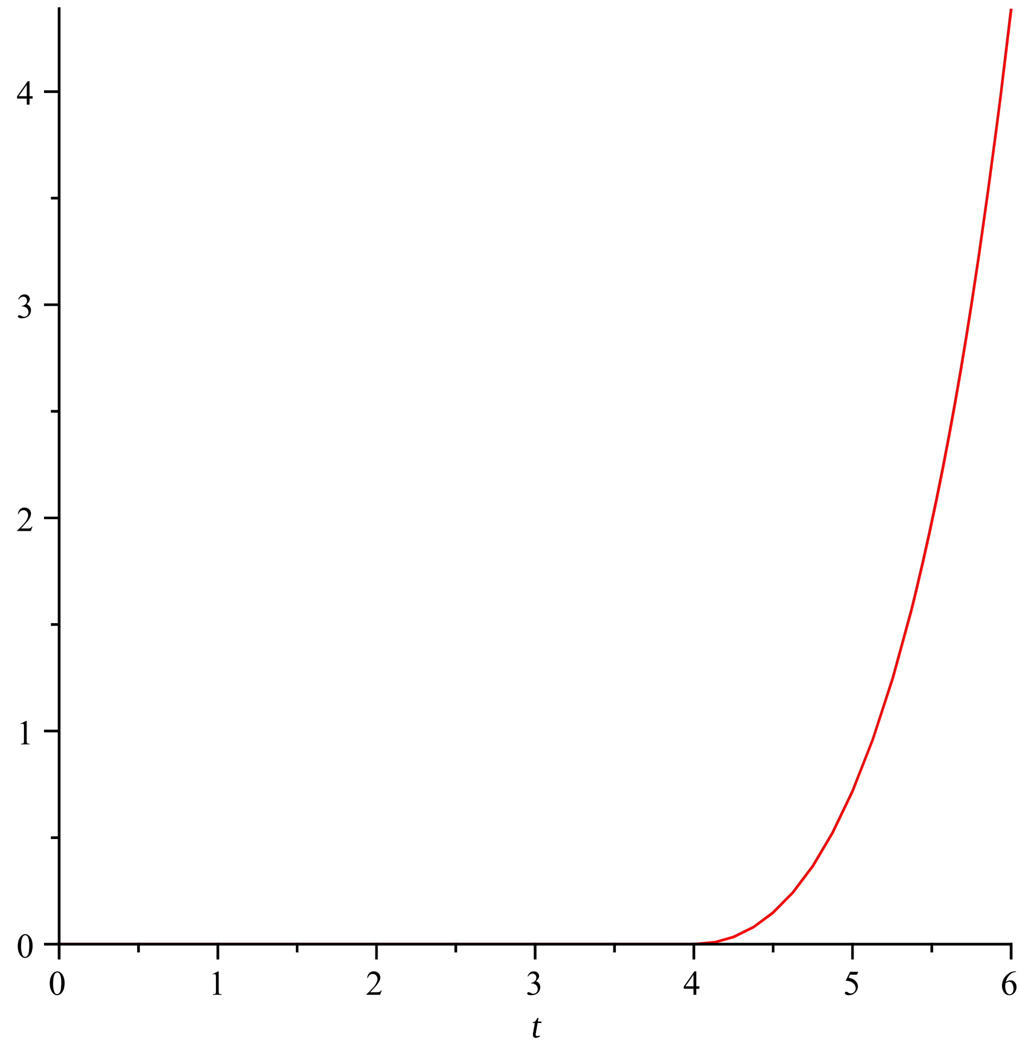
> $Condiciones := y(0) = 0, \mathrm{D}(y)(0) = 0, \mathrm{D}(\mathrm{D}(y))(0) = 0$

$$Condiciones := y(0) = 0, \mathrm{D}(y)(0) = 0, \mathrm{D}^{(2)}(y)(0) = 0 \quad \textbf{(57)}$$

> $SolucionParticular := dsolve(\{Ecuacion, Condiciones\})$

$$SolucionParticular := y(t) = \mathrm{Heaviside}(t-4)\, e^{t-4} + 3\, \mathrm{Heaviside}(t-4) - \mathrm{Heaviside}(t-4)\, t \quad \textbf{(58)}$$

