UNAM
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
SEGUNDO EXAMEN FINAL
SEMESTRE 2020-1
ECUACIONES DIFERENCIALES
GRUPO 12

6 DICIEMBRE 2019

```
> restart
```

1. Resuelva

```
> Ecuacion := x·diff(y(x), x) − y(x) = x··3·exp(3·x)
```

$$Ecuacion := x\left(\frac{d}{dx}\,y(x)\right) - y(x) = x^3\,e^{3x} \tag{1}$$

```
> Condicion := y(1) = 0
```

$$Condicion := y(1) = 0 \tag{2}$$

Solución i)

```
> SolucionParticular := expand(dsolve({Ecuacion, Condicion})); evalf(%, 2)
```

$$SolucionParticular := y(x) = \frac{1}{3}\,\left(e^x\right)^3 x^2 - \frac{1}{9}\,\left(e^x\right)^3 x - \frac{2}{9}\,e^3 x$$

$$y(x) = 0.33\,\left(e^x\right)^3 x^2 - 0.11\,\left(e^x\right)^3 x - 4.4\,x \tag{3}$$

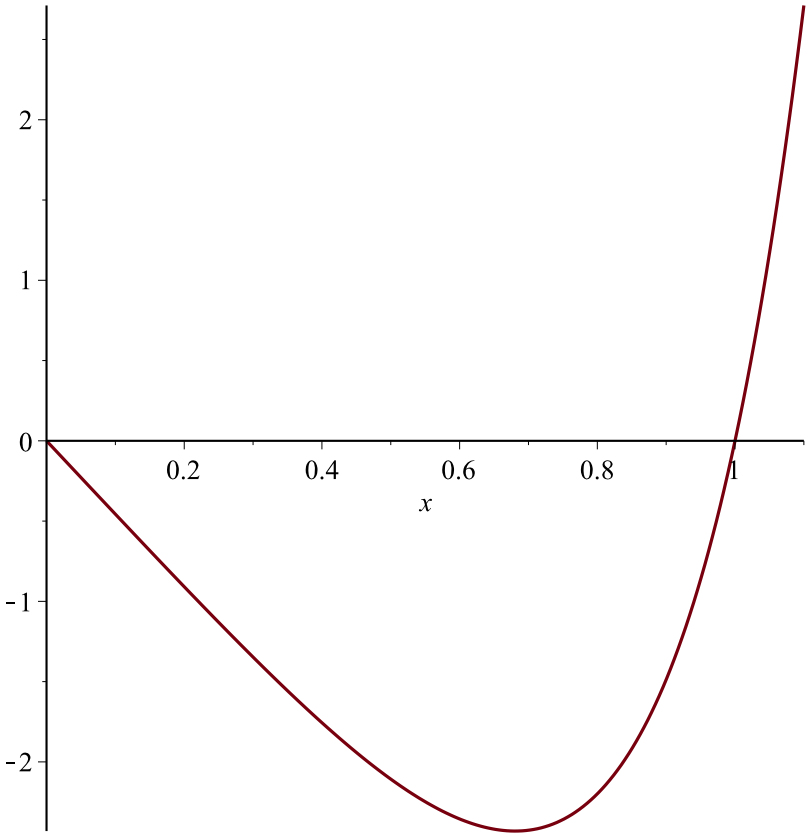
```
> CompUno := simplify(eval(subs(y(x) = rhs(SolucionParticular), lhs(Ecuacion)
      − rhs(Ecuacion) = 0)))
```

$$CompUno := 0 = 0 \tag{4}$$

```
> CompDos := simplify(subs(x = 1, SolucionParticular))
```

$$CompDos := y(1) = 0 \tag{5}$$

```
> plot(rhs(SolucionParticular), x = 0..1.1)
```



solucion ii)

> $Ecuacion$

$$x\left(\frac{d}{dx}y(x)\right) - y(x) = x^3 e^{3x} \quad (6)$$

> $EcuaDos := expand\left(\frac{lhs(Ecuacion)}{x} = \frac{rhs(Ecuacion)}{x}\right)$

$$EcuaDos := \frac{d}{dx}y(x) - \frac{y(x)}{x} = \left(e^x\right)^3 x^2 \quad (7)$$

