

NOMBRE DEL ALUMNO

SERIE 3 (CAPÍTULO III)
SEMESTRE 2024-1

Octubre 29 de 2023

> restart

1) OBTENER LA TRANSFORMADA INVERSA DE LAPLACE

a) UTILIZANDO EL TEOREMA DE LA CONVOLUCIÓN DE LA SIGUIENTE FUNCIÓN

> $F := \frac{1}{s \cdot (s \cdot 2 + 1)}$

$$F := \frac{1}{s(s^2 + 1)} \quad (1)$$

>

RESPUESTA

> $G := \frac{1}{s}; H := \frac{1}{s^2 + 1}$

$$G := \frac{1}{s}$$

$$H := \frac{1}{s^2 + 1} \quad (2)$$

> with(inttrans) :

> $g := \text{invlaplace}(G, s, t)$

$$g := 1 \quad (3)$$

> $h := \text{invlaplace}(H, s, t)$

$$h := \sin(t) \quad (4)$$

> $f := \text{int}(\sin(\text{tau}) \cdot 1, \text{tau} = 0..t)$

$$f := 1 - \cos(t) \quad (5)$$

> $ff := \text{invlaplace}(F, s, t)$

$$ff := 1 - \cos(t) \quad (6)$$

> restart

b) UTILIZANDO DIVERSAS PROPIEDADES

> $G := \frac{(\exp(-4 \cdot s) + s - 3)}{s \cdot 2 - 6 \cdot s - 7}$

$$G := \frac{e^{-4s} + s - 3}{s^2 - 6s - 7} \quad (7)$$

RESPUESTA

> $H := \frac{\exp(-4 \cdot s)}{s^2 - 6 \cdot s - 7}$

$$H := \frac{e^{-4s}}{s^2 - 6s - 7} \quad (8)$$

$$> J := \frac{s - 3}{s^2 - 6s - 7}$$

$$J := \frac{s - 3}{s^2 - 6s - 7} \quad (9)$$

> with(inttrans) :

> h := invlaplace(H, s, t)

$$h := \frac{\text{Heaviside}(t - 4) e^{3t - 12} \sinh(4t - 16)}{4} \quad (10)$$

> j := invlaplace(J, s, t)

$$j := e^{3t} \cosh(4t) \quad (11)$$

> g := h + j

$$g := \frac{\text{Heaviside}(t - 4) e^{3t - 12} \sinh(4t - 16)}{4} + e^{3t} \cosh(4t) \quad (12)$$

>

> restart

2) OBTENER LA SOLUCIÓN DEL SIGUIENTE PROBLEMA DE CONDICIONES INICIALES UTILIZANDO LA TRANSFORMADA DE LAPLACE

> diff(y(t), t\$2) - 5·diff(y(t), t) + 4·y(t) = 4·exp(4·t); y(0) = 0; D(y)(0) = 2

$$\frac{d^2}{dt^2} y(t) - 5 \frac{d}{dt} y(t) + 4 y(t) = 4 e^{4t}$$

$$y(0) = 0$$

$$D(y)(0) = 2 \quad (13)$$

RESPUESTA

> ECUA := diff(y(t), t\$2) - 5·diff(y(t), t) + 4·y(t) = 4·exp(4·t)

$$ECUA := \frac{d^2}{dt^2} y(t) - 5 \frac{d}{dt} y(t) + 4 y(t) = 4 e^{4t} \quad (14)$$

> CondIni := y(0) = 0, D(y)(0) = 2

$$CondIni := y(0) = 0, D(y)(0) = 2 \quad (15)$$

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(lhs(ECUA), t, s) = laplace(rhs(ECUA), t, s))

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) - 2 - 5s \mathcal{L}(y(t), t, s) + 4 \mathcal{L}(y(t), t, s) = \frac{4}{s - 4} \quad (16)$$

> SolPartTL := simplify(isolate(EcuaTL, laplace(y(t), t, s)))

(17)

$$SolPartTL := \mathcal{L}(y(t), t, s) = \frac{-4 + 2s}{(s-4)^2 (s-1)} \quad (17)$$

