

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL (TEMAS 3 Y 4)
SEMESTRE 2019-1

2018 NOVIEMBRE 22

> restart

1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE (**sin usar dsolve**):

a) (20/100 puntos) OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS

b) (15/100 puntos) GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA; PARA UN INTERVALO DE $0 < t < 3$

$$\frac{d^2}{dt^2} y(t) + 4 y(t) = 64 (t - 2) \text{Heaviside}(t - 2) \sin(2 t - 4)$$

$$y(0) = 2$$

$$D(y)(0) = 0$$

(1)

> restart

RESPUESTA 1a)

> Ecuacion := $\frac{d^2}{dt^2} y(t) + 4 y(t) = 64 (t - 2) \text{Heaviside}(t - 2) \sin(2 t - 4)$; Condiciones
:= $y(0) = 2, D(y)(0) = 0$

$$\text{Ecuacion} := \frac{d^2}{dt^2} y(t) + 4 y(t) = 64 (t - 2) \text{Heaviside}(t - 2) \sin(2 t - 4)$$

$$\text{Condiciones} := y(0) = 2, D(y)(0) = 0$$

(2)

> with(inttrans) :

> TransLapEcuacion := simplify(subs(Condiciones, laplace(Ecuacion, t, s)))

$$\text{TransLapEcuacion} := s^2 \text{laplace}(y(t), t, s) - 2 s + 4 \text{laplace}(y(t), t, s) = \frac{256 e^{-2s} s}{(s^2 + 4)^2}$$

(3)

> TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))

$$\text{TransLapSolucion} := \text{laplace}(y(t), t, s) = \frac{2 s (128 e^{-2s} + s^4 + 8 s^2 + 16)}{(s^2 + 4)^3}$$

(4)

> SolucionParticular := invlaplace(TransLapSolucion, s, t)

$$\text{SolucionParticular} := y(t) = 2 \cos(2 t) + 4 (t - 2) (\sin(2 t - 4) - 2 \cos(2 t - 4) (t - 2)) \text{Heaviside}(t - 2)$$

(5)

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RESPUESTA 1b)

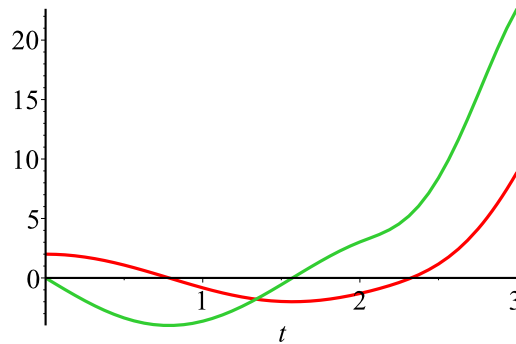
> DerSolucionParticular := diff(SolucionParticular, t)

$$\text{DerSolucionParticular} := \frac{d}{dt} y(t) = -4 \sin(2 t) + 4 (\sin(2 t - 4) - 2 \cos(2 t - 4) (t - 2)) \text{Heaviside}(t - 2)$$

(6)

$-2) \cdot \text{Heaviside}(t-2) + 16(t-2)^2 \sin(2t-4) \text{Heaviside}(t-2) + 4(t-2)(\sin(2t-4) - 2 \cos(2t-4)(t-2)) \text{Dirac}(t-2)$

> `plot([rhs(SolucionParticular), rhs(DerSolucionParticular)], t=0..3)`

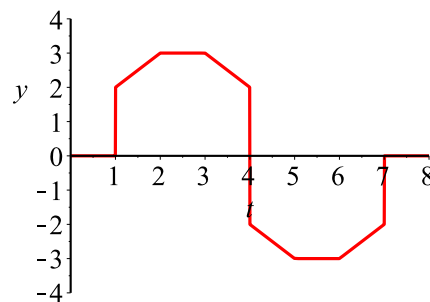


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FIN RESPUESTA 1)

> `restart`

2) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:



a) (15/100 puntos) OBTENER SU TRANSFORMADA DE LAPLACE.

