EXAMEN

```
> restart
```

1)

```
> Ecua := diff(y(t), t$3) + diff(y(t), t$2) - diff(y(t), t) - y(t) = exp(t)
```

$$Ecua := \frac{d^3}{dt^3} y(t) + \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - y(t) = e^t \quad (1)$$

E.D.O.(3).L.cc.NH-----tipo=ordinaria----V.D.="y"---V.I.= "t"------ORDEN=3

```
>
> Ecua := (x·diff(diff(y(x), x), x) - x·diff(y(x), x)) = 6·x + x·sin(3 x)
```

$$Ecua := x \left(\frac{d^2}{dx^2} y(x) \right) - x \left(\frac{d}{dx} y(x) \right) = 6\,x + x \sin(3\,x) \quad (2)$$

E.D.O.(2).L.cv.NH------tipo=ordinaria----V.D.="y"-----V.I.= "x" -------ORDEN=2

```
> restart
```

2)

```
>
> Ecua := (x + sqrt(x·y))·y' + x - y = x^(-1/2)·y^(3/2)
```

$$Ecua := \left(x + \sqrt{x\,y(x)}\right) \left(\frac{d}{dx} y(x) \right) + x - y(x) = \frac{y(x)^{3/2}}{\sqrt{x}} \quad (3)$$

```
> EcuaDos := x - y(x) - y(x)^(3/2)/sqrt(x) + (x + sqrt(x·y(x)))·(diff(y(x), x)) = 0
```

$$EcuaDos := x - y(x) - \frac{y(x)^{3/2}}{\sqrt{x}} + \left(x + \sqrt{x\,y(x)}\right) \left(\frac{d}{dx} y(x) \right) = 0 \quad (4)$$

```
> with(DEtools):
> odeadvisor(EcuaDos)
```

$$[[\_homogeneous, class\ G]] \quad (5)$$

COEFICIENTES HOMOGENEOS

```
> EcuaTres := simplify(isolate(eval(subs(y(x) = u(x)·x, EcuaDos)), diff(u(x), x)))
```

$$EcuaTres := \frac{d}{dx} u(x) = \frac{u(x)\,x\sqrt{u(x)\,x} - u(x)\sqrt{x}\,\sqrt{x^2\,u(x)} - x^{3/2}}{x^{3/2}\left(x + \sqrt{x^2\,u(x)}\right)} \quad (6)$$

VARIABLES SEPARABLES

```
> N := simplify(x^(3/2)·(x + x·sqrt(u(x))))
```

$$N := x^{5/2}\left(\sqrt{u(x)} + 1\right) \quad (7)$$

```
> M := expand(-(u(x)·x·sqrt(u(x)·x) - u(x)·sqrt(x)·sqrt(x^2·u(x)) - x^(3/2)))
```

$$M := x^{3/2} + u(x)\sqrt{x}\,\sqrt{x^2\,u(x)} - u(x)\,x\sqrt{u(x)\,x} \quad (8)$$

```
> MM := -u(x)·x·sqrt(x)·sqrt(u(x)) + u(x)·sqrt(x)·x·sqrt(u(x)) + x^(3/2)
```

$$MM := x^{3/2} \quad (9)$$

> $P := x^{3/2}; Q := 1; R := x^{5/2}; S := \left(\sqrt{u} + 1\right)$

$$P := x^{3/2}$$

$$Q := 1$$

$$R := x^{5/2}$$

$$S := \sqrt{u} + 1 \tag{10}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = \_C1$

$$SolGral := \ln(x) + u + \frac{2\,u^{3/2}}{3} = \_C1 \tag{11}$$

> $SolGralDos := expand\left(subs\left(u = \frac{y}{x}, SolGral\right)\right)$

$$SolGralDos := \ln(x) + \frac{y}{x} + \frac{2\left(\frac{y}{x}\right)^{3/2}}{3} = \_C1 \tag{12}$$

> $SolGralTres := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \_C1$

$$SolGralTres := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \_C1 \tag{13}$$

> $DerSolGralTres := isolate(diff(SolGralTres, x), diff(y(x), x))$

$$DerSolGralTres := \frac{d}{dx}\,y(x) = \frac{-\left(-\frac{1}{x} + \frac{y(x)}{x^2}\right)x^2 - \sqrt{\frac{y(x)}{x}}\,y(x)}{-\sqrt{\frac{y(x)}{x}}\,x - x} \tag{14}$$