> $p := -\frac{1}{x}; q := rhs(EcuaDos)$

$$p := -\frac{1}{x}$$

$$q := \left(e^x\right)^3 x^2 \quad (8)$$

> $SolGral := y(x) = expand(C[1]\cdot\exp(-int(p,x)) + \exp(-int(p,x))\cdot int(\exp(int(p,x))\cdot q, x))$

$$SolGral := y(x) = C_1 x + \frac{1}{3}\left(e^x\right)^3 x^2 - \frac{1}{9}\left(e^x\right)^3 x \quad (9)$$

> $Para := isolate(subs(x=1, rhs(SolGral) = 0), C[1])$

(10)

$$Para := C_1 = -\frac{2}{9}\,(e)^3 \tag{10}$$

> $SolPart := subs(C[1] = rhs(Para), SolGral); evalf(\%, 2)$

$$SolPart := y(x) = -\frac{2}{9}\,(e)^3 x + \frac{1}{3}\,\left(e^x\right)^3 x^2 - \frac{1}{9}\,\left(e^x\right)^3 x$$

$$y(x) = 0.33\,\left(e^x\right)^3 x^2 - 0.11\,\left(e^x\right)^3 x - 4.4\,x \tag{11}$$

> $CompTres := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$CompTres := 0 = 0 \tag{12}$$

> $CompCuatro := simplify(subs(x = 1, SolPart))$

$$CompCuatro := y(1) = 0 \tag{13}$$

fin solucion 1)

> $restart$

2. Resuelva

> $Ecuacion := x\cdot\cdot 2\cdot diff(y(x), x\$2) - (x\cdot\cdot 2 + 2\cdot x)\cdot diff(y(x), x) + (x+2)\cdot y(x) = x\cdot\cdot 3$

$$Ecuacion := x^2\left(\frac{d^2}{dx^2}\,y(x)\right) - \left(x^2 + 2\,x\right)\left(\frac{d}{dx}\,y(x)\right) + (x+2)\,y(x) = x^3 \tag{14}$$

solucion i)

> $SolucionGeneral := dsolve(Ecuacion)$

$$SolucionGeneral := y(x) = x\,\_C2 + x\,e^x\,\_C1 - x^2 \tag{15}$$

> $CompUno := simplify(eval(subs(y(x) = rhs(SolucionGeneral), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$CompUno := 0 = 0 \tag{16}$$

solucion ii)

> $yy[1] := x; yy[2] := x\cdot\exp(x)$

$$yy_1 := x$$

$$yy_2 := x\,e^x \tag{17}$$

> $EcuacionNormal := expand\left(\frac{lhs(Ecuacion)}{x\cdot\cdot 2}\right) = \frac{rhs(Ecuacion)}{x\cdot\cdot 2}$

$$EcuacionNormal := \frac{d^2}{dx^2}\,y(x) - \left(\frac{d}{dx}\,y(x)\right) - \frac{2\left(\frac{d}{dx}\,y(x)\right)}{x} + \frac{y(x)}{x} + \frac{2\,y(x)}{x^2} = x \tag{18}$$

> $EcuacionHom := lhs(EcuacionNormal) = 0$

$$EcuacionHom := \frac{d^2}{dx^2}\,y(x) - \left(\frac{d}{dx}\,y(x)\right) - \frac{2\left(\frac{d}{dx}\,y(x)\right)}{x} + \frac{y(x)}{x} + \frac{2\,y(x)}{x^2} = 0 \tag{19}$$

> $Q := rhs(EcuacionNormal)$

$$Q := x \tag{20}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} x & x\,e^x \\ 1 & e^x + x\,e^x \end{bmatrix} \tag{21}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x \end{bmatrix} \tag{22}$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} -1 & \frac{1}{e^x} \end{bmatrix} \tag{23}$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -1$$

$$Bprima := \frac{1}{e^x} \tag{24}$$

> $A := int(Aprima, x) + C[1]; B := int(Bprima, x) + C[2]$

$$A := -x + C_1$$

$$B := -\frac{1}{e^x} + C_2 \tag{25}$$

> $SolNoHom := y(x) = expand(A \cdot yy[1] + B \cdot yy[2])$

$$SolNoHom := y(x) = -x^2 + x\,C_1 - x + x\,e^x\,C_2 \tag{26}$$

> $CompDos := simplify(eval(subs(y(x) = rhs(SolNoHom), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$CompDos := 0 = 0 \tag{27}$$

fin solucion 2)

