>

UNAM
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
PRIMER EXAMEN FINAL
SEMESTRE 2020-1
ECUACIONES DIFERENCIALES
GRUPO 12

29 NOVIEMBRE 2019

> *restart*

1. Resuelva

> $Ecuacion := 2 \cdot x \cdot \cdot 2 \cdot diff(y(x), x) = y(x) \cdot (x + y(x))$

$$Ecuacion := 2\,x^2 \left(\frac{d}{dx} y(x) \right) = y(x)\ (x + y(x)) \quad \textbf{(1)}$$

> $Condicion := y(1) = -1$

$$Condicion := y(1) = -1 \quad \textbf{(2)}$$

solucion i)

> $SolPart := dsolve(\{Ecuacion, Condicion\})$

$$SolPart := y(x) = \frac{x^{3/2}}{\sqrt{x} - 2\,x} \quad \textbf{(3)}$$

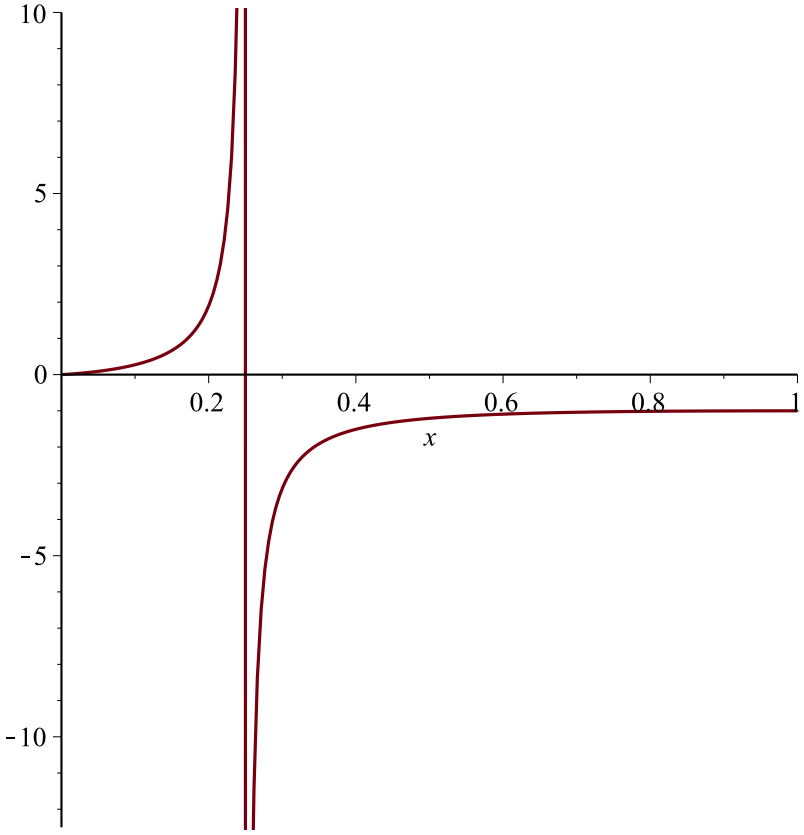
> $Comprobacion := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion := 0 = 0 \quad \textbf{(4)}$$

> $ComprobacionDos := subs(x = 1, SolPart)$

$$ComprobacionDos := y(1) = -1 \quad \textbf{(5)}$$

> $plot(rhs(SolPart), x = 0..1)$



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solucion ii)

> $with(DEtools)$:

> $Ecuacion$

$$2\,x^2\left(\frac{d}{dx}\,y(x)\right)=y(x)\,(x+y(x)) \tag{6}$$

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_Bernoulli] \tag{7}$$

> $EcuacionDos := simplify(isolate(eval(subs(y(x)=u(x)\cdot x, Ecuacion)), diff(u(x),x)))$

$$EcuacionDos := \frac{d}{dx}\,u(x)=\frac{1}{2}\,\frac{u(x)\,(u(x)-1)}{x} \tag{8}$$

> $P := \frac{1}{x}; Q := \frac{1}{2}\,u\,(u-1)$

$$P := \frac{1}{x}$$

$$Q := \frac{1}{2}\,u\,(u-1) \tag{9}$$

> $SolGral := int(P, x) + int\left(\frac{1}{Q}, u\right) = C[1]$

$$SolGral := \ln(x) + 2\ln(u-1) - 2\ln(u) = C_1 \quad (10)$$

> $SolGralDos := subs\left(u = \frac{y}{x}, SolGral\right)$

$$SolGralDos := \ln(x) + 2\ln\left(\frac{y}{x} - 1\right) - 2\ln\left(\frac{y}{x}\right) = C_1 \quad (11)$$