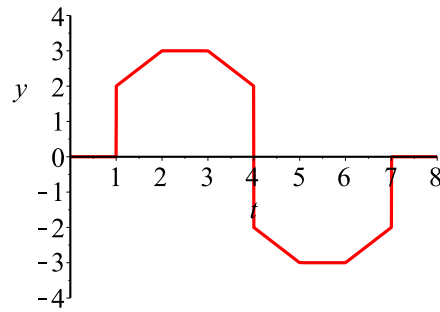
b) (25/100 puntos) OBTENER Y GRAFICAR SU SERIE COSENO DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

> `restart`

RESPUESTA 2a)

> `f:= 2·Heaviside(t-1) + (t-1)·Heaviside(t-1) - (t-2)·Heaviside(t-2) - (t-3)·Heaviside(t-3) + (t-4)·Heaviside(t-4) - 4·Heaviside(t-4) - (t-4)·Heaviside(t-4) + (t-5)·Heaviside(t-5) + (t-6)·Heaviside(t-6) - (t-7)·Heaviside(t-7) + 2·Heaviside(t-7); plot(f, t=0..8, y=-4..4)`

$f := 2 \text{Heaviside}(t-1) + (t-1) \text{Heaviside}(t-1) - (t-2) \text{Heaviside}(t-2) - (t-3) \text{Heaviside}(t-3) - 4 \text{Heaviside}(t-4) + (t-4) \text{Heaviside}(t-4) + (t-5) \text{Heaviside}(t-5) - (t-6) \text{Heaviside}(t-6) - (t-7) \text{Heaviside}(t-7) + 2 \text{Heaviside}(t-7)$



> with(inttrans) :

> F := laplace(f, t, s)

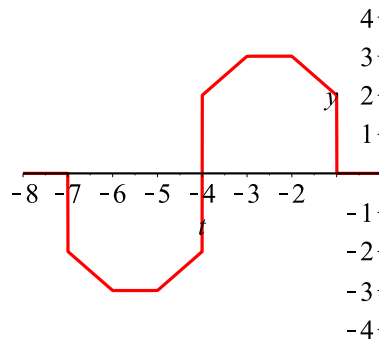
$$F := \frac{e^{-s} - e^{-7s} + e^{-6s} + e^{-5s} - e^{-3s} - e^{-2s}}{s^2} + \frac{2(e^{-s} + e^{-7s} - 2e^{-4s})}{s}$$

(7)

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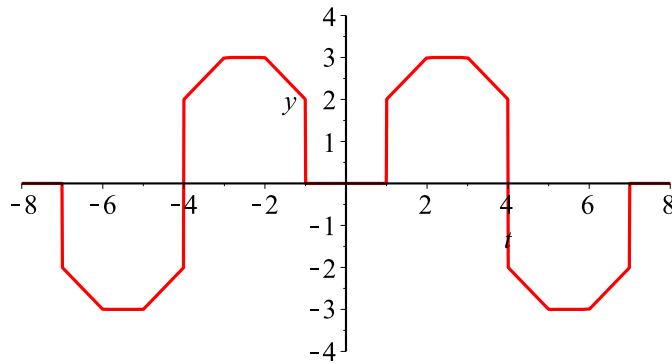
RESPUESTA 2b)

> g := -2 · Heaviside(t + 7) - (t + 7) · Heaviside(t + 7) + (t + 6) · Heaviside(t + 6) + (t + 5) · Heaviside(t + 5) - (t + 4) · Heaviside(t + 4) + 4 · Heaviside(t + 4) + (t + 4) · Heaviside(t + 4) - (t + 3) · Heaviside(t + 3) - (t + 2) · Heaviside(t + 2) + (t + 1) · Heaviside(t + 1) - 2 · Heaviside(t + 1) : plot(g, t=-8..0, y=-4..4)