> $restart$

3. Utilizando transformada de Laplace obtener i(t)=q'(t)

> $Ecuacion := diff(q(t), t\$2) - 2 \cdot diff(q(t), t) + q(t) = \frac{1}{4} \cdot \cos(t)$

$$Ecuacion := \frac{d^2}{dt^2} q(t) - 2\left(\frac{d}{dt} q(t)\right) + q(t) = \frac{1}{4}\cos(t) \tag{28}$$

> $Condiciones := q(0) = 0, D(q)(0) = 0$

$$Condiciones := q(0) = 0, D(q)(0) = 0 \tag{29}$$

solución

> $with(inttrans)$:

> $LapEcua := subs(Condiciones, laplace(Ecuacion, t, s))$

$$LapEcua := s^2\,laplace(q(t), t, s) - 2\,s\,laplace(q(t), t, s) + laplace(q(t), t, s) = \frac{1}{4}\,\frac{s}{s^2+1} \tag{30}$$

> $LapSol := isolate(LapEcua, laplace(q(t), t, s))$

$$LapSol := laplace(q(t), t, s) = \frac{1}{4}\,\frac{s}{(s^2+1)\,(s^2-2\,s+1)} \tag{31}$$

> $SolPart := invlaplace(LapSol, s, t)$

$$SolPart := q(t) = \frac{1}{8}\,t\,e^t - \frac{1}{8}\sin(t) \tag{32}$$

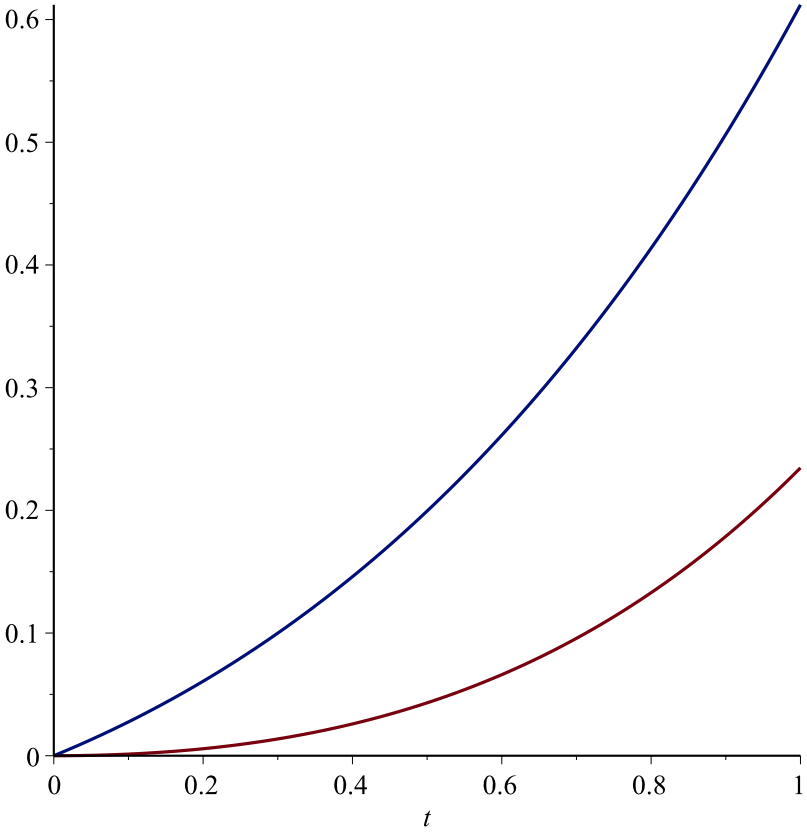
> $Solucion := i(t) = rhs(diff(SolPart, t))$

$$Solucion := i(t) = \frac{1}{8}\,e^t + \frac{1}{8}\,t\,e^t - \frac{1}{8}\cos(t) \tag{33}$$

