FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN PARCIAL
SEMESTRE 2020-1

2019 NOVIEMBRE 21

> *restart*

1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE (**sin usar dsolve**):
a) **(15/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS
b) **(15/100 puntos)** GRAFICAR -JUNTAS- LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA PARA UN INTERVALO DE 0 < t < 1

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> $\frac{d^2}{dt^2} y(t) - 10\left(\frac{d}{dt} y(t)\right) + 25\, y(t) = 49 \cdot \text{Heaviside}(t-5)\sin(3t-15); y(0) = -1; D(y)(0) = 1$

$$\frac{d^2}{dt^2} y(t) - 10\left(\frac{d}{dt} y(t)\right) + 25\, y(t) = 49\, \text{Heaviside}(t-5)\sin(3t-15)$$

$$y(0) = -1$$

$$D(y)(0) = 1 \quad (1)$$

**RESPUESTA 1a)**

> $Ecuacion := \frac{d^2}{dt^2} y(t) - 10\left(\frac{d}{dt} y(t)\right) + 25\, y(t) = 49 \cdot \text{Heaviside}(t-5)\sin(3t-15)$

$$Ecuacion := \frac{d^2}{dt^2} y(t) - 10\left(\frac{d}{dt} y(t)\right) + 25\, y(t) = 49\, \text{Heaviside}(t-5)\sin(3t-15) \quad (2)$$

> $Condiciones := y(0) = -1, D(y)(0) = 1$

$$Condiciones := y(0) = -1, D(y)(0) = 1 \quad (3)$$

> $with(inttrans):$

> $EcuaTransLap := subs(Condiciones, laplace(Ecuacion, t, s))$

$$EcuaTransLap := s^2\, laplace(y(t), t, s) - 11 + s - 10\, s\, laplace(y(t), t, s) + 25\, laplace(y(t), t, s) = \frac{147\, e^{-5s}}{s^2+9} \quad (4)$$

> $SolTransLap := simplify(isolate(EcuaTransLap, laplace(y(t), t, s)))$

$$SolTransLap := laplace(y(t), t, s) = \frac{-s^3 + 11 s^2 + 147\, e^{-5s} - 9s + 99}{(s^2+9)(s^2-10s+25)} \quad (5)$$

> $Solucion := invlaplace(SolTransLap, s, t)$

$$Solucion := y(t) = e^{5t}(-1+6t) + \frac{147}{578}(-\text{Heaviside}(5-t)+1)\, e^{5t-25}(17t-90) + \frac{49}{578}\text{Heaviside}(t-5)(15\cos(3t-15) + 8\sin(3t-15)) \quad (6)$$

> $Comprobacion := simplify(eval(subs(y(t) = rhs(Solucion), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

(7)

$$Comprobacion := 0 = 0 \qquad (7)$$

> $ComprobacionDos := simplify(subs(t = 0, Solucion))$

$$ComprobacionDos := y(0) = -1 \qquad (8)$$

> $ComprobacionTres := D(y)(0) = eval(simplify(subs(t = 0, rhs(diff(Solucion, t)))))$

$$ComprobacionTres := D(y)(0) = 1 \qquad (9)$$

**FIN RESPUESTA 1a)**

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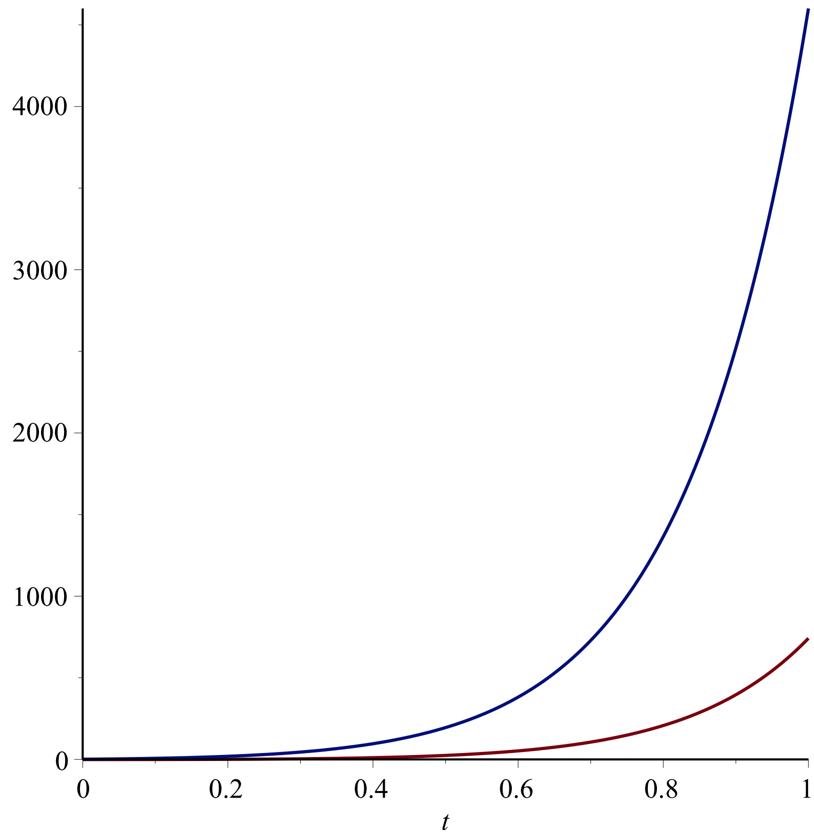
**RESPUESTA 1b)**