> $Condicion$

$$y(1) = -1 \quad (12)$$

> $Para := simplify(isolate(subs(x = 1, y = -1, SolGralDos), C[1]))$

$$Para := C_1 = 2\ln(2) \quad (13)$$

> $SolPartDos := isolate(subs(C[1] = rhs(Para), SolGralDos), y)$

$$SolPartDos := y = \frac{x^{3/2}}{\sqrt{x} - 2} \quad (14)$$

>

>

fin solucion 1

> $restart$

2. Obtenga la solucion general

> $Ecuacion := diff(y(x), x\$3) + 3\cdot diff(y(x), x\$2) + 4\cdot diff(y(x), x) + 12\cdot y(x) = -13\cdot\exp(-3\cdot x)$

$$Ecuacion := \frac{d^3}{dx^3} y(x) + 3\left(\frac{d^2}{dx^2} y(x)\right) + 4\left(\frac{d}{dx} y(x)\right) + 12\, y(x) = -13\, e^{-3x} \quad (15)$$

solucion

> $SolGral := simplify(dsolve(Ecuacion))$

$$SolGral := y(x) = (\_C1\cos(2x)\, e^{3x} + \_C3\sin(2x)\, e^{3x} + \_C2 - x)\, e^{-3x} \quad (16)$$

> $Comprobacion := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion := 0 = 0 \quad (17)$$

> $EcuaHom := lhs(Ecuacion) = 0$

$$EcuaHom := \frac{d^3}{dx^3} y(x) + 3\left(\frac{d^2}{dx^2} y(x)\right) + 4\left(\frac{d}{dx} y(x)\right) + 12\, y(x) = 0 \quad (18)$$

> $Q := rhs(Ecuacion)$

$$Q := -13\, e^{-3x} \quad (19)$$

> $EcuaCarac := m\cdot\cdot 3 + 3\cdot m\cdot\cdot 2 + 4\cdot m + 12 = 0$

$$EcuaCarac := m^3 + 3m^2 + 4m + 12 = 0 \quad (20)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -3, 2I, -2I \quad (21)$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \cos(\mathrm{Im}(Raiz[2])\cdot x); yy[3] := \sin(\mathrm{Im}(Raiz[2])\cdot x)$

$$yy_1 := e^{-3x}$$

$$yy_2 := \cos(2x)$$

$$yy_3 := \sin(2x) \quad (22)$$

```
> with(linalg) :
> WW := wronskian([yy[1], yy[2], yy[3]], x)
```

$$WW := \begin{bmatrix} e^{-3x} & \cos(2x) & \sin(2x) \\ -3e^{-3x} & -2\sin(2x) & 2\cos(2x) \\ 9e^{-3x} & -4\cos(2x) & -4\sin(2x) \end{bmatrix} \tag{23}$$

```
> BB := array([0, 0, Q])
```

$$BB := \begin{bmatrix} 0 & 0 & -13e^{-3x} \end{bmatrix} \tag{24}$$

```
> PRIMA := simplify(linsolve(WW, BB))
```

$$PRIMA := \begin{bmatrix} -1 & \frac{1}{2}(2\cos(2x) + 3\sin(2x))e^{-3x} & -\frac{1}{2}(-2\sin(2x) + 3\cos(2x))e^{-3x} \end{bmatrix} \tag{25}$$

```
> Aprima := PRIMA[1]; Bprima := PRIMA[2]; Dprima := PRIMA[3]
```

$$Aprima := -1$$

$$Bprima := \frac{1}{2}(2\cos(2x) + 3\sin(2x))e^{-3x}$$

$$Dprima := -\frac{1}{2}(-2\sin(2x) + 3\cos(2x))e^{-3x} \tag{26}$$

```
> AA := int(Aprima, x) + C[1]
```

$$AA := -x + C_1 \tag{27}$$

```
> BB := int(Bprima, x) + C[2]
```

$$BB := \frac{2}{13}(-3\cos(x) + 2\sin(x))e^{-3x}\cos(x) + \frac{3}{13(e^x)^3} + \frac{3}{26}e^{-3x}(-2\cos(2x) - 3\sin(2x)) + C_2 \tag{28}$$

```
> DD := int(Dprima, x) + C[3]
```

$$DD := \frac{1}{13}e^{-3x}(-2\cos(2x) - 3\sin(2x)) - \frac{3}{13}(-3\cos(x) + 2\sin(x))e^{-3x}\cos(x) - \frac{9}{26(e^x)^3} + C_3 \tag{29}$$

```
> SolNoHom := y(x) = simplify(subs(cos(x)^2 = 1/2 + 1/2·cos(2·x), cos(x) = 1/(2·sin(x))·sin(2·x),
      expand(simplify(expand(AA·yy[1] + BB·yy[2] + DD·yy[3])))))
```

$$SolNoHom := y(x) = \frac{1}{13}\left(13C_3e^{3x}\sin(2x) + 13C_2e^{3x}\cos(2x) - 6 - 13x + 13C_1\right)e^{-3x} \tag{30}$$

```
>
```

fin solucion 2

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> restart
```

3 Resuelva por Laplace