> $comprobación := eval(subs(q(t) = rhs(SolPart), lhs(Ecuacion) - rhs(Ecuacion) = 0))$

$$comprobación := 0 = 0 \quad \textbf{(34)}$$

> $plot([rhs(SolPart), rhs(Solucion)], t = 0..1)$



fin solucion 3)

> $restart$

4.- Solución del Sistema

> $Sistema := diff(x(t), t) + 2 \cdot diff(y(t), t) = \exp(t), 2 \cdot diff(x(t), t) + diff(y(t), t) = \sin(t)$:
$Sistema[1]; Sistema[2]$

$$\frac{d}{dt} x(t) + 2 \left(\frac{d}{dt} y(t) \right) = e^t$$

$$2 \left(\frac{d}{dt} x(t) \right) + \frac{d}{dt} y(t) = \sin(t) \quad \textbf{(35)}$$

> $Cond := x(0) = 1, y(0) = -1$

$$Cond := x(0) = 1, y(0) = -1 \quad \textbf{(36)}$$

solución

> $Solucion := dsolve(\{Sistema, Cond\}) : Solucion[1]; Solucion[2]$

$$x(t) = -\frac{1}{3} e^t - \frac{2}{3} \cos(t) + 2$$

(37)

$$y(t) = \frac{2}{3}\, e^{t} + \frac{1}{3}\, \cos(t) - 2 \tag{37}$$

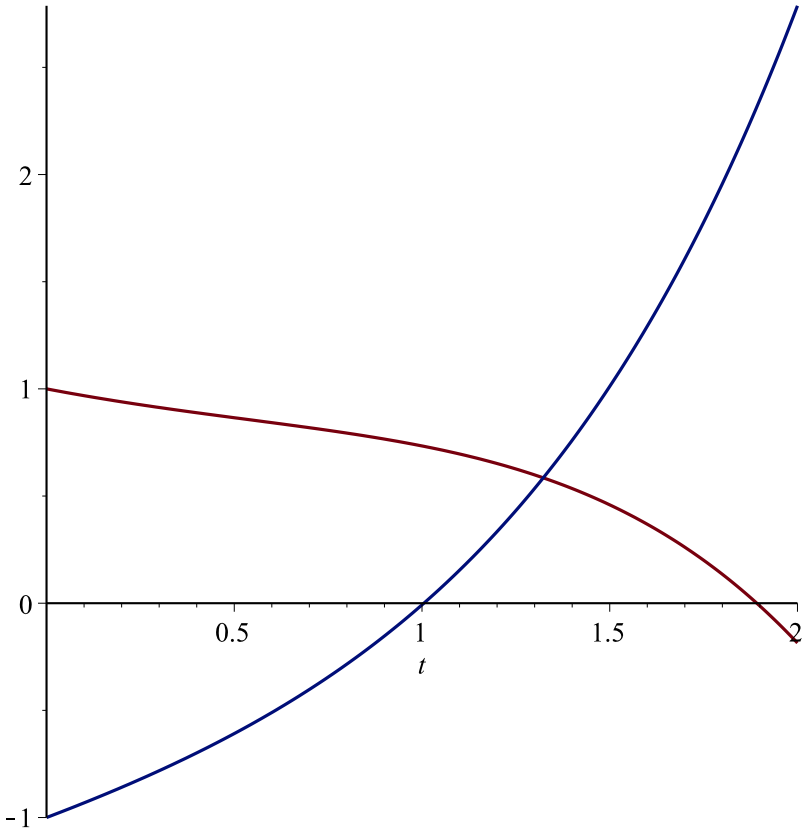
```
> CompUno := eval(subs(x(t) = rhs(Solucion[1]), y(t) = rhs(Solucion[2]), lhs(Sistema[1])
      - rhs(Sistema[1]) = 0))
```

$$CompUno := 0 = 0 \tag{38}$$

```
> CompDos := eval(subs(x(t) = rhs(Solucion[1]), y(t) = rhs(Solucion[2]), lhs(Sistema[2])
      - rhs(Sistema[2]) = 0))
```

$$CompDos := 0 = 0 \tag{39}$$

```
> plot([rhs(Solucion[1]), rhs(Solucion[2])], t = 0..2)
```