> $plot(rhs(SolucionParticular), t = 0..6)$



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Por Transformada de Laplace

> $Ecuacion$

$$\mathrm{D}^{(3)}(y)(t) - \mathrm{D}^{(2)}(y)(t) = \mathrm{Dirac}(t-4) \quad \textbf{(59)}$$

> $Condiciones$

$$y(0) = 0, \mathrm{D}(y)(0) = 0, \mathrm{D}^{(2)}(y)(0) = 0 \quad \textbf{(60)}$$

> $with(inttrans):$

> $TransLapEcua := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcua := s^3\, laplace(y(t), t, s) - s^2\, laplace(y(t), t, s) = e^{-4 s} \quad \textbf{(61)}$$

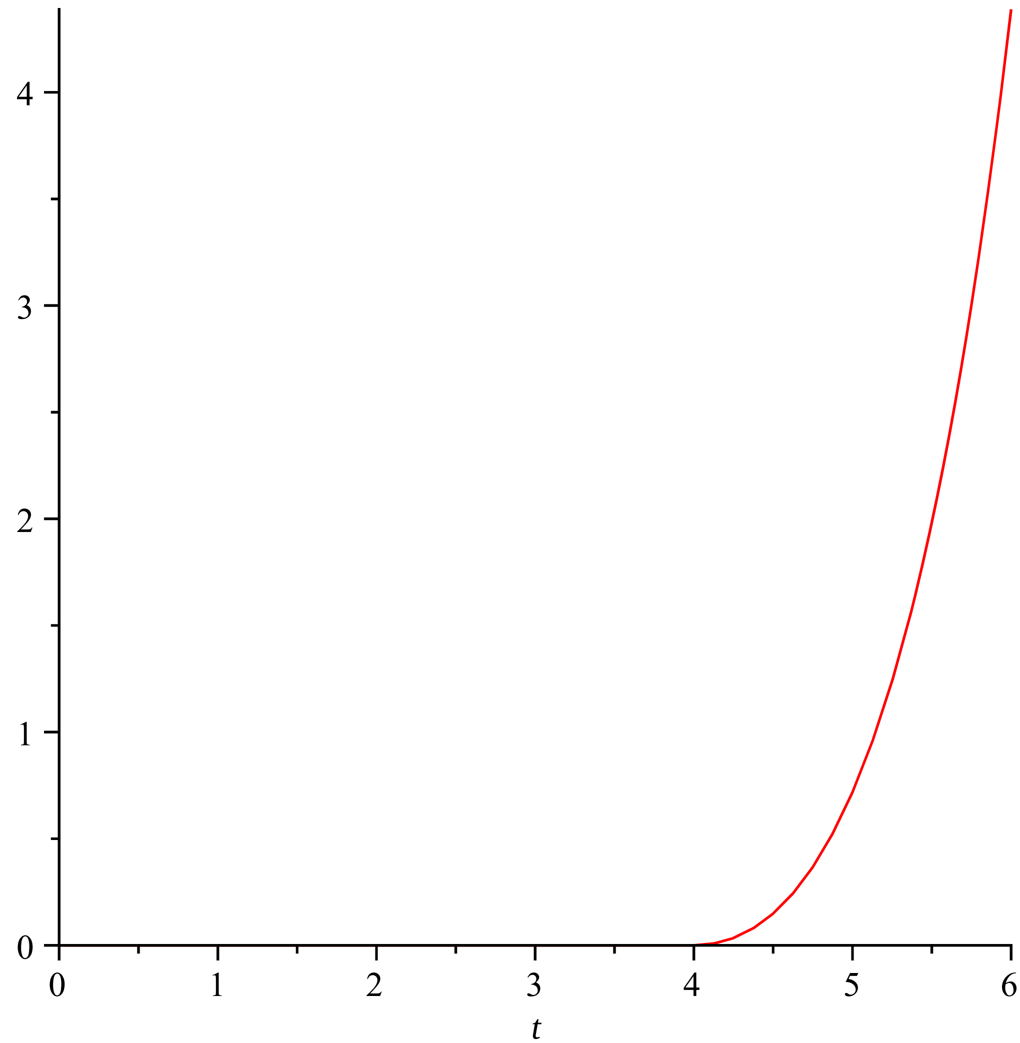
> $TransLapSol := isolate(TransLapEcua, laplace(y(t), t, s))$

$$TransLapSol := laplace(y(t), t, s) = \frac{e^{-4s}}{s^3 - s^2} \quad \textbf{(62)}$$

