>

SOLUCIÓN

UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE CIENCIAS APLICADAS
DEPARTAMENTO DE ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL

INGENIERÍA

SEMESTRE 2015 -1
DURACIÓN MÁXIMA 2.0 HORAS

TIPO 1
4 DE DICIEMBRE DE 2014

> *restart*

1. Obtener una función $N(x,y)$ de modo que la siguiente ecuación diferencial sea exacta

$$\left(ye^{xy}+y^2-\frac{1}{x^2}y\right)dx+N(x,y)\,dy=0$$

2 PUNTOS

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Respuesta 1)

> $Ecuacion := \left(y(x)\cdot\exp(x\cdot y(x)) + y(x)\cdot\cdot 2 - \frac{y(x)}{x\cdot\cdot 2}\right) + N(x,y)\cdot diff(y(x),x) = 0$

$$Ecuacion := y(x)\,e^{x\,y(x)} + y(x)^2 - \frac{y(x)}{x^2} + N(x,y)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (1)$$

> $M := y\cdot\exp(x\cdot y) + y\cdot\cdot 2 - \frac{y}{x\cdot\cdot 2}$

$$M := y\,e^{xy} + y^2 - \frac{y}{x^2} \quad (2)$$

> $DerMy := expand(diff(M, y))$

$$DerMy := e^{xy} + y\,x\,e^{xy} + 2\,y - \frac{1}{x^2} \quad (3)$$

> $N := expand(int(DerMy, x))$

$$N := x\,e^{xy} + 2\,x\,y + \frac{1}{x} \quad (4)$$

> $comprobacion_0 := simplify(diff(N,x) - diff(M,y)) = 0$

$$comprobacion_0 := 0 = 0 \quad (5)$$

> $IntMx := int(M, x)$

$$IntMx := e^{xy} + y^2\,x + \frac{y}{x} \quad (6)$$

> $NN := expand(diff(IntMx, y))$

(7)

$$NN := x\,e^{xy} + 2\,x\,y + \frac{1}{x} \quad (7)$$

> *SolucionGeneral* := *IntMx* + *int*((*N* − *diff* (*IntMx*, *y*)), *y*) = C_1

$$SolucionGeneral := e^{xy} + y^2\,x + \frac{y}{x} = C_1 \quad (8)$$

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Fin respuesta 1)

> *restart*

2. Determinar la ecuación diferencial que tiene por solución general a la función

$$y(t) = Ae^{-t}\cos t + Be^{-t}\,sen\,t + 3\,sen\,t - \cos t$$

2 PUNTOS

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Respuesta 2)

> *SolucionGeneral* := *y*(*t*) = *A*·exp(−*t*)cos(*t*) + *B*·exp(−*t*)·sin(*t*) + 3·sin(*t*) − cos(*t*)

$$SolucionGeneral := y(t) = A\,e^{-t}\cos(t) + B\,e^{-t}\sin(t) + 3\sin(t) - \cos(t) \quad (9)$$

> *SolHom* := *y*(*t*) = *A*·exp(−*t*)cos(*t*) + *B*·exp(−*t*)·sin(*t*)

$$SolHom := y(t) = A\,e^{-t}\cos(t) + B\,e^{-t}\sin(t) \quad (10)$$

> *SolPart* := *y*(*t*) = 3·sin(*t*) − cos(*t*)

$$SolPart := y(t) = 3\sin(t) - \cos(t) \quad (11)$$

> *EcuacCarac* := *expand*((*m* − (−1 + *I*))·(*m* − (−1 − *I*))) = 0

$$EcuacCarac := m^2 + 2\,m + 2 = 0 \quad (12)$$

> *EcuaHom* := *diff* (*y*(*t*), *t*$2) + 2·*diff* (*y*(*t*), *t*) + 2·*y*(*t*) = 0

$$EcuaHom := \frac{d^2}{dt^2}\,y(t) + 2\left(\frac{d}{dt}\,y(t)\right) + 2\,y(t) = 0 \quad (13)$$