> h := f + g; plot(h, t=-8..8, y=-4..4)

h := 2 Heaviside(t - 1) + (t - 1) Heaviside(t - 1) - (t - 2) Heaviside(t - 2) - (t - 3) Heaviside(t - 3) - 4 Heaviside(t - 4) + (t - 5) Heaviside(t - 5) + (t - 6) Heaviside(t - 6) - (t - 7) Heaviside(t - 7) + 2 Heaviside(t - 7) - 2 Heaviside(t + 7) - (t + 7) Heaviside(t + 7) + (t + 6) Heaviside(t + 6) + (t + 5) Heaviside(t + 5) + 4 Heaviside(t + 4) - (t + 3) Heaviside(t + 3) - (t + 2) Heaviside(t + 2) + (t + 1) Heaviside(t + 1) - 2 Heaviside(t + 1)



> L := 8

L := 8

(8)

> $a_0 := \left(\frac{1}{L}\right) \cdot \text{int}(h, t=-L..L); C := \frac{a_0}{2}$

$a_0 := 0$

C := 0

(9)

> $a_n := \text{simplify}\left(\left(\frac{1}{L}\right) \cdot \text{int}\left(h \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t=-L..L\right)\right)$

$$a_n := -\frac{1}{n^2 \pi^2} \left(4 \left(-4 \cos\left(\frac{1}{4} n \pi\right) + 4 \cos\left(\frac{1}{8} n \pi\right) + n \pi \sin\left(\frac{1}{8} n \pi\right) + 4 \cos\left(\frac{5}{8} n \pi\right) + 4 \cos\left(\frac{3}{4} n \pi\right) - 4 \cos\left(\frac{7}{8} n \pi\right) + n \pi \sin\left(\frac{7}{8} n \pi\right) - 2 \sin\left(\frac{1}{2} n \pi\right) n \pi - 4 \cos\left(\frac{3}{8} n \pi\right) \right) \right)$$

(10)

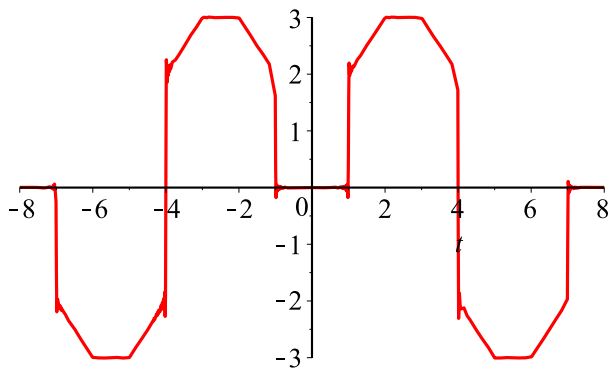
> $b_n := \text{simplify}\left(\left(\frac{1}{L}\right) \cdot \text{int}\left(h \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t=-L..L\right)\right)$

$b_n := 0$

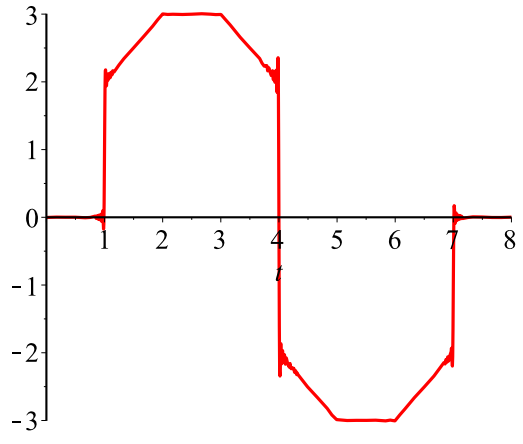
(11)

> $STF_{500} := \text{sum}\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n=1..500\right) :$