> $SolucionParticular_2 := invlaplace(TransLapSol, s, t)$

$$SolucionParticular_2 := y(t) = -\text{Heaviside}(t-4)\left(t - 3 - e^{t-4}\right) \quad \textbf{(63)}$$

> $plot(rhs(SolucionParticular_2), t = 0..6)$



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Fin respuesta 4)

> *restart*

5. Desarrollar $f(x) = e^{-x}$ para $0 < x < 1$ en serie de cosenos

2 PUNTOS

>

Respuesta 5)

> $f := \exp(-x)$

$$f := e^{-x} \quad \textbf{(64)}$$

> $L := 1$

$$L := 1 \quad (65)$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(f, x = -L..L)$

$$a_0 := e - e^{-1} \quad (66)$$

> $c := \frac{a_0}{2}$

$$c := \frac{1}{2} e - \frac{1}{2} e^{-1} \quad (67)$$

> $a_n := subs\left(\cos(n \cdot \mathrm{Pi}) = (-1) \cdot \cdot n, \sin(n \cdot \mathrm{Pi}) = 0, \left(\frac{1}{L}\right) \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

$$a_n := -\frac{-e(-1)^n + e^{-1}(-1)^n}{1 + n^2 \pi^2} \quad (68)$$

> $b_n := subs\left(\cos(n \cdot \mathrm{Pi}) = (-1) \cdot \cdot n, \sin(n \cdot \mathrm{Pi}) = 0, \left(\frac{1}{L}\right) \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

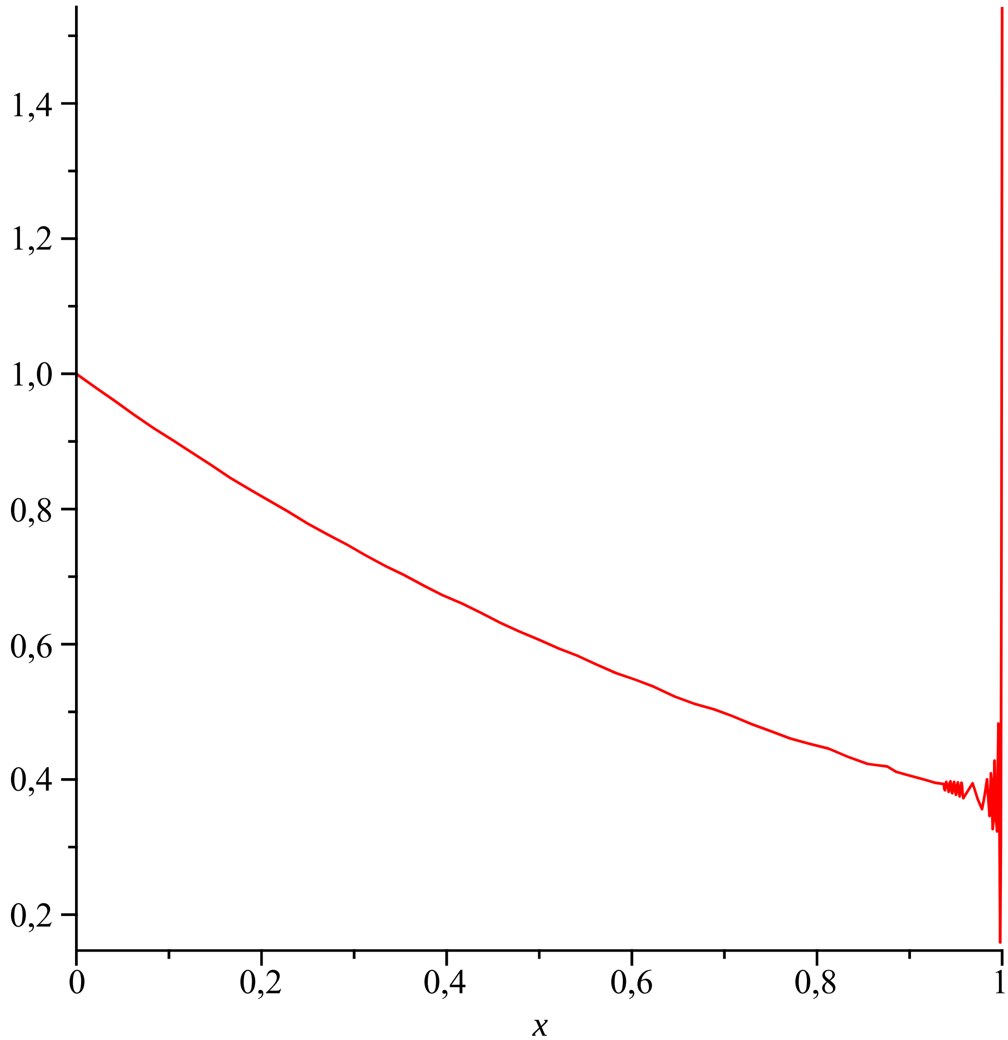
$$b_n := \frac{e\, n \pi (-1)^n - e^{-1} n \pi (-1)^n}{1 + n^2 \pi^2} \quad (69)$$