> *Q* := *eval*(*subs*(*y*(*t*) = *rhs*(*SolPart*), *lhs*(*EcuaHom*)))

$$Q := 5\sin(t) + 5\cos(t) \quad (14)$$

> *EcuacionNoHomogenea* := *lhs*(*EcuaHom*) = *Q*

$$EcuacionNoHomogenea := \frac{d^2}{dt^2}\,y(t) + 2\left(\frac{d}{dt}\,y(t)\right) + 2\,y(t) = 5\sin(t) + 5\cos(t) \quad (15)$$

> *Solucion* := *dsolve*(*EcuacionNoHomogenea*)

$$Solucion := y(t) = e^{-t}\sin(t)\,\_C2 + e^{-t}\cos(t)\,\_C1 + 3\sin(t) - \cos(t) \quad (16)$$

> *SolucionGeneral*

$$y(t) = A\,e^{-t}\cos(t) + B\,e^{-t}\sin(t) + 3\sin(t) - \cos(t) \quad (17)$$

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Fin respuesta 2)

> *restart*

3. Determinar la solución del sistema de ecuaciones diferenciales

$$\frac{dx}{dt} = -y + t$$

$$\frac{dy}{dt} = x - t$$

2 PUNTOS

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Respuesta 3)

> $Sistema := diff(x(t), t) = -y(t) + t, diff(y(t), t) = x(t) - t : Sistema_1; Sistema_2;$

$$\frac{d}{dt} x(t) = -y(t) + t$$

$$\frac{d}{dt} y(t) = x(t) - t \qquad \textbf{(18)}$$

> $Solucion := dsolve(\{Sistema\}) : Solucion_1; Solucion_2$

$$x(t) = \sin(t)\ \_C2 + \cos(t)\ \_C1 + 1 + t$$

$$y(t) = -\cos(t)\ \_C2 + \sin(t)\ \_C1 - 1 + t \qquad \textbf{(19)}$$

> $comprobacion_1 := eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2), (lhs(Sistema_1) - rhs(Sistema_1)))) = 0$

$$comprobacion_1 := 0 = 0 \qquad \textbf{(20)}$$

> $comprobacion_2 := eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2), (lhs(Sistema_2) - rhs(Sistema_2)))) = 0$

$$comprobacion_2 := 0 = 0 \qquad \textbf{(21)}$$

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opción 2

> $AA := array([[0, -1], [1, 0]])$

$$AA := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \textbf{(22)}$$

> $BB := array([t, -t])$

$$BB := \begin{bmatrix} t & -t \end{bmatrix} \qquad \textbf{(23)}$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{bmatrix} \qquad \textbf{(24)}$$

> $Xcero := array([C_1, C_2])$

$$Xcero := \begin{bmatrix} C_1 & C_2 \end{bmatrix} \qquad \textbf{(25)}$$

> $MatExpTau := map(rcurry(eval, t = 't - \text{tau}'), MatExp)$

$$MatExpTau := \begin{bmatrix} \cos(t-\tau) & -\sin(t-\tau) \\ \sin(t-\tau) & \cos(t-\tau) \end{bmatrix} \quad \textbf{(26)}$$

> *BBtau* := *map*(*rcurry*(*eval*, *t* ='tau'), *BB*)

$$BBtau := \begin{bmatrix} \tau & -\tau \end{bmatrix} \quad \textbf{(27)}$$

> *ProdMatTau* := *evalm*(*MatExpTau* &* *BBtau*)

$$ProdMatTau := \begin{bmatrix} \cos(t-\tau)\,\tau + \sin(t-\tau)\,\tau & \sin(t-\tau)\,\tau - \cos(t-\tau)\,\tau \end{bmatrix} \quad \textbf{(28)}$$