```
> Ecuacion := diff(y(t), t$2) + 3·diff(y(t), t) + 2·y(t) = -2·Dirac(t - 1)
```

$$Ecuacion := \frac{d^2}{dt^2}y(t) + 3\left(\frac{d}{dt}y(t)\right) + 2y(t) = -2\,\mathrm{Dirac}(t-1) \tag{31}$$

```
> Condiciones := y(0) = 1, D(y)(0) = 0
```

$$Condiciones := y(0) = 1, D(y)(0) = 0 \quad (32)$$

```
> with(inttrans) :
> LapEcua := subs(Condiciones, laplace(Ecuacion, t, s))
```

$$LapEcua := s^2\, laplace(y(t), t, s) - 3 - s + 3\, s\, laplace(y(t), t, s) + 2\, laplace(y(t), t, s) = -2\, e^{-s} \quad (33)$$

```
> LapSolucion := isolate(LapEcua, laplace(y(t), t, s))
```

$$LapSolucion := laplace(y(t), t, s) = \frac{-2\, e^{-s} + s + 3}{s^2 + 3\, s + 2} \quad (34)$$

```
> SolPart := invlaplace(LapSolucion, s, t)
```

$$SolPart := y(t) = -e^{-2t} + 2\, e^{-t} + 2\, \text{Heaviside}(t - 1)\, \left(e^{-2t+2} - e^{1-t}\right) \quad (35)$$

```
> Comprobacion := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecuacion) - rhs(Ecuacion)
      = 0)))
```

$$Comprobacion := 0 = 0 \quad (36)$$

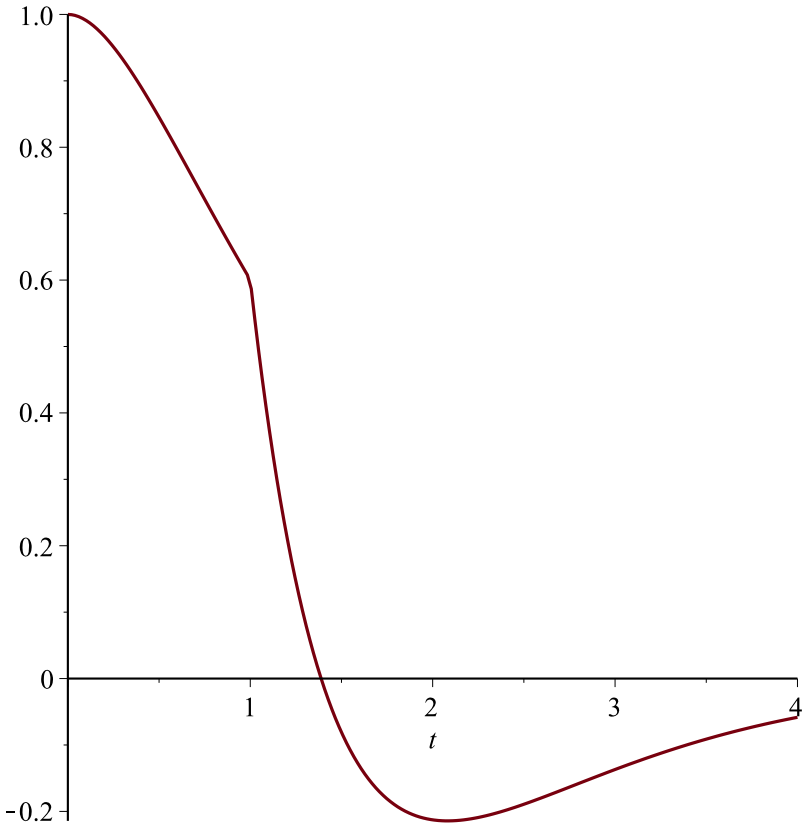
```
> ComprobacionDos := simplify(subs(t = 0, SolPart))
```

$$ComprobacionDos := y(0) = 1 \quad (37)$$

```
> ComprobacionTres := D(y)(0) = simplify(eval(subs(t = 0, rhs(diff(SolPart, t)))))
```

$$ComprobacionTres := D(y)(0) = 0 \quad (38)$$

