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DEPARTAMENTO DE MATEMATICAS APLICADAS
1325_24-2_2F_JS1
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL COLEGIADO

> restart

1) Solución General

> $Ecua := y' = \frac{y - x + 8}{y - x + 4}$

$$Ecua := \frac{d}{dx}\, y(x) = \frac{y(x) - x + 8}{y(x) - x + 4} \quad (1)$$

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> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[[\_homogeneous, class\ C], \_rational, [\_Abel, 2nd\ type, class\ A]] \quad (2)$$

> $EcuaDos := simplify(isolate(eval(subs(y(x) = u(x) + x, Ecua)), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx}\, u(x) = \frac{4}{u(x) + 4} \quad (3)$$

> $M := -\frac{1}{4}; N := \frac{1}{u + 4}$

$$M := -\frac{1}{4}$$

$$N := \frac{1}{u + 4} \quad (4)$$

> $SolGral := simplify\left(int\left(\frac{1}{N}, u\right) + int\left(\frac{1}{M}, x\right)\right) = \_C1$

$$SolGral := \frac{1}{2}\, u^2 + 4\, u - 4\, x = \_C1 \quad (5)$$

> $SolFinal := subs(u = y(x) - x, SolGral)$

$$SolFinal := \frac{(y(x) - x)^2}{2} + 4\, y(x) - 8\, x = \_C1 \quad (6)$$

> $DerSolFinal := simplify(isolate(simplify(diff(SolFinal, x)), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{y(x) - x + 8}{y(x) - x + 4} \quad (7)$$

> $Ecua$

$$\frac{d}{dx}\, y(x) = \frac{y(x) - x + 8}{y(x) - x + 4} \quad (8)$$

> $Comprobar := (rhs(DerSolFinal) - rhs(Ecua) = 0)$

$$Comprobar := 0 = 0 \quad (9)$$

> restart

2)

> $Ecua := diff(y(x), x\$a) + x^b \cdot y(x)^c = Q(x)$

$$Ecua := \frac{d^a}{dx^a}\, y(x) + x^b\, y(x)^c = Q(x) \quad (10)$$

> $a := 2;\ b := 0;\ c := 1$

$$a := 2$$
$$b := 0$$
$$c := 1 \quad (11)$$

> $Ecua$

$$\frac{d^2}{dx^2}\, y(x) + y(x) = Q(x) \quad (12)$$

> $EcuaUno := subs(Q(x) = x, Ecua)$

$$EcuaUno := \frac{d^2}{dx^2}\, y(x) + y(x) = x \quad (13)$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) + y(x) = 0 \quad (14)$$

> $Q := rhs(EcuaUno)$

$$Q := x \quad (15)$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \quad (16)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := \mathrm{I}, -\mathrm{I} \quad (17)$$

> $yy[1] := \cos(\mathrm{Im}(Raiz[1]) \cdot x);\ yy[2] := \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(x)$$
$$yy_2 := \sin(x) \quad (18)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad (19)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x \end{bmatrix} \quad (20)$$

> $Xcero := array([\_C1, \_C2])$

$$Xcero := \begin{bmatrix} \_C1 & \_C2 \end{bmatrix} \quad (21)$$

> $WWtau := map(rcurry(eval, x = 'x - \mathrm{tau}'), WW)$

$$WWtau := \begin{bmatrix} \cos(-x + \tau) & -\sin(-x + \tau) \\ \sin(-x + \tau) & \cos(-x + \tau) \end{bmatrix} \quad (22)$$

> $BBtau := map(rcurry(eval, x='tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & \tau \end{bmatrix} \quad (23)$$

> $ProdTau := evalm(WWtau \&* BBtau)$

$$ProdTau := \begin{bmatrix} -\sin(-x+\tau)\,\tau & \cos(-x+\tau)\,\tau \end{bmatrix} \quad (24)$$

> $SolGralUnoHom := evalm(WW \&* Xcero)$

$$SolGralUnoHom := \begin{bmatrix} \cos(x)\,\_C1 + \sin(x)\,\_C2 & -\sin(x)\,\_C1 + \cos(x)\,\_C2 \end{bmatrix} \quad (25)$$