> *SolPart* := invlaplace(*SolPartTL*, *s*, *t*)

$$SolPart := y(t) = -\frac{2e^t}{9} + \frac{2e^{4t}(6t+1)}{9} \quad (18)$$

>

> *restart*

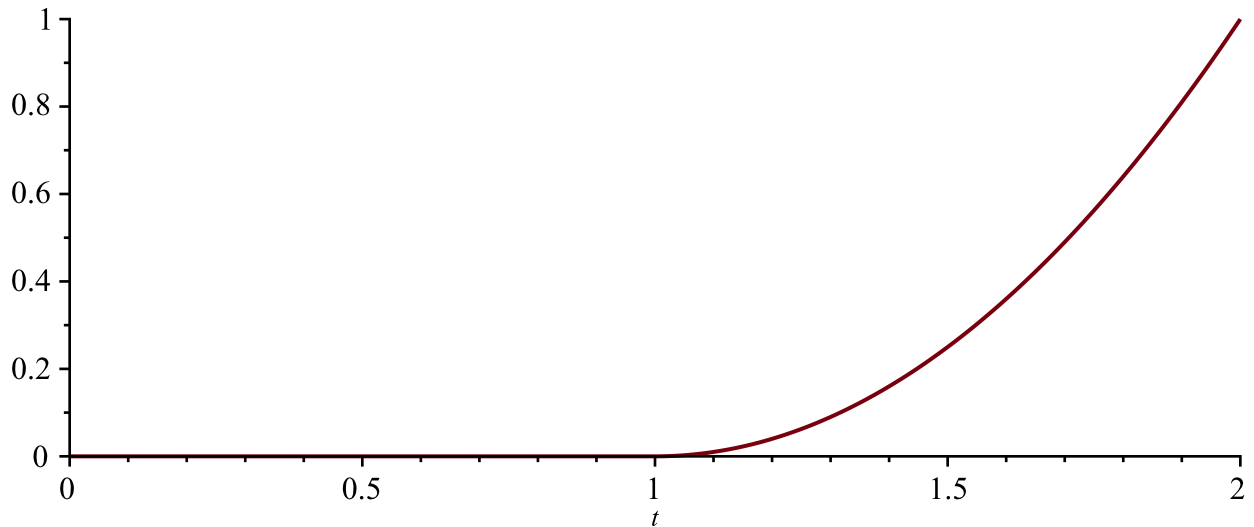
3) DADA LA FUNCIÓN

> *f* := (*t* - 1) · 2 · Heaviside(*t* - 1)

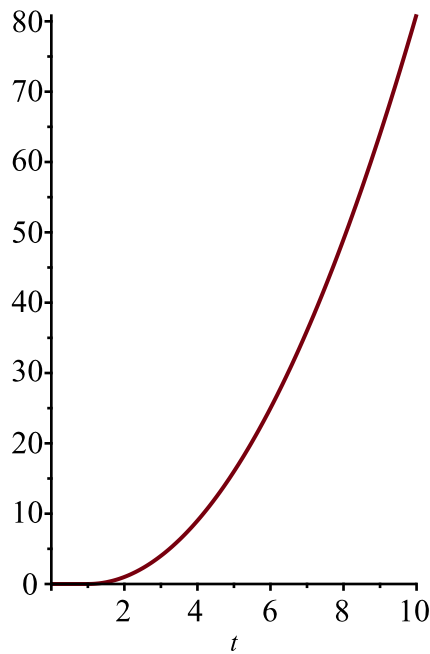
$$f := (t-1)^2 \text{Heaviside}(t-1) \quad (19)$$

a) GRAFIQUE LA FUNCIÓN PARA $0 < t < 10$

> *plot*(*f*, *t* = 0 .. 2)



> *plot*(*f*, *t* = 0 .. 10)



>

b) OBTENER LA TRANSFORMADA DE LA FUNCIÓN

> with(inttrans) :

> F := laplace(f, t, s)

$$F := \frac{2 e^{-s}}{s^3}$$

(20)

>

> restart

4)

a) OBTENER LA MATRIZ A CUYA MATRIZ EXPONENCIAL ESTÁ DADA

> MatExp := $\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} e^{-t} + \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} e^{3t}$

$$MatExp := \begin{bmatrix} \frac{e^{-t}}{2} + \frac{e^{3t}}{2} & -\frac{e^{-t}}{2} + \frac{e^{3t}}{2} \\ -\frac{e^{-t}}{2} + \frac{e^{3t}}{2} & \frac{e^{-t}}{2} + \frac{e^{3t}}{2} \end{bmatrix}$$