```
> plot(rhs(SolPart), t = 0..4)
```



>

fin solucion 3

> *restart*

4. Resuelva el Sistema

> $Sist := diff(x(t), t) - 2\cdot y(t) = 0,\ diff(y(t), t) - 2\cdot x(t) = 0 : Sist[1]; Sist[2]$

$$\frac{d}{dt} x(t) - 2\,y(t) = 0$$

$$\frac{d}{dt} y(t) - 2\,x(t) = 0 \qquad \textbf{(39)}$$

> $Cond := x(0) = 1,\ y(0) = -1$

$$Cond := x(0) = 1,\ y(0) = -1 \qquad \textbf{(40)}$$

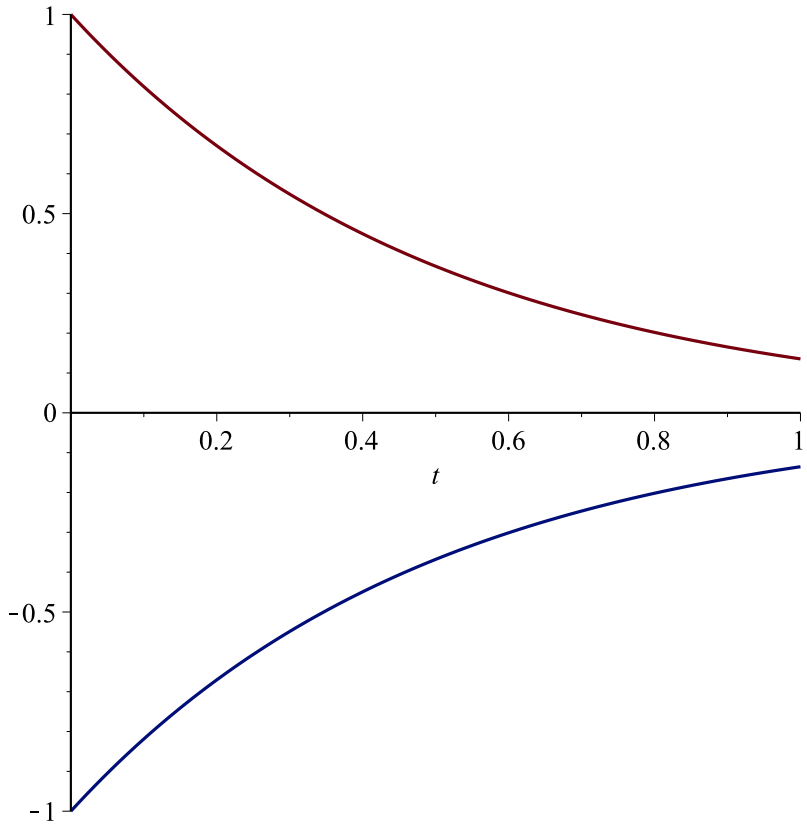
>

solución i)

> $Sol := dsolve(\{Sist, Cond\})$

$$Sol := \{x(t) = e^{-2t},\ y(t) = -e^{-2t}\} \qquad \textbf{(41)}$$

> $plot([rhs(Sol[1]), rhs(Sol[2])], t = 0..1)$



> $ComprobacionUno := simplify(eval(subs(x(t) = rhs(Sol[1]), y(t) = rhs(Sol[2]), Sist[1])))$

$$ComprobacionUno := 0 = 0 \quad \textbf{(42)}$$

> $ComprobacionDos := simplify(eval(subs(x(t) = rhs(Sol[1]), y(t) = rhs(Sol[2]), Sist[2])))$

$$ComprobacionDos := 0 = 0 \quad \textbf{(43)}$$

> $ComprobacionTres := simplify(subs(t = 0, Sol[1]))$

$$ComprobacionTres := x(0) = 1 \quad \textbf{(44)}$$