> $STF := c + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..infinity\right)$

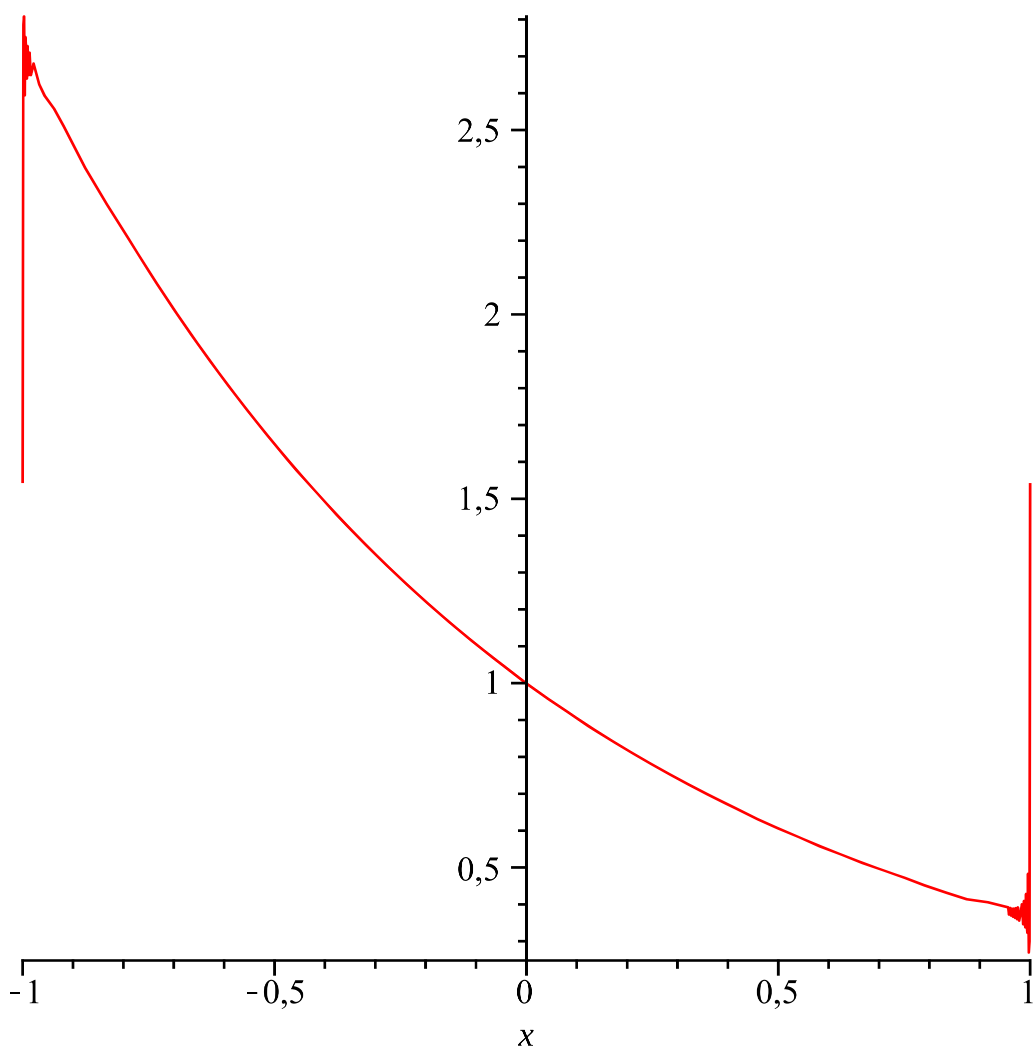
$$STF := \frac{1}{2} e - \frac{1}{2} e^{-1} + \sum_{n=1}^{\infty} \left(-\frac{\left(-e(-1)^n + e^{-1}(-1)^n\right) \cos(n \pi x)}{1 + n^2 \pi^2} + \frac{\left(e\, n \pi (-1)^n - e^{-1} n \pi (-1)^n\right) \sin(n \pi x)}{1 + n^2 \pi^2} \right) \quad (70)$$