> *IntTau* := *map*(*int*, *ProdMatTau*, tau = 0 ..*t*)

$$IntTau := \begin{bmatrix} -\cos(t) - \sin(t) + 1 + t & -\sin(t) + \cos(t) - 1 + t \end{bmatrix} \quad \textbf{(29)}$$

> *SolGral* := *evalm*(*evalm*(*MatExp* &* *Xcero*) + *IntTau*) : *SolUno* := $x(t) = SolGral_1$; *SolDos* := $y(t) = SolGral_2$

$$SolUno := x(t) = \cos(t)\,C_1 - \sin(t)\,C_2 - \cos(t) - \sin(t) + 1 + t$$

$$SolDos := y(t) = \sin(t)\,C_1 + \cos(t)\,C_2 - \sin(t) + \cos(t) - 1 + t \quad \textbf{(30)}$$

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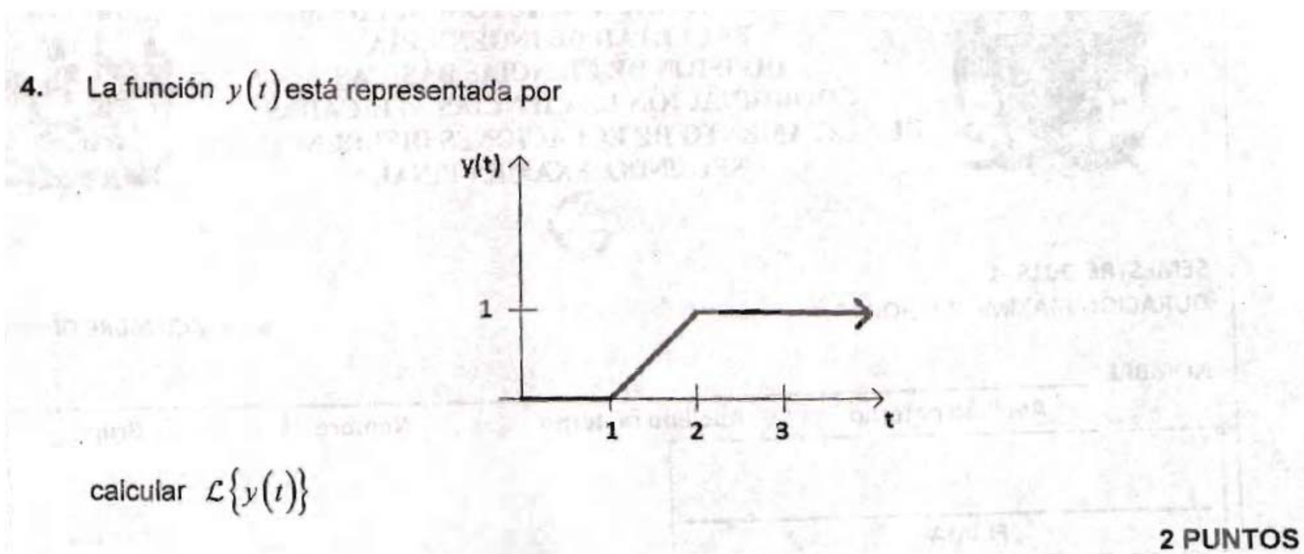
Fin respuesta 3)