> $ComprobacionCuatro := simplify(subs(t = 0, Sol[2]))$

$$ComprobacionCuatro := y(0) = -1 \quad \textbf{(45)}$$

>

solucion ii)

> $AA := array([[0, -2], [-2, 0]])$

$$AA := \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} \quad \textbf{(46)}$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

(47)

$$MatExp := \begin{bmatrix} \frac{1}{2}e^{-2t} + \frac{1}{2}e^{2t} & -\frac{1}{2}e^{2t} + \frac{1}{2}e^{-2t} \\ -\frac{1}{2}e^{2t} + \frac{1}{2}e^{-2t} & \frac{1}{2}e^{-2t} + \frac{1}{2}e^{2t} \end{bmatrix} \quad (47)$$

> $BB := array([1, -1])$

$$BB := \begin{bmatrix} 1 & -1 \end{bmatrix} \quad (48)$$

> $SolPart := evalm(MatExp \&* BB)$

$$SolPart := \begin{bmatrix} e^{2t} & -e^{2t} \end{bmatrix} \quad (49)$$

> $Solucion[1] := x(t) = SolPart[1]$

$$Solucion_1 := x(t) = e^{2t} \quad (50)$$

> $Solucion[2] := y(t) = SolPart[2]$

$$Solucion_2 := y(t) = -e^{2t} \quad (51)$$

>

solución iii)

> $with(inttrans)$:

> $LapSist[1] := subs(Cond, laplace(Sist[1], t, s))$

$$LapSist_1 := s\,laplace(x(t), t, s) - 1 - 2\,laplace(y(t), t, s) = 0 \quad (52)$$