(21)

RESPUESTA

> DerMatExp := map(diff, MatExp, t)

$$DerMatExp := \begin{bmatrix} -\frac{e^{-t}}{2} + \frac{3e^{3t}}{2} & \frac{e^{-t}}{2} + \frac{3e^{3t}}{2} \\ \frac{e^{-t}}{2} + \frac{3e^{3t}}{2} & -\frac{e^{-t}}{2} + \frac{3e^{3t}}{2} \end{bmatrix}$$

(22)

> $InvMatExp := map(rcurry(eval, t = -t'), MatExp)$

$$InvMatExp := \begin{bmatrix} \frac{e^t}{2} + \frac{e^{-3t}}{2} & -\frac{e^t}{2} + \frac{e^{-3t}}{2} \\ -\frac{e^t}{2} + \frac{e^{-3t}}{2} & \frac{e^t}{2} + \frac{e^{-3t}}{2} \end{bmatrix} \quad (23)$$

> $AA := simplify(evalm(DerMatExp \&* InvMatExp))$

$$AA := \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad (24)$$

>

b) CON LA MATRIZ **A** OBTENIDA EN EL INCISO [a)] PROPONER UN SISTEMA HOMOGÉNEO DE ECUACIONES DIFERENCIALES CON **x(t) & y(t)** COMO INCÓGNITAS

> $Sistema := diff(x(t), t) = x(t) + 2 \cdot y(t), diff(y(t), t) = 2 \cdot x(t) + y(t) : Sistema[1]; Sistema[2]$

$$\frac{d}{dt} x(t) = x(t) + 2 y(t)$$

$$\frac{d}{dt} y(t) = 2 x(t) + y(t) \quad (25)$$

>

c) OBTENER LA SOLUCIÓN GENERAL DEL SISTEMA OBTENIDO EN EL INCISO [b)] MEDIANTE **dsolve**

> $SolGral := dsolve(\{Sistema\}) : SolGral[1]; SolGral[2]$

$$x(t) = c_1 e^{3t} + c_2 e^{-t}$$

$$y(t) = c_1 e^{3t} - c_2 e^{-t} \quad (26)$$

>

> **restart:**

5) DADA LA ECUACIÓN DIFERENCIAL DE CUARTO ORDEN SIGUIENTE:

>

a) OBTENER UN SISTEMA DE ECUACIONES DIFERENCIALES EQUIVALENTE (CON TODO Y CONDICIONES INICIALES)

> $Sistema := diff(y[1](t), t) = y[2](t), diff(y[2](t), t) = y[3](t), diff(y[3](t), t) = y[4](t),$
 $diff(y[4](t), t) = 4 \cdot y[1](t) - 5 \cdot y[3](t) + 5 e^{-2t} \sin(3t) : Sistema[1]; Sistema[2];$
 $Sistema[3]; Sistema[4];$

$$\frac{d}{dt} y_1(t) = y_2(t)$$

$$\frac{d}{dt} y_2(t) = y_3(t)$$

$$\frac{d}{dt} y_3(t) = y_4(t)$$

$$\frac{d}{dt} y_4(t) = 4 y_1(t) - 5 y_3(t) + 5 e^{-2t} \sin(3t) \quad (27)$$

> *CondIni* := *y*[1](0) = -5, *y*[2](0) = -3, *y*[3](0) = 4, *y*[4](0) = 2 : *CondIni*[1]; *CondIni*[2];
CondIni[3]; *CondIni*[4];

$$y_1(0) = -5$$

$$y_2(0) = -3$$

$$y_3(0) = 4$$

$$y_4(0) = 2 \quad (28)$$

>

b) MOSTRAR LA REPRESENTACIÓN MATRICIAL DEL MISMO SISTEMA

> *Y* := *array*([*y*[1](*t*), *y*[2](*t*), *y*[3](*t*), *y*[4](*t*)])

$$Y := \begin{bmatrix} y_1(t) & y_2(t) & y_3(t) & y_4(t) \end{bmatrix} \quad (29)$$

> *DerY* := *map*(*diff*, *Y*, *t*)

$$DerY := \begin{bmatrix} \frac{d}{dt} y_1(t) & \frac{d}{dt} y_2(t) & \frac{d}{dt} y_3(t) & \frac{d}{dt} y_4(t) \end{bmatrix} \quad (30)$$