> $plot([rhs(Solucion), rhs(diff(Solucion, t))], t = 0..1)$




**FIN RESPUESTA 1b)**

> $restart$

2) DE LA FUNCIÓN DIBUJADA:
a) **(15/100 puntos)** OBTENER SU TRANSFORMADA DE LAPLACE
b) **(25/100 puntos)** GRAFICAR - JUNTAS - EN EL INTERVALO 1.8 < x < 2.2 A LA FUNCION Y SU SERIE TRIGONOMÉTRICA DE FOURIER OBTENIDA CALCULANDO SUS PRIMEROS 500 TÉRMINOS

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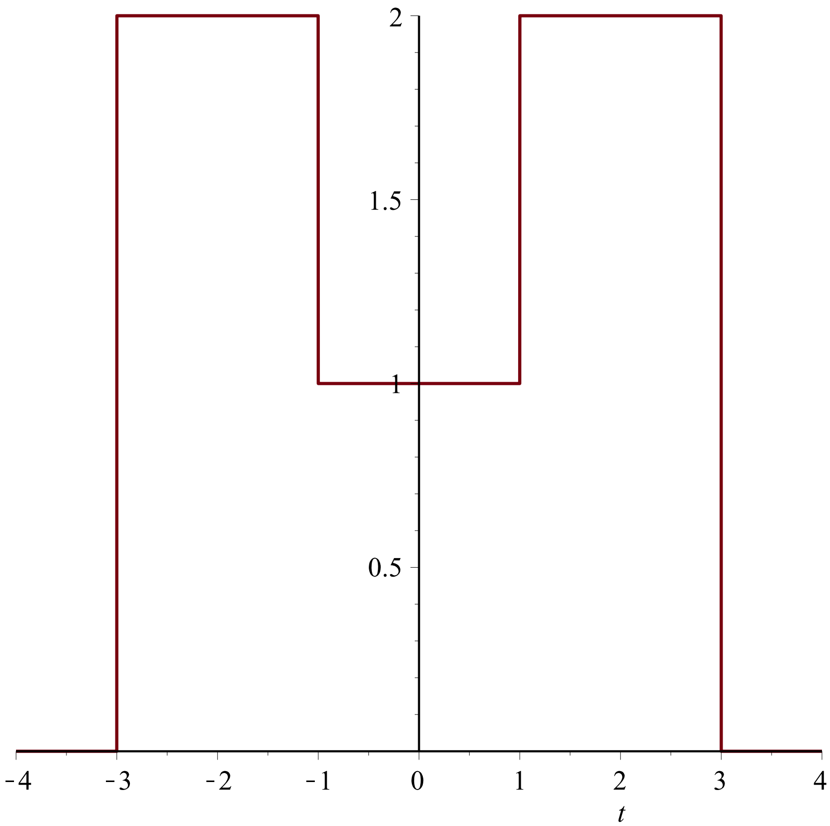
**RESPUESTA 2a)**

> $f := 2\cdot\text{Heaviside}(t+3) - \text{Heaviside}(t+1) + \text{Heaviside}(t-1) - 2\cdot\text{Heaviside}(t-3) : plot(f, t = -4..4)$