> $SolGralUnoNoHom := map(int, ProdTau, \mathrm{tau}=0..x)$

$$SolGralUnoNoHom := \begin{bmatrix} -\sin(x)+x & 1-\cos(x) \end{bmatrix} \quad (26)$$

> $SolGralUno[1] := y(x) = SolGralUnoHom[1] + SolGralUnoNoHom[1]$

$$SolGralUno_1 := y(x) = \cos(x)\,\_C1 + \sin(x)\,\_C2 - \sin(x) + x \quad (27)$$

> $SolFinal := y(x) = \_C10 \cdot yy[1] + \_C20 \cdot yy[2] + x$

$$SolFinal := y(x) = \_C10\cos(x) + \_C20\sin(x) + x \quad (28)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), EcuaUno)))$

$$Comprobar := x = x \quad (29)$$

> $EcuaDos := subs\left(Q(x) = \frac{1}{8 \cdot \sin(x)}, Ecua\right)$

$$EcuaDos := \frac{d^2}{dx^2}\,y(x) + y(x) = \frac{1}{8\sin(x)} \quad (30)$$

> $SolFinalDos := y(x) = \_C1 \cdot \cos(x) + \_C2 \cdot \sin(x) - \frac{x \cdot \cos(x)}{8} + \frac{1}{8} \cdot \sin(x) \log(\sin(x))$

$$SolFinalDos := y(x) = \sin(x)\,c_2 + \cos(x)\,c_1 + \frac{\sin(x)\ln(\sin(x))}{8} - \frac{x\cos(x)}{8} \quad (31)$$

> $DerSolFinalDos := simplify(eval(subs(y(x) = rhs(SolFinalDos), lhs(EcuaDos) - rhs(EcuaDos) = 0)))$

$$DerSolFinalDos := 0 = 0 \quad (32)$$

> $dsolve(EcuaDos)$

$$y(x) = \sin(x)\,c_2 + \cos(x)\,c_1 + \frac{\sin(x)\ln(\sin(x))}{8} - \frac{x\cos(x)}{8} \quad (33)$$

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3)

> $Ecua := diff(y(t), t) + 4 \cdot y(t) = \exp(-4\,t)$

$$Ecua := \frac{d}{dt}\,y(t) + 4\,y(t) = e^{-4\,t} \quad (34)$$

> $CondIni := y(0) = 8$

$$CondIni := y(0) = 8 \quad (35)$$

> $with(inttrans):$

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s\,\mathcal{L}(y(t),t,s) - 8 + 4\,\mathcal{L}(y(t),t,s) = \frac{1}{s+4} \quad \textbf{(36)}$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := \mathcal{L}(y(t),t,s) = \frac{\frac{1}{s+4}+8}{s+4} \quad \textbf{(37)}$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \mathrm{e}^{-4\,t}\,(8+t) \quad \textbf{(38)}$$

> restart

4)

> $Sistema := diff(y(t), t) - 2\,y(t) = 4, diff(x(t), t) + y(t) - x(t) - 2\cdot\mathrm{Dirac}(t-1):$
> $Sistema[1]; Sistema[2]$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,y(t) - 2\,y(t) = 4$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,x(t) + y(t) - x(t) - 2\,\mathrm{Dirac}(t-1) \quad \textbf{(39)}$$

> $CondIni := x(0) = 0, y(0) = 1$

$$CondIni := x(0) = 0, y(0) = 1 \quad \textbf{(40)}$$

> $with(inttrans):$

> $SistUnoTL := subs(CondIni, laplace(Sistema[1], t, s))$

$$SistUnoTL := s\,\mathcal{L}(y(t),t,s) - 1 - 2\,\mathcal{L}(y(t),t,s) = \frac{4}{s} \quad \textbf{(41)}$$

> $SolYTL := isolate(SistUnoTL, laplace(y(t), t, s))$

$$SolYTL := \mathcal{L}(y(t),t,s) = \frac{\frac{4}{s}+1}{s-2} \quad \textbf{(42)}$$