> *AA* := *array*([[0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1], [4, 0, -5, 0]])

$$AA := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & -5 & 0 \end{bmatrix} \quad (31)$$

> *BB* := *array*([0, 0, 0, 5 e^{-2t} sin(3t)])

$$BB := \begin{bmatrix} 0 & 0 & 0 & 5 e^{-2t} \sin(3t) \end{bmatrix} \quad (32)$$

> *Xcero* := *array*([-5, -3, 4, 2])

$$Xcero := \begin{bmatrix} -5 & -3 & 4 & 2 \end{bmatrix} \quad (33)$$

c) OBTENER LA MATRIZ EXPONENCIAL QUE NOS PERMITA RESOLVERLO

> *with*(*linalg*) :

> *MatExp* := *exponential*(*AA*, *t*) : *evalf*(*MatExp*[1, 1], 2); *evalf*(*MatExp*[2, 1], 2);
evalf(*MatExp*[3, 1], 2); *evalf*(*MatExp*[4, 1], 2)

$$\begin{aligned} & 0.43 e^{0.85t} + 0.43 e^{-0.85t} + 0.11 \cos(2.4t) \\ & -0.26 \sin(2.4t) + 0.37 e^{0.85t} - 0.37 e^{-0.85t} \\ & -0.63 \cos(2.4t) + 0.31 e^{0.85t} + 0.31 e^{-0.85t} \end{aligned}$$

$$1.5 \sin(2.4 t) + 0.30 e^{0.85 t} - 0.30 e^{-0.85 t} \quad (34)$$

> evalf(MatExp[1, 2], 2); evalf(MatExp[2, 2], 2); evalf(MatExp[3, 2], 2); evalf(MatExp[4, 2], 2)

$$\begin{aligned} &0.047 \sin(2.4 t) + 0.53 e^{0.85 t} - 0.53 e^{-0.85 t} \\ &0.43 e^{0.85 t} + 0.43 e^{-0.85 t} + 0.11 \cos(2.4 t) \\ &-0.26 \sin(2.4 t) + 0.37 e^{0.85 t} - 0.37 e^{-0.85 t} \\ &-0.63 \cos(2.4 t) + 0.31 e^{0.85 t} + 0.31 e^{-0.85 t} \end{aligned} \quad (35)$$

> evalf(MatExp[1, 3], 2); evalf(MatExp[2, 3], 2); evalf(MatExp[3, 3], 2); evalf(MatExp[4, 3], 2)

$$\begin{aligned} &-0.16 \cos(2.4 t) + 0.078 e^{0.85 t} + 0.078 e^{-0.85 t} \\ &0.37 \sin(2.4 t) + 0.065 e^{0.85 t} - 0.065 e^{-0.85 t} \\ &0.054 e^{0.85 t} + 0.054 e^{-0.85 t} + 0.90 \cos(2.4 t) \\ &-2.1 \sin(2.4 t) + 0.048 e^{0.85 t} - 0.048 e^{-0.85 t} \end{aligned} \quad (36)$$

> evalf(MatExp[1, 4], 2); evalf(MatExp[2, 4], 2); evalf(MatExp[3, 4], 2); evalf(MatExp[4, 4], 2)

$$\begin{aligned} &-0.065 \sin(2.4 t) + 0.094 e^{0.85 t} - 0.094 e^{-0.85 t} \\ &-0.16 \cos(2.4 t) + 0.078 e^{0.85 t} + 0.078 e^{-0.85 t} \\ &0.37 \sin(2.4 t) + 0.065 e^{0.85 t} - 0.065 e^{-0.85 t} \\ &0.054 e^{0.85 t} + 0.054 e^{-0.85 t} + 0.90 \cos(2.4 t) \end{aligned} \quad (37)$$