> *restart*



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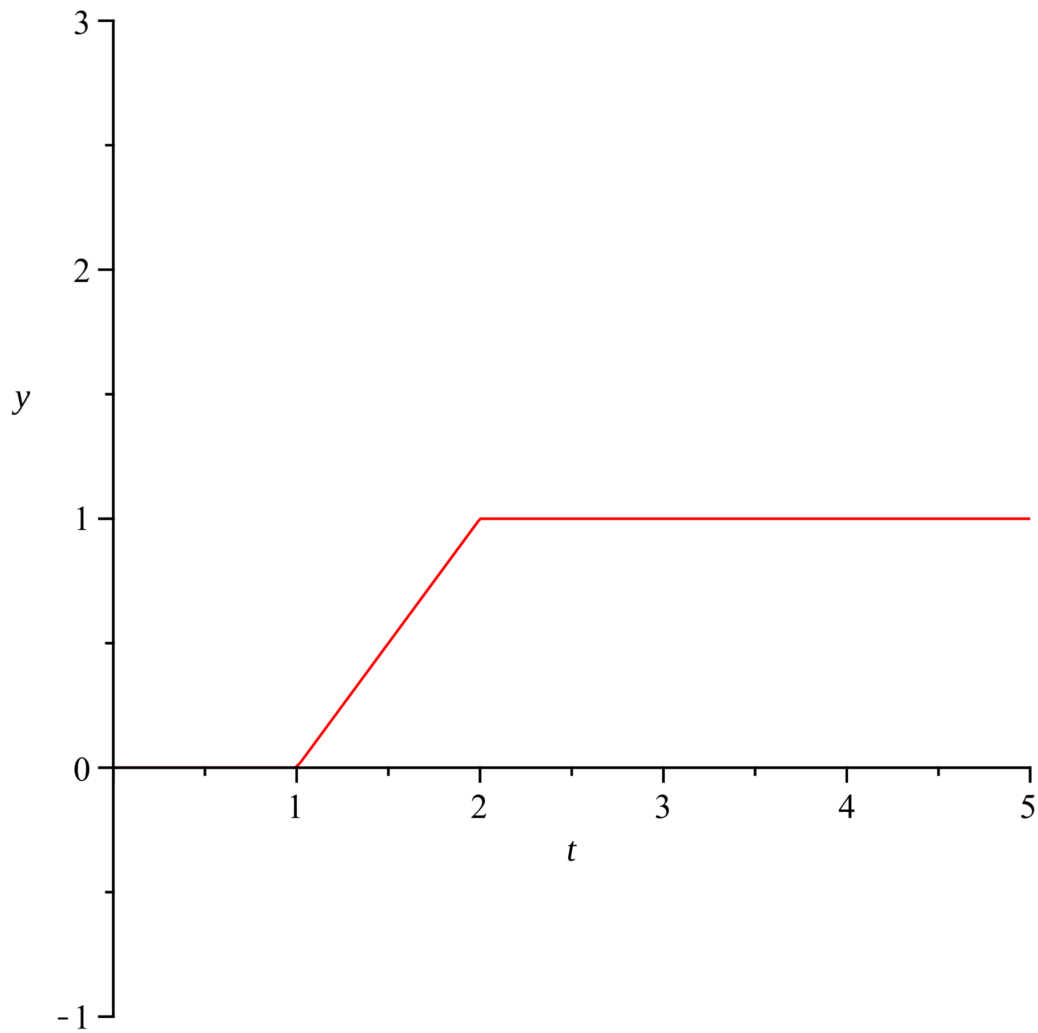
Respuesta 4)

> *Sol* := $y(t) = (t-1)\cdot\text{Heaviside}(t-1) - (t-2)\cdot\text{Heaviside}(t-2)$: *plot*(*rhs*(*Sol*), *t* = 0 ..5, *y* =-1 ..3)



> *with*(*inttrans*) :

> *Transformada* := $Y(s) = laplace(rhs(Sol), t, s)$

$$Transformada := Y(s) = \frac{e^{-s} - e^{-2s}}{s^2} \quad \textbf{(31)}$$

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Fin respuesta 4)

> *restart*

5. Determinar la solución de la ecuación diferencial

$$\frac{\partial^2 u(x,y)}{\partial y^2} = u(x,y) - \frac{\partial u(x,y)}{\partial x}$$

considérese una constante de separación positiva.