fin solucion 4

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> restart
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5.- Resuelva para alpha = 1

```
> Ecua := diff(u(x, t), t$3) = 4·diff(u(x, t), x, t)
```

$$Ecua := \frac{\partial^3}{\partial t^3}\, u(x, t) = 4 \left(\frac{\partial^2}{\partial x\, \partial t}\, u(x, t) \right) \tag{40}$$

```
> EcuaDos := eval(subs(u(x, t) = F(x)·G(t), Ecua))
```

$$EcuaDos := F(x) \left(\frac{d^3}{dt^3}\, G(t) \right) = 4 \left(\frac{d}{dx}\, F(x) \right) \left(\frac{d}{dt}\, G(t) \right) \tag{41}$$

solucion i)

> $EcuaTres := \frac{lhs(EcuaDos)}{F(x)\cdot diff(G(t),t)} = \frac{rhs(EcuaDos)}{F(x)\cdot diff(G(t),t)}$

$$EcuaTres := \frac{\frac{d^3}{dt^3} G(t)}{\frac{d}{dt} G(t)} = \frac{4\left(\frac{d}{dx} F(x)\right)}{F(x)} \quad (42)$$

> $EcuaTresX := rhs(EcuaTres) = 1;\ EcuaTresT := lhs(EcuaTres) = 1$

$$EcuaTresX := \frac{4\left(\frac{d}{dx} F(x)\right)}{F(x)} = 1$$

$$EcuaTresT := \frac{\frac{d^3}{dt^3} G(t)}{\frac{d}{dt} G(t)} = 1 \quad (43)$$

> $SolTresX := dsolve(EcuaTresX);\ SolTresT := dsolve(EcuaTresT)$

$$SolTresX := F(x) = \_C1\, e^{\frac{1}{4}x}$$

$$SolTresT := G(t) = \_C1 + \_C2\, e^{t} + \_C3\, e^{-t} \quad (44)$$

> $SolGral := u(x,t) = subs(\_C1 = 1, rhs(SolTresX))\cdot rhs(SolTresT)$

$$SolGral := u(x,t) = e^{\frac{1}{4}x}\left(\_C1 + \_C2\, e^{t} + \_C3\, e^{-t}\right) \quad (45)$$

> $CompUno := eval(subs(u(x,t) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0))$

$$CompUno := 0 = 0 \quad (46)$$

solucion ii)

> $EcuaCuatro := \frac{lhs(EcuaDos)}{4\cdot F(x)\cdot diff(G(t),t)} = \frac{rhs(EcuaDos)}{4\cdot F(x)\cdot diff(G(t),t)}$

$$EcuaCuatro := \frac{1}{4}\,\frac{\frac{d^3}{dt^3} G(t)}{\frac{d}{dt} G(t)} = \frac{\frac{d}{dx} F(x)}{F(x)} \quad (47)$$

> $EcuaCuatroX := rhs(EcuaCuatro) = 1;\ EcuaCuatroT := lhs(EcuaCuatro) = 1$

$$EcuaCuatroX := \frac{\frac{d}{dx} F(x)}{F(x)} = 1$$

$$EcuaCuatroT := \frac{1}{4}\,\frac{\frac{d^3}{dt^3} G(t)}{\frac{d}{dt} G(t)} = 1 \quad (48)$$

> $SolCuatroX := dsolve(EcuaCuatroX);\ SolCuatroT := dsolve(EcuaCuatroT)$

$$SolCuatroX := F(x) = \_C1\, e^{x}$$

$$SolCuatroT := G(t) = \_C1 + \_C2\, e^{-2t} + \_C3\, e^{2t} \quad (49)$$

```
> SolGralDos := u(x, t) = subs(_C1 = 1, rhs(SolCuatroX)) · rhs(SolCuatroT)
```

$$SolGralDos := u(x,t) = e^{x}\left(\_C1 + \_C2\, e^{-2t} + \_C3\, e^{2t}\right) \quad \textbf{(50)}$$

```
> CompDos := simplify(eval(subs(u(x, t) = rhs(SolGralDos), lhs(Ecua) − rhs(Ecua) = 0)))
```

$$CompDos := 0 = 0 \quad \textbf{(51)}$$

fin solución 5)

```
> restart
```

Fin solución examen

```
>
>
```