>

d) OBTENER LA SOLUCIÓN PARTICULAR DADAS LAS CONDICIONES SEÑALADAS UTILIZANDO EL MÉTODO DE MATRIZ EXPONENCIAL

> SolHom := evalm(MatExp &* Xcero) : Y[1] = evalf(SolHom[1], 2); Y[2] = evalf(SolHom[2], 2); Y[3] = evalf(SolHom[3], 2); Y[4] = evalf(SolHom[4], 2);

$$y_1(t) = -3.2 e^{0.85 t} - 0.4 e^{-0.85 t} - 1.2 \cos(2.4 t) - 0.27 \sin(2.4 t)$$

$$y_2(t) = 2.8 \sin(2.4 t) - 2.8 e^{0.85 t} + 0.25 e^{-0.85 t} - 0.68 \cos(2.4 t)$$

$$y_3(t) = 6.8 \cos(2.4 t) - 2.3 e^{0.85 t} - 0.4 e^{-0.85 t} + 1.5 \sin(2.4 t)$$

$$y_4(t) = -16. \sin(2.4 t) - 2.1 e^{0.85 t} + 0.46 e^{-0.85 t} + 3.7 \cos(2.4 t) \quad (38)$$

> Comprobar := simplify(map(rcurry(eval, t=0'), SolHom))

$$\text{Comprobar} := \begin{bmatrix} -5 & -3 & 4 & 2 \end{bmatrix} \quad (39)$$

> MatExpTau := map(rcurry(eval, t=t-tau'), MatExp) :

> BBtau := map(rcurry(eval, t=tau'), BB)

$$\text{BBtau} := \begin{bmatrix} 0 & 0 & 0 & 5 e^{-2 \tau} \sin(3 \tau) \end{bmatrix} \quad (40)$$

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> ProdTau := evalm(MatExpTau &* BBtau) :
> SolNoHom := map(int, ProdTau, tau = 0..t) : Y[1] = evalf(SolNoHom[1], 2); Y[2]
= evalf(SolNoHom[2], 2); Y[3] = evalf(SolNoHom[3], 2); Y[4] = evalf(SolNoHom[4],
2);

$$y_1(t) = 0.082 e^{0.85t} - 0.13 e^{-0.85t} + 0.062 \cos(2.4t) - 0.029 \sin(3.t) e^{-2.t} \\ - 0.012 \cos(3.t) e^{-2.t} - 0.053 \sin(2.4t)$$


$$y_2(t) = -0.15 \sin(2.4t) + 0.071 e^{0.85t} + 0.12 e^{-0.85t} - 0.12 \cos(2.4t) + 0.093 \sin(3.t) e^{-2.t} \\ - 0.064 \cos(3.t) e^{-2.t}$$


$$y_3(t) = -0.37 \cos(2.4t) + 0.053 e^{0.85t} - 0.086 e^{-0.85t} + 0.40 \cos(3.t) e^{-2.t} \\ + 0.0038 \sin(3.t) e^{-2.t} + 0.28 \sin(2.4t)$$


$$y_4(t) = 0.88 \sin(2.4t) + 0.05 e^{0.85t} + 0.09 e^{-0.85t} + 0.68 \cos(2.4t) - 0.80 \cos(3.t) e^{-2.t} \\ - 1.2 \sin(3.t) e^{-2.t} \quad (41)$$


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> ComprobarDos := simplify(map(rcurry(eval, t=0'), SolNoHom))
ComprobarDos := [ 0 0 0 0 ] \quad (42)

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> SolPart := Y[1] = simplify(evalf(SolHom[1], 2) + evalf(SolNoHom[1], 2)), Y[2]
= simplify(evalf(SolHom[2], 2) + evalf(SolNoHom[2], 2)), Y[3]
= simplify(evalf(SolHom[3], 2) + evalf(SolNoHom[3], 2)), Y[4]
= simplify(evalf(SolHom[4], 2) + evalf(SolNoHom[4], 2)) : SolPart[1]; SolPart[2];
SolPart[3]; SolPart[4];

$$y_1(t) = (-0.029 \sin(3.t) - 0.012 \cos(3.t)) e^{-2.t} - 3.118 e^{0.85t} - 0.53 e^{-0.85t} - 1.138 \cos(2.4t) \\ - 0.323 \sin(2.4t)$$


$$y_2(t) = (0.093 \sin(3.t) - 0.064 \cos(3.t)) e^{-2.t} + 2.65 \sin(2.4t) - 2.729 e^{0.85t} + 0.37 e^{-0.85t} \\ - 0.80 \cos(2.4t)$$


$$y_3(t) = (0.40 \cos(3.t) + 0.0038 \sin(3.t)) e^{-2.t} + 6.43 \cos(2.4t) - 2.247 e^{0.85t} - 0.486 e^{-0.85t} \\ + 1.78 \sin(2.4t)$$


$$y_4(t) = (-0.80 \cos(3.t) - 1.2 \sin(3.t)) e^{-2.t} - 15.12 \sin(2.4t) - 2.05 e^{0.85t} + 0.55 e^{-0.85t} \\ + 4.38 \cos(2.4t) \quad (43)$$