> $with(inttrans) :$

> $F := laplace(f, t, s)$

$$F := \frac{1 + e^{-s} - 2\,e^{-3\,s}}{s} \quad \textbf{(10)}$$

**FIN RESPUESTA 2a)**

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**RESPUESTA 2b)**

> $L := 4$

$$L := 4 \quad \textbf{(11)}$$

POR SIMETRÍA DEBE CALCULARSE UNA SERIE SENO DE FOURIER

> $a[0] := \frac{1}{L}\cdot int(f, t = -L..L);\ C := \frac{a[0]}{2}$

$$a_0 := \frac{5}{2}$$

**(12)**

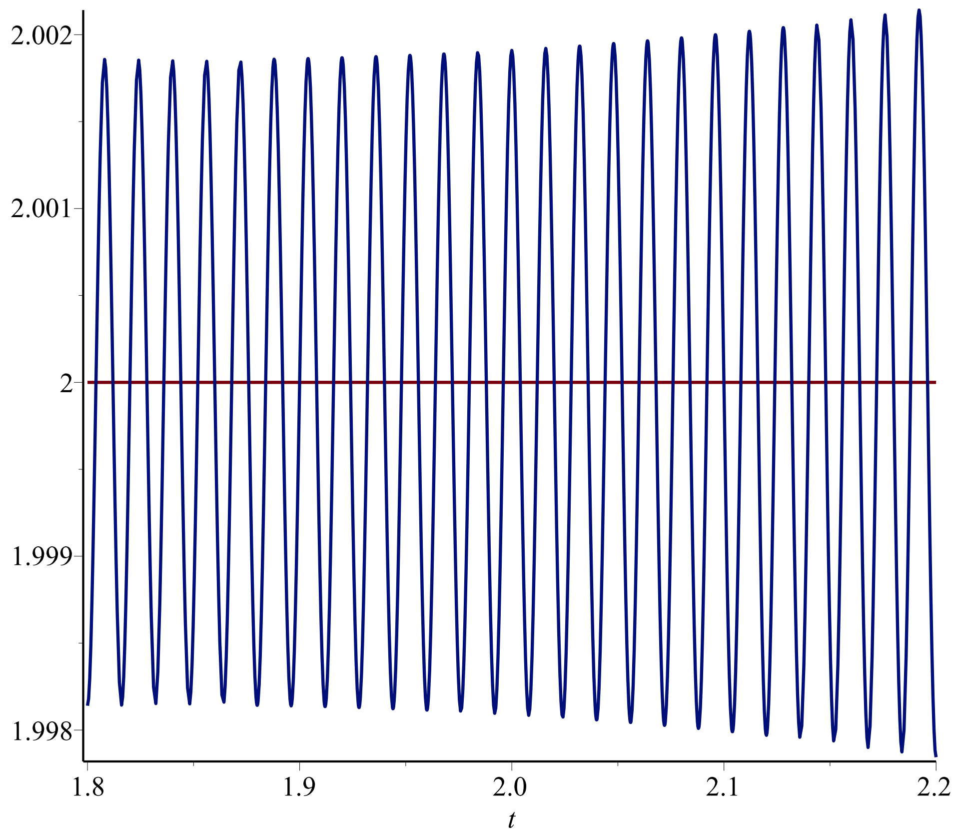
$$C := \frac{5}{4} \qquad (12)$$

> $a[n] := \left(\frac{1}{L}\right) \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)$

$$a_n := \frac{4 \sin\left(\frac{3}{4} n \pi\right)}{n \pi} - \frac{2 \sin\left(\frac{1}{4} n \pi\right)}{n \pi} \qquad (13)$$

> $STF[500] := C + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..500\right) :$

> $plot([f, STF[500]], t = 1.8..2.2)$




**FIN RESPUESTA 2b)**

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3) **(30/100 puntos)** OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA CONSTANTE DE SEPARACIÓN **POSITIVA**:

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> $\frac{\partial^2}{\partial x^2} z(x,y) - y^2 \left(\frac{\partial}{\partial y} z(x,y)\right) = 5 \cdot z(x,y)$

$$\frac{\partial^2}{\partial x^2} z(x,y) - y^2 \left(\frac{\partial}{\partial y} z(x,y)\right) = 5\, z(x,y) \quad \textbf{(14)}$$

**RESPUESTA 3)**

> $Ecuacion := \frac{\partial^2}{\partial x^2} z(x,y) - y^2 \left(\frac{\partial}{\partial y} z(x,y)\right) = 5 \cdot z(x,y)$

$$Ecuacion := \frac{\partial^2}{\partial x^2} z(x,y) - y^2 \left(\frac{\partial}{\partial y} z(x,y)\right) = 5\, z(x,y) \quad \textbf{(15)}$$

> $EcuacionDos := eval(subs(z(x,y) = F(x) \cdot G(y), Ecuacion))$

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x)\right) G(y) - y^2 F(x) \left(\frac{d}{dy} G(y)\right) = 5\, F(x)\, G(y) \quad \textbf{(16)}$$