2 PUNTOS

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Respuesta 5)

> *Ecuacion* := *diff* $(u(x, y), y\$2) = u(x, y) - $ *diff* $(u(x, y), x)$

(32)

$$Ecuacion := \frac{\partial^2}{\partial y^2} u(x,y) = u(x,y) - \left(\frac{\partial}{\partial x} u(x,y)\right) \quad \textbf{(32)}$$

> EcuacionDos := eval(subs(u(x, y) = F(x)·G(y), Ecuacion))

$$EcuacionDos := F(x)\left(\frac{d^2}{dy^2} G(y)\right) = F(x)\, G(y) - \left(\frac{d}{dx} F(x)\right) G(y) \quad \textbf{(33)}$$

> $EcuacionTres := \frac{lhs(EcuacionDos)}{F(x)\cdot G(y)} = simplify\left(\frac{rhs(EcuacionDos)}{F(x)\cdot G(y)}\right)$

$$EcuacionTres := \frac{\frac{d^2}{dy^2} G(y)}{G(y)} = \frac{F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} \quad \textbf{(34)}$$

> EcuacionX := rhs(EcuacionTres) = alpha; EcuacionY := lhs(EcuacionTres) = alpha

$$EcuacionX := \frac{F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = \alpha$$

$$EcuacionY := \frac{\frac{d^2}{dy^2} G(y)}{G(y)} = \alpha \quad \textbf{(35)}$$

> SolucionXpos := dsolve(subs(alpha = beta· ·2, EcuacionX)); SolucionYpos := dsolve(subs(alpha = beta· ·2, EcuacionY))

$$SolucionXpos := F(x) = \_C1\, e^{-(\beta-1)(\beta+1)x}$$

$$SolucionYpos := G(y) = \_C1\, e^{-\beta y} + \_C2\, e^{\beta y} \quad \textbf{(36)}$$

> SolucionGeneral := u(x, y) = subs(_C1 = 1, rhs(SolucionXpos))·rhs(SolucionYpos)

$$SolucionGeneral := u(x,y) = e^{-(\beta-1)(\beta+1)x}\left(\_C1\, e^{-\beta y} + \_C2\, e^{\beta y}\right) \quad \textbf{(37)}$$

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opción dos

> $EcuacionCuatro := simplify\left(\frac{lhs(EcuacionDos) - F(x)\cdot G(y)}{F(x)\cdot G(y)}\right) = simplify\left(\frac{rhs(EcuacionDos) - F(x)\cdot G(y)}{F(x)\cdot G(y)}\right)$

$$EcuacionCuatro := \frac{\frac{d^2}{dy^2} G(y) - G(y)}{G(y)} = -\frac{\frac{d}{dx} F(x)}{F(x)} \quad \textbf{(38)}$$

> EcuacionXX := rhs(EcuacionCuatro) = alpha; EcuacionYY := lhs(EcuacionCuatro) = alpha

$$EcuacionXX := -\frac{\frac{d}{dx} F(x)}{F(x)} = \alpha$$

$$EcuacionYY := \frac{\frac{d^2}{dy^2} G(y) - G(y)}{G(y)} = \alpha \quad \textbf{(39)}$$

> SolucionXXpos := dsolve(subs(alpha = beta· ·2, EcuacionXX)); SolucionYYpos := dsolve(subs(alpha = beta· ·2, EcuacionYY))

$$SolucionXXpos := F(x) = \_C1\, e^{-\beta^2 x}$$

$$SolucionYYpos := G(y) = \_C1 \sin\left(\sqrt{-1-\beta^2}\, y\right) + \_C2 \cos\left(\sqrt{-1-\beta^2}\, y\right) \quad \textbf{(40)}$$

> $SolucionGeneralDos := u(x, y) = subs(\_C1 = 1, rhs(SolucionXXpos)) \cdot rhs(SolucionYYpos)$

$$SolucionGeneralDos := u(x, y) = e^{-\beta^2 x} \left(\_C1 \sin\left(\sqrt{-1-\beta^2}\, y\right) + \_C2 \cos\left(\sqrt{-1-\beta^2}\, y\right)\right) \quad \textbf{(41)}$$

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Fin respuesta 5)

> *restart*

Fin examen