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> restart:

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6) DADO EL SISTEMA, Y CON LAS CONDICIONES: $\mathbf{x}(0) = 3$; $\mathbf{y}(0) = -4$; $\mathbf{z}(0) = 6$

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RESPUESTA

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> Sistema := \frac{d}{dt} x(t) = x(t) - y(t) + z(t), \frac{d}{dt} y(t) = -x(t) + y(t) + z(t) + 2 e^t, \frac{d}{dt} z(t) = x(t)

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+ $y(t) - z(t) + e^{3t}$: $Sistema[1]; Sistema[2]; Sistema[3];$

$$\frac{d}{dt} x(t) = x(t) - y(t) + z(t)$$

$$\frac{d}{dt} y(t) = -x(t) + y(t) + z(t) + 2e^t$$

$$\frac{d}{dt} z(t) = x(t) + y(t) - z(t) + e^{3t} \quad (44)$$

> $CondIni := x(0) = 3, y(0) = -4, z(0) = 6$

$CondIni := x(0) = 3, y(0) = -4, z(0) = 6 \quad (45)$

>

a) OBTENER LA SOLUCIÓN PARTICULAR UTILIZANDO **dsolve**

> $SolPart := dsolve(\{Sistema, CondIni\}) : SolPart[1]; SolPart[2]; SolPart[3];$

$$x(t) = \frac{2te^t}{3} + \frac{e^{3t}}{10} + \frac{47e^t}{18} - \frac{199e^{-2t}}{90} + \frac{5e^{2t}}{2}$$

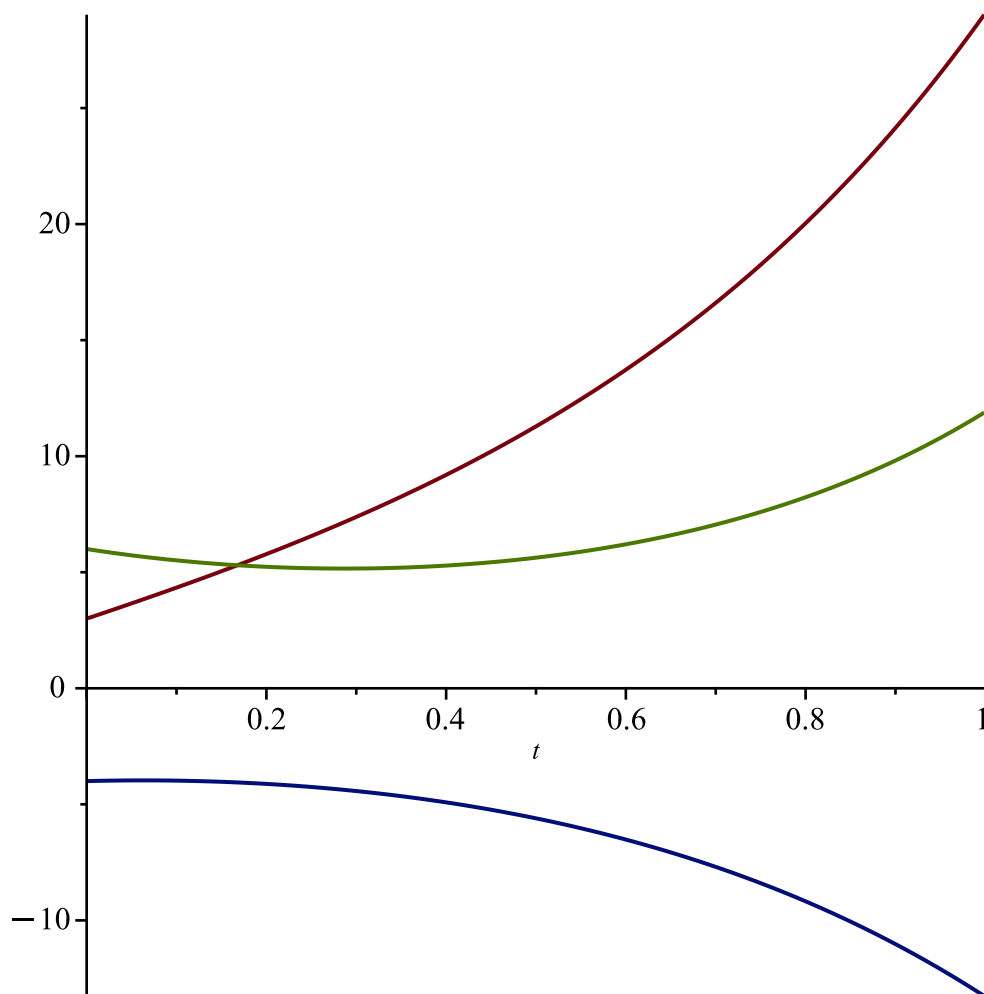
$$y(t) = \frac{e^{3t}}{10} + \frac{2te^t}{3} + \frac{11e^t}{18} - \frac{5e^{2t}}{2} - \frac{199e^{-2t}}{90}$$