> $DerSolCuatro := diff(y(x), x) = \frac{expand\left(-\left(-\frac{1}{x} + \frac{y(x)}{x^2}\right)x^2 - \frac{\mathrm{sqrt}(y(x))}{\mathrm{sqrt}(x)}\,y(x)\right)}{expand\left(-\sqrt{\frac{y(x)\cdot x^2}{x}} - x\right)}$

$$DerSolCuatro := \frac{d}{dx}\,y(x) = \frac{x - y(x) - \frac{y(x)^{3/2}}{\sqrt{x}}}{-\sqrt{x\,y(x)} - x} \tag{15}$$

> $Ecua$

$$\left(x + \sqrt{x\,y(x)}\right)\left(\frac{d}{dx}\,y(x)\right) + x - y(x) = \frac{y(x)^{3/2}}{\sqrt{x}} \tag{16}$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

(17)

$$DerEcua := \frac{d}{dx} y(x) = \frac{-x + y(x) + \frac{y(x)^{3/2}}{\sqrt{x}}}{x + \sqrt{x\,y(x)}} \quad (17)$$

> $ComprobarUno := simplify(rhs(DerSolCuatro) - rhs(DerEcua)) = 0$

$$ComprobarUno := 0 = 0 \quad (18)$$

> $CondIniUno := y(1) = 4$

$$CondIniUno := y(1) = 4 \quad (19)$$

> $ParaUno := simplify(subs(x = 1, y = 4, SolGralDos))$

$$ParaUno := \frac{28}{3} = \_C1 \quad (20)$$

> $SolPartFinalUno := subs(\_C1 = lhs(ParaUno), SolGralDos)$

$$SolPartFinalUno := \ln(x) + \frac{y}{x} + \frac{2\left(\frac{y}{x}\right)^{3/2}}{3} = \frac{28}{3} \quad (21)$$

> $SolFinalUno := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \frac{28}{3}$

$$SolFinalUno := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \frac{28}{3} \quad (22)$$

> $CondIniDos := y(1) = 1$

$$CondIniDos := y(1) = 1 \quad (23)$$

> $ParaDos := simplify(subs(x = 1, y = 1, SolGralDos))$

$$ParaDos := \frac{5}{3} = \_C1 \quad (24)$$

> $SolFinalPartDos := subs(\_C1 = lhs(ParaDos), SolGralDos)$

$$SolFinalPartDos := \ln(x) + \frac{y}{x} + \frac{2\left(\frac{y}{x}\right)^{3/2}}{3} = \frac{5}{3} \quad (25)$$

> $SolFinalDos := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \frac{5}{3}$

$$SolFinalDos := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \frac{5}{3} \quad (26)$$

> $CondiIniTres := y(1) = 16$

$$CondiIniTres := y(1) = 16 \quad (27)$$

> $ParaTres := simplify(subs(x = 1, y = 16, SolGralDos))$

$$ParaTres := \frac{176}{3} = \_C1 \quad (28)$$

```
> SolFinalPartTres := subs(_C1 = lhs(ParaTres), SolGralDos)
```

$$SolFinalPartTres := \ln(x) + \frac{y}{x} + \frac{2\left(\frac{y}{x}\right)^{3/2}}{3} = \frac{176}{3} \quad (29)$$

```
> SolFinalTres := ln(x) + y(x)/x + 2*(y(x)/x)^(3/2)/3 = 176/3
```

$$SolFinalTres := \ln(x) + \frac{y(x)}{x} + \frac{2\left(\frac{y(x)}{x}\right)^{3/2}}{3} = \frac{176}{3} \quad (30)$$

```
> restart
```

3)

```
> Ecua := diff(r(theta), theta) + r(theta)·tan(theta) − sec(theta) = 0
```

$$Ecua := \frac{d}{d\theta} r(\theta) + r(\theta)\tan(\theta) - \sec(\theta) = 0 \quad (31)$$

```
> p := tan(theta); q := sec(theta)
```

$$p := \tan(\theta)$$

$$q := \sec(\theta) \quad (32)$$

```
> SolGral := r(theta) = _C1·exp(−int(p, theta)) + exp(−int(p, theta))·int(exp(int(p, theta))
      ·q, theta)
```

$$SolGral := r(\theta) = \_C1\cos(\theta) + \cos(\theta)\tan(\theta) \quad (33)$$

```
> Comprobar := simplify(eval(subs(r(theta) = rhs(SolGral), Ecua)))
```

$$Comprobar := 0 = 0 \quad (34)$$

```
> restart
```

4)

```
> Ecua := y''−2·y'=2·exp(2 x) + 4
```

$$Ecua := \frac{d^2}{dx^2} y(x) - 2\frac{d}{dx} y(x) = 2e^{2x} + 4 \quad (35)$$

```
>
> Q := rhs(Ecua)
```