> $LapSist[2] := subs(Cond, laplace(Sist[2], t, s))$

$$LapSist_2 := s\,laplace(y(t), t, s) + 1 - 2\,laplace(x(t), t, s) = 0 \quad (53)$$

> $LapSistema := LapSist[1], LapSist[2]$

$$LapSistema := s\,laplace(x(t), t, s) - 1 - 2\,laplace(y(t), t, s) = 0, s\,laplace(y(t), t, s) + 1 - 2\,laplace(x(t), t, s) = 0 \quad (54)$$

> $LapSol := solve(\{LapSistema\}, \{laplace(x(t), t, s), laplace(y(t), t, s)\})$

$$LapSol := \left\{laplace(x(t), t, s) = \frac{1}{s+2}, laplace(y(t), t, s) = -\frac{1}{s+2}\right\} \quad (55)$$

> $SolPart[1] := invlaplace(LapSol[1], s, t)$

$$SolPart_1 := x(t) = e^{-2t} \quad (56)$$

> $SolPart[2] := invlaplace(LapSol[2], s, t)$

$$SolPart_2 := y(t) = -e^{-2t} \quad (57)$$

>

fin solucion 4

> $restart$

>

5. Obtener la STF en senos

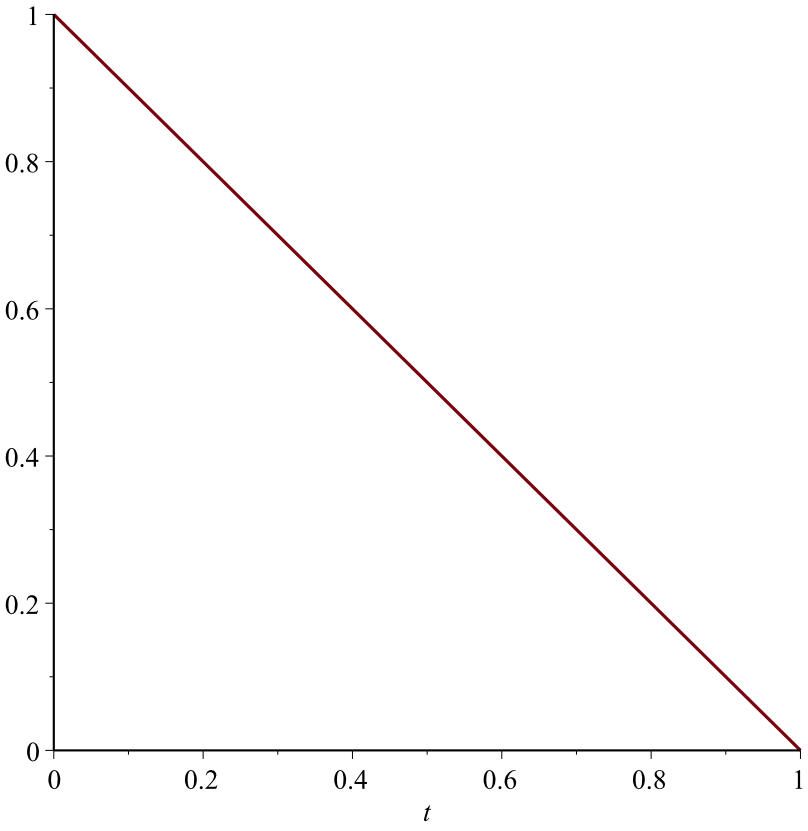
>

solución i)

> $ff := 1 - t$

$$ff := 1 - t \quad (58)$$

> $plot(ff, t = 0..1)$



> $L := 1$

$$L := 1 \tag{59}$$

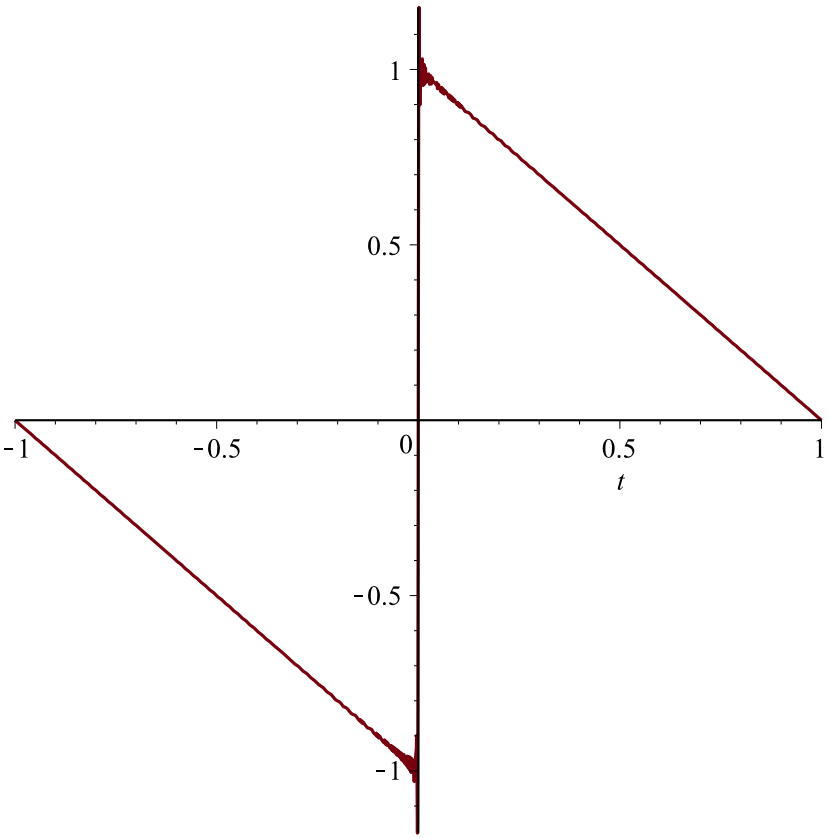
>