> $STF_{500} := c + sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..500\right):$

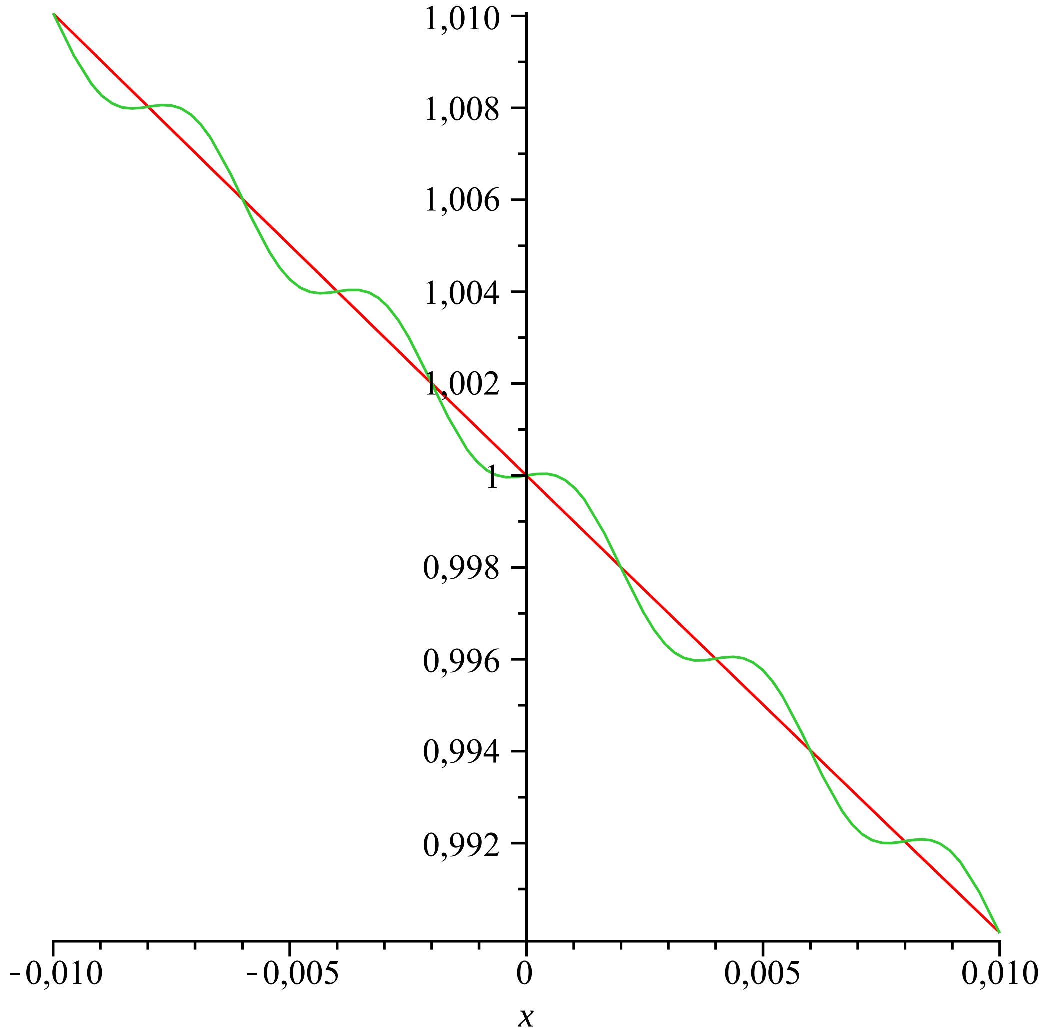
> $plot(STF_{500}, x = 0..1)$



> $plot(STF_{500}, x=-L..L)$



> $plot\left(\left[f, STF_{500}\right], x = -0.01\,..0.01\right)$



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Fin respuesta 5)

> *restart*

Fin examen

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