$$Q := 2e^{2x} + 4 \quad (36)$$

```
> EcuaCarac := m^2−2·m = 0
```

$$EcuaCarac := m^2 - 2m = 0 \quad (37)$$

```
> Raiz := solve(EcuaCarac)
```

$$Raiz := 0, 2 \quad (38)$$

```
> yy[1] := exp(Raiz[1]); yy[2] := exp(Raiz[2]·x)
```

$$yy_1 := 1$$

$$yy_2 := e^{2x} \quad (39)$$

```
> with(linalg) :
```

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} 1 & e^{2x} \\ 0 & 2e^{2x} \end{bmatrix} \quad (40)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 2e^{2x} + 4 \end{bmatrix} \quad (41)$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -e^{2x} - 2 & 1 + 2e^{-2x} \end{bmatrix} \quad (42)$$

> $SolGral := y(x) = expand((int(ParaVar[1], x) + \_C1) \cdot yy[1] + (int(ParaVar[2], x) + \_C2) \cdot yy[2])$

$$SolGral := y(x) = -2x - \frac{(e^x)^2}{2} + \_C1 + (e^x)^2 x - 1 + (e^x)^2 \_C2 \quad (43)$$

> $SolGralDos := y(x) = \_C1 + \_C2 \cdot \exp(2x) + \left(-\frac{1}{2} + x\right) \cdot \exp(2x) - 2 \cdot x - 1$

$$SolGralDos := y(x) = \_C1 + \_C2\, e^{2x} + \left(-\frac{1}{2} + x\right) e^{2x} - 2x - 1 \quad (44)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralDos), Ecua)))$

$$Comprobar := 2e^{2x} + 4 = 2e^{2x} + 4 \quad (45)$$

> $CondIni := y(0) = 1, D(y)(0) = 0$

$$CondIni := y(0) = 1, D(y)(0) = 0 \quad (46)$$

> $SolIni := simplify(subs(x = 0, rhs(SolGralDos) = 1))$

$$SolIni := \_C1 + \_C2 - \frac{3}{2} = 1 \quad (47)$$

> $DerSolIni := simplify(subs(x = 0, rhs(diff(SolGralDos, x)) = 0))$

$$DerSolIni := 2\,\_C2 - 2 = 0 \quad (48)$$

> $Sistema := SolIni, DerSolIni : Sistema[1]; Sistema[2]$

$$\_C1 + \_C2 - \frac{3}{2} = 1$$

$$2\,\_C2 - 2 = 0 \quad (49)$$

> $Para := solve([Sistema])$

$$Para := \left\{\_C1 = \frac{3}{2}, \_C2 = 1\right\} \quad (50)$$

> $SolPart := expand(subs(\_C1 = rhs(Para[1]), \_C2 = rhs(Para[2]), SolGralDos))$

$$SolPart := y(x) = \frac{1}{2} + \frac{(e^x)^2}{2} + (e^x)^2 x - 2x \quad (51)$$

>

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolPart), Ecua)))$

$$ComprobarDos := 2e^{2x} + 4 = 2e^{2x} + 4 \quad (52)$$

>

> $CondIniTres := y(0) = 2, D(y)(0) = 1$

$$CondIniTres := y(0) = 2, D(y)(0) = 1 \quad (53)$$

> $SolIniTres := simplify(subs(x = 0, rhs(SolGralDos) = 2))$

$$SolIniTres := \_C1 + \_C2 - \frac{3}{2} = 2 \quad (54)$$

> $DerSolIniTres := simplify(subs(x = 0, rhs(diff(SolGralDos, x)) = 1))$

$$DerSolIniTres := 2\,\_C2 - 2 = 1 \quad (55)$$

> $SistemaTres := SolIniTres, DerSolIniTres : SistemaTres[1]; SistemaTres[2]$

$$\_C1 + \_C2 - \frac{3}{2} = 2$$

$$2\,\_C2 - 2 = 1 \quad (56)$$

> $ParaTres := solve([SistemaTres])$

$$ParaTres := \left\{\_C1 = 2, \_C2 = \frac{3}{2}\right\} \quad (57)$$

> $SolPartTres := expand(subs(\_C1 = rhs(ParaTres[1]), \_C2 = rhs(ParaTres[2]), SolGralDos))$

$$SolPartTres := y(x) = 1 + \left(e^{x}\right)^2 + \left(e^{x}\right)^2 x - 2x \quad (58)$$

>

> $ComprobarTres := simplify(eval(subs(y(x) = rhs(SolPartTres), Ecua)))$

$$ComprobarTres := 2\,e^{2x} + 4 = 2\,e^{2x} + 4 \quad (59)$$

>

> $CondIniCuatro := y(0) = 3, D(y)