> $SolY := invlaplace(SolYTL, s, t)$

$$SolY := y(t) = 3\,\mathrm{e}^{2\,t} - 2 \quad \textbf{(43)}$$

> $SistDos := subs(y(t) = rhs(SolY), Sistema[2])$

$$SistDos := \frac{\mathrm{d}}{\mathrm{d}t}\,x(t) + 3\,\mathrm{e}^{2\,t} - 2 - x(t) - 2\,\mathrm{Dirac}(t-1) \quad \textbf{(44)}$$

> $SolXTL := isolate(subs(CondIni, laplace(SistDos, t, s)), laplace(x(t), t, s))$

$$SolXTL := \mathcal{L}(x(t),t,s) = \frac{-\frac{3}{s-2}+\frac{2}{s}+2\,\mathrm{e}^{-s}}{s-1} \quad \textbf{(45)}$$

> $SolX := invlaplace(SolXTL, s, t)$

$$SolX := x(t) = -2 - 3\,\mathrm{e}^{2\,t} + 5\,\mathrm{e}^{t} + 2\,(1-\mathrm{Heaviside}(1-t))\,\mathrm{e}^{t-1} \quad \textbf{(46)}$$

> restart

5)

> $Ecua := diff(u(x,y), x\$2) + y\cdot diff(u(x,y), y) = 0$

(47)

$$Ecua := \frac{\partial^2}{\partial x^2} u(x,y) + y \left(\frac{\partial}{\partial y} u(x,y) \right) = 0 \qquad \textbf{(47)}$$

> $EcuaSep := eval(subs(u(x,y) = F(x) \cdot G(y), Ecua))$

$$EcuaSep := \left(\frac{d^2}{dx^2} F(x) \right) G(y) + y\, F(x) \left(\frac{d}{dy} G(y) \right) = 0 \qquad \textbf{(48)}$$

> $EcuaSeparada := \frac{\left(lhs(EcuaSep) - y\, F(x) \left(\frac{d}{dy} G(y) \right) \right)}{F(x) \cdot G(y)} = \frac{\left(rhs(EcuaSep) - y\, F(x) \left(\frac{d}{dy} G(y) \right) \right)}{F(x) \cdot G(y)}$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\frac{y \left(\frac{d}{dy} G(y) \right)}{G(y)} \qquad \textbf{(49)}$$

> $EcuaX := lhs(EcuaSeparada) = -3$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -3 \qquad \textbf{(50)}$$

> $EcuaY := rhs(EcuaSeparada) = -3$

$$EcuaY := -\frac{y \left(\frac{d}{dy} G(y) \right)}{G(y)} = -3 \qquad \textbf{(51)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 \sin\left(\sqrt{3}\, x\right) + c_2 \cos\left(\sqrt{3}\, x\right) \qquad \textbf{(52)}$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = c_1\, y^3 \qquad \textbf{(53)}$$

> $SolFinal := u(x,y) = rhs(SolX) \cdot subs(c_1 = 1, rhs(SolY))$

$$SolFinal := u(x,y) = \left(c_1 \sin\left(\sqrt{3}\, x\right) + c_2 \cos\left(\sqrt{3}\, x\right) \right) y^3 \qquad \textbf{(54)}$$

> $Ecua$

$$\frac{\partial^2}{\partial x^2} u(x,y) + y \left(\frac{\partial}{\partial y} u(x,y) \right) = 0 \qquad \textbf{(55)}$$

> $SolDos := u(x,y) = (\_C1 \cdot \exp(\mathrm{sqrt}(3) \cdot x) + \_C2 \cdot \exp(-\mathrm{sqrt}(3) \cdot x)) \cdot y^{(-3)}$

$$SolDos := u(x,y) = \frac{c_1\, e^{\sqrt{3}\, x} + c_2\, e^{-\sqrt{3}\, x}}{y^3} \qquad \textbf{(56)}$$

> $Comprobar := simplify(eval(subs(u(x,y) = rhs(SolDos), Ecua)))$

$$Comprobar := 0 = 0 \qquad \textbf{(57)}$$

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FIN RESPUESTA

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