$$z(t) = \frac{3e^{3t}}{10} + \frac{2te^t}{3} + \frac{23e^t}{18} + \frac{199e^{-2t}}{45} \quad (46)$$

>

b) GRAFICAR LA SOLUCIÓN DEL SISTEMA OBTENIDA EN EL INCISO [a] (FUNCIONES JUNTAS EN UN SOLO GRÁFICO) CON UN INTERVALO $0 < t < 1$

> $plot([rhs(SolPart[1]), rhs(SolPart[2]), rhs(SolPart[3])], t=0..1)$



>

c) ESTABLECER LA MATRIZ A DEL MISMO SISTEMA Y RESOLVERLO, TAMBIÉN, CON LA MATRIZ EXPONENCIAL

> $AA := \text{array}([[1, -1, 1], [-1, 1, 1], [1, 1, -1]])$

$$AA := \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad (47)$$

> $X_{\text{cero}} := \text{array}([3, -4, 6])$

$$X_{\text{cero}} := \begin{bmatrix} 3 & -4 & 6 \end{bmatrix} \quad (48)$$

> $BB := \text{array}([0, 2 \cdot \exp(t), \exp(3 \cdot t)])$

$$BB := \begin{bmatrix} 0 & 2 e^t & e^{3t} \end{bmatrix} \quad (49)$$

> $\text{with}(\text{linalg}) :$

> $\text{MatExp} := \text{exponential}(AA, t) : \text{MatExp}[1, 1]$

$$\frac{e^t}{3} + \frac{e^{2t}}{2} + \frac{e^{-2t}}{6} \quad (50)$$

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> SolHom := evalm(MatExp &* Xcero) : SolHom[1]; x(0) = simplify(subs(t=0, SolHom[1]));
y(0) = simplify(subs(t=0, SolHom[2])); z(0) = simplify(subs(t=0, SolHom[3]))

$$\frac{5 e^t}{3} + \frac{7 e^{2t}}{2} - \frac{13 e^{-2t}}{6}$$


$$x(0) = 3$$


$$y(0) = -4$$


$$z(0) = 6$$

(51)

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> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp) :
> BBtau := map(rcurry(eval, t='tau'), BB)

$$BBtau := \begin{bmatrix} 0 & 2 e^\tau & e^{3\tau} \end{bmatrix}$$

(52)

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> ProdTau := evalm(MatExpTau &* BBtau) :
> SolNoHom := map(int, ProdTau, tau=0..t) :
> SolPart := x(t) = SolHom[1] + SolNoHom[1]; y(t) = SolHom[2] + SolNoHom[2]; z(t)
= SolHom[3] + SolNoHom[3];

$$SolPart := x(t) = \frac{2 t e^t}{3} + \frac{e^{3t}}{10} + \frac{47 e^t}{18} - \frac{199 e^{-2t}}{90} + \frac{5 e^{2t}}{2}$$


$$y(t) = \frac{e^{3t}}{10} + \frac{2 t e^t}{3} + \frac{11 e^t}{18} - \frac{5 e^{2t}}{2} - \frac{199 e^{-2t}}{90}$$


$$z(t) = \frac{3 e^{3t}}{10} + \frac{2 t e^t}{3} + \frac{23 e^t}{18} + \frac{199 e^{-2t}}{45}$$

(53)

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> restart:
[SOLUCIÓN

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[FIN DE LA SERIE

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