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0,\ \frac{2}{L} \cdot int\left(ff \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = 0..L\right)\right)$

$$b_n := \frac{2}{n\,\pi} \tag{60}$$

> $STF500 := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..500\right) :$

> $plot(STF500, t = -1..1)$



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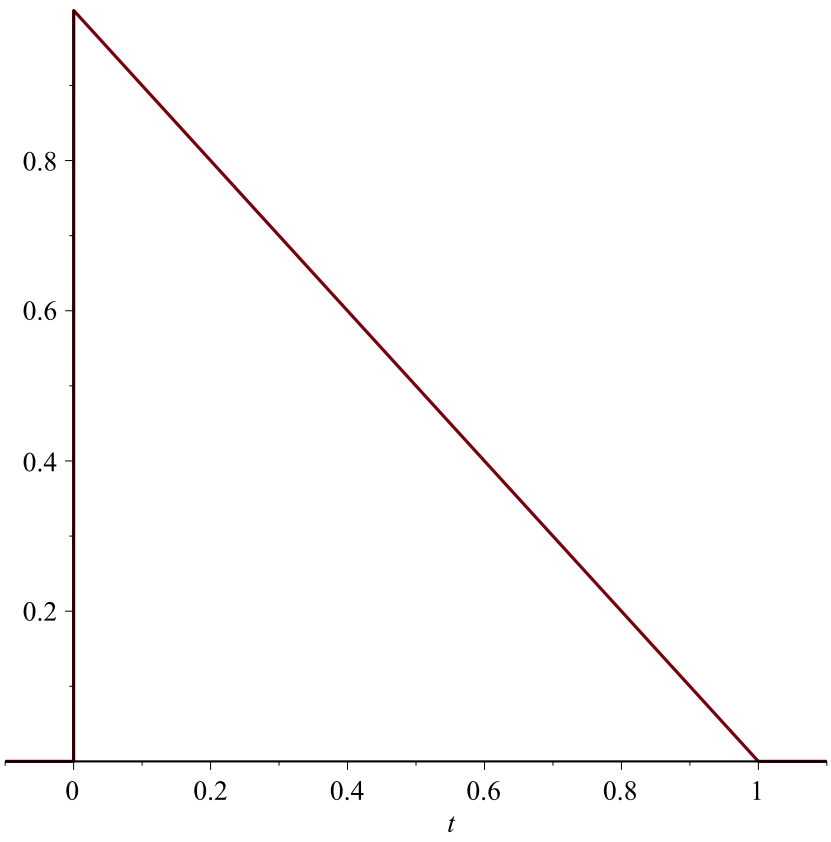
solución ii)

> $f := \text{Heaviside}(t) - t \cdot \text{Heaviside}(t) + t \cdot \text{Heaviside}(t-1) - \text{Heaviside}(t-1); plot(f, t = -0.1..1.1)$

$$f := \text{Heaviside}(t) - t\,\text{Heaviside}(t) + t\,\text{Heaviside}(t-1) - \text{Heaviside}(t-1)$$

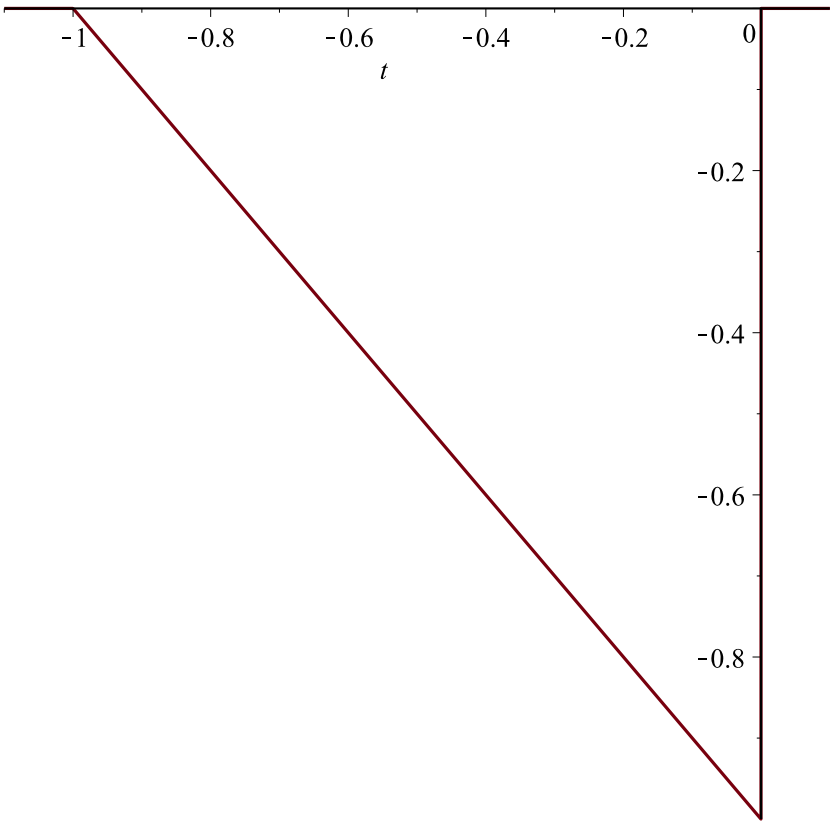


> $L := 1$

$$L := 1 \tag{61}$$

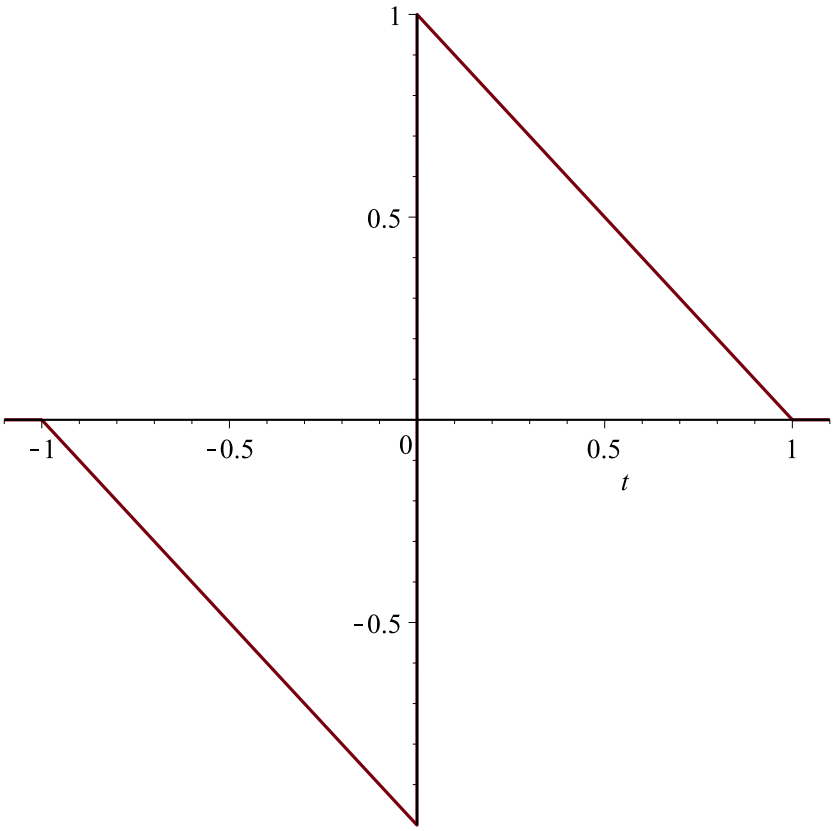
> $g := -\text{Heaviside}(t+1) - t \cdot \text{Heaviside}(t+1) + t \cdot \text{Heaviside}(t) + \text{Heaviside}(t); plot(g, t = -1.1..0.1)$

$$g := -\text{Heaviside}(t+1) - t\,\text{Heaviside}(t+1) + t\,\text{Heaviside}(t) + \text{Heaviside}(t)$$