**alternativa uno**

> $EcuacionTres := \frac{\left(lhs(EcuacionDos) + y^2 F(x)\left(\frac{d}{dy} G(y)\right)\right)}{F(x) \cdot G(y)} = simplify\left(\frac{\left(rhs(EcuacionDos) + y^2 F(x)\left(\frac{d}{dy} G(y)\right)\right)}{F(x) \cdot G(y)}\right)$

$$EcuacionTres := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\left(\frac{d}{dy} G(y)\right) y^2 + 5\, G(y)}{G(y)} \quad \textbf{(17)}$$

> $EcuacionX := lhs(EcuacionTres) = \mathrm{alpha}; EcuacionY := rhs(EcuacionTres) = \mathrm{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionY := \frac{\left(\frac{d}{dy} G(y)\right) y^2 + 5\, G(y)}{G(y)} = \alpha \quad \textbf{(18)}$$

> $SolucionXpos := dsolve(subs(\mathrm{alpha} = \mathrm{beta} \cdot \cdot 2, EcuacionX)); SolucionYpos := dsolve(subs(\mathrm{alpha} = \mathrm{beta} \cdot \cdot 2, EcuacionY))$

$$SolucionXpos := F(x) = \_C1\, e^{\beta x} + \_C2\, e^{-\beta x}$$

$$SolucionYpos := G(y) = \_C1\, e^{-\frac{\beta^2 - 5}{y}} \quad \textbf{(19)}$$

> $SolucionGralPos := z(x,y) = rhs(SolucionXpos) \cdot subs(\_C1 = 1, rhs(SolucionYpos))$

$$SolucionGralPos := z(x,y) = \left(\_C1\, e^{\beta x} + \_C2\, e^{-\beta x}\right) e^{-\frac{\beta^2 - 5}{y}} \quad \textbf{(20)}$$

> $ComprobacionTres := simplify(eval(subs(z(x,y) = rhs(SolucionGralPos), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$ComprobacionTres := 0 = 0 \quad \textbf{(21)}$$

**alternativa dos**

> $EcuacionCuatro := simplify\left(\frac{\left(lhs(EcuacionDos) + y^2 F(x) \left(\frac{d}{dy} G(y)\right) - F(x) \cdot G(y)\right)}{5 \cdot F(x) \cdot G(y)}\right) = simplify\left(\frac{\left(rhs(EcuacionDos) + y^2 F(x) \left(\frac{d}{dy} G(y)\right) - F(x) \cdot G(y)\right)}{5 \cdot F(x) \cdot G(y)}\right)$

$$EcuacionCuatro := \frac{1}{5} \frac{\frac{d^2}{dx^2} F(x) - F(x)}{F(x)} = \frac{1}{5} \frac{\left(\frac{d}{dy} G(y)\right) y^2 + 4 G(y)}{G(y)} \quad \textbf{(22)}$$

> $EcuacionXX := lhs(EcuacionCuatro) = \text{alpha}; EcuacionYY := rhs(EcuacionCuatro) = \text{alpha}$

$$EcuacionXX := \frac{1}{5} \frac{\frac{d^2}{dx^2} F(x) - F(x)}{F(x)} = \alpha$$

$$EcuacionYY := \frac{1}{5} \frac{\left(\frac{d}{dy} G(y)\right) y^2 + 4 G(y)}{G(y)} = \alpha \quad \textbf{(23)}$$

> $SolucionXXpos := dsolve(subs(\text{alpha} = \text{beta} \cdot {}^2, EcuacionXX)); SolucionYYpos := dsolve(subs(\text{alpha} = \text{beta} \cdot {}^2, EcuacionYY))$

$$SolucionXXpos := F(x) = \_C1 \sin\left(\sqrt{-5 \beta^2 - 1}\ x\right) + \_C2 \cos\left(\sqrt{-5 \beta^2 - 1}\ x\right)$$

$$SolucionYYpos := G(y) = \_C1\ e^{-\frac{5 \beta^2 - 4}{y}} \quad \textbf{(24)}$$

> $SolucionGralPosPos := z(x, y) = rhs(SolucionXXpos) \cdot subs(\_C1 = 1, rhs(SolucionYYpos))$

$$SolucionGralPosPos := z(x, y) = \left(\_C1 \sin\left(\sqrt{-5 \beta^2 - 1}\ x\right) + \_C2 \cos\left(\sqrt{-5 \beta^2 - 1}\ x\right)\right) e^{-\frac{5 \beta^2 - 4}{y}} \quad \textbf{(25)}$$

> $ComprobacionCuatro := simplify(eval(subs(z(x, y) = rhs(SolucionGralPosPos), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$ComprobacionCuatro := 0 = 0 \quad \textbf{(26)}$$

**FIN RESPUESTA 3)**

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**FIN EXAMEN**