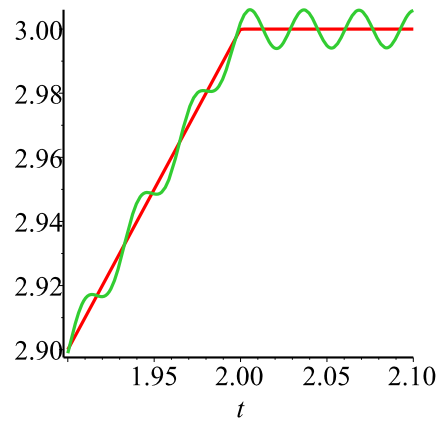
> $\text{plot}(STF_{500}, t=-8..8)$



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> plot(STF_500, t=0..8)
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> plot([f, STF_500], t=1.9..2.1)
```



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[FIN RESPUESTA 2)

> restart

3) (35/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA CONSTANTE DE SEPARACIÓN NEGATIVA:

$$\frac{\partial^2}{\partial x^2} y(x, t) + t^2 \left(\frac{\partial}{\partial t} y(x, t) \right) = \frac{\partial}{\partial x} y(x, t) \quad (12)$$

> restart

RESPUESTA 3)

> Ecuacion := $\frac{\partial^2}{\partial x^2} y(x, t) + t^2 \left(\frac{\partial}{\partial t} y(x, t) \right) = \frac{\partial}{\partial x} y(x, t)$

$$Ecuacion := \frac{\partial^2}{\partial x^2} y(x, t) + t^2 \left(\frac{\partial}{\partial t} y(x, t) \right) = \frac{\partial}{\partial x} y(x, t) \quad (13)$$

> EcuacionDos := eval(subs(y(x, t) = F(x) · G(t), Ecuacion))

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x) \right) G(t) + t^2 F(x) \left(\frac{d}{dt} G(t) \right) = \left(\frac{d}{dx} F(x) \right) G(t) \quad (14)$$

> EcuacionTres := lhs(EcuacionDos) - t^2 F(x) (d/dt G(t)) - (d/dx F(x)) G(t)
= rhs(EcuacionDos) - t^2 F(x) (d/dt G(t)) - (d/dx F(x)) G(t)

$$EcuacionTres := \left(\frac{d^2}{dx^2} F(x) \right) G(t) - \left(\frac{d}{dx} F(x) \right) G(t) = -t^2 F(x) \left(\frac{d}{dt} G(t) \right) \quad (15)$$

> EcuacionSeparada := simplify((lhs(EcuacionTres) = rhs(EcuacionTres)) / (F(x) · G(t)))

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x) \right)}{F(x)} = - \frac{t^2 \left(\frac{d}{dt} G(t) \right)}{G(t)} \quad (16)$$

> EcuacionX := lhs(EcuacionSeparada) = -beta · 2; EcuacionT := rhs(EcuacionSeparada) = -beta · 2

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x) \right)}{F(x)} = -\beta^2$$

$$EcuacionT := - \frac{t^2 \left(\frac{d}{dt} G(t) \right)}{G(t)} = -\beta^2 \quad (17)$$

> SolucionX := dsolve(EcuacionX); SolucionT := dsolve(EcuacionT)

$$SolucionX := F(x) = _C1 e^{\left(\frac{1}{2} + \frac{1}{2} \sqrt{1-4\beta^2}\right)x} + _C2 e^{\left(\frac{1}{2} - \frac{1}{2} \sqrt{1-4\beta^2}\right)x}$$

$$SolucionT := G(t) = _C1 e^{-\frac{\beta^2}{t}} \quad (18)$$

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> *SolucionNegativa* := $y(x, t) = rhs(SolucionX) \cdot subs(_C1 = 1, rhs(SolucionT))$

$$SolucionNegativa := y(x, t) = \left(_C1 e^{\left(\frac{1}{2} + \frac{1}{2} \sqrt{1-4\beta^2}\right)x} + _C2 e^{\left(\frac{1}{2} - \frac{1}{2} \sqrt{1-4\beta^2}\right)x} \right) e^{-\frac{\beta^2}{t}}$$

(19)

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FIN RESPUESTA 3)

> *restart*

FIN DEL SOLUCIÓN

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