> $h := f + g; plot(h, t = -1.1 .. 1.1)$

$$h := 2\,\text{Heaviside}(t) + t\,\text{Heaviside}(t-1) - \text{Heaviside}(t-1) - \text{Heaviside}(t+1) - t\,\text{Heaviside}(t+1)$$



> $a[0] := \frac{1}{L} \cdot int(h, t = -L..L)$

$$a_0 := 0 \quad \textbf{(62)}$$

> $a[n] := \frac{1}{L} \cdot int\left(h \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)$

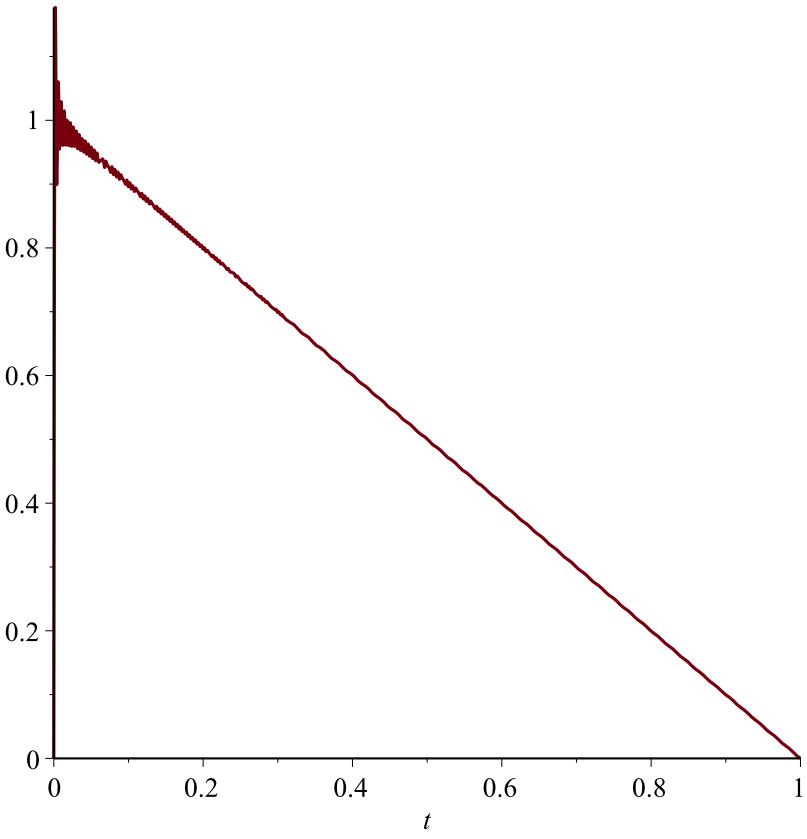
$$a_n := 0 \quad \textbf{(63)}$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \frac{1}{L} \cdot int\left(h \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$

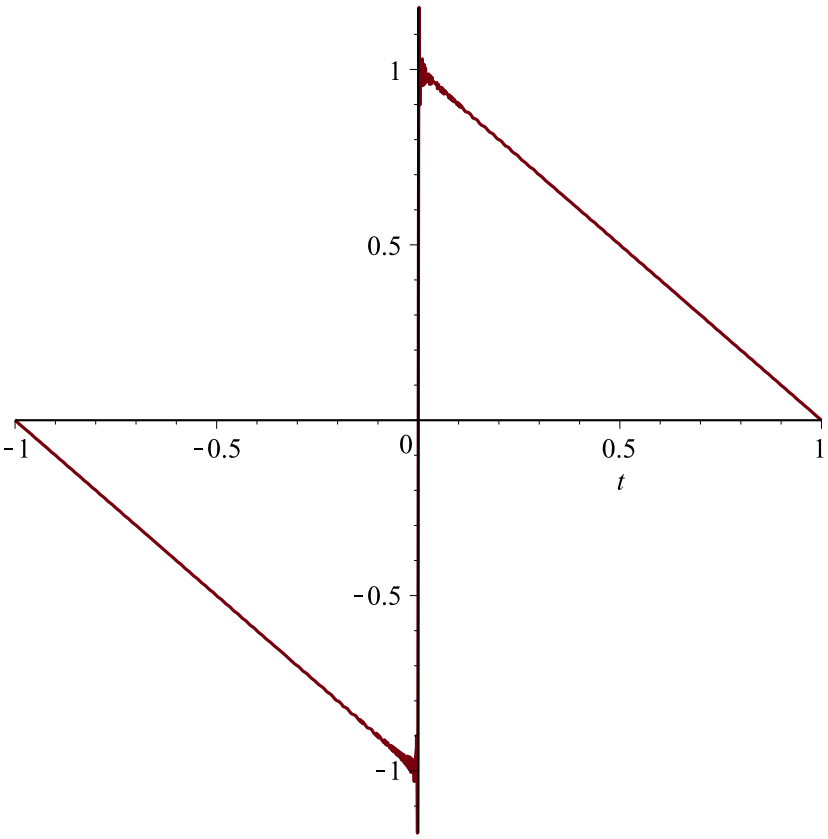
$$b_n := \frac{2}{n\,\pi} \quad \textbf{(64)}$$

> $STF500 := sum\left(b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..500\right) :$

> $plot(STF500, t = 0..1)$



> $plot(STF500, t = -1..1)$



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fin respuesta 5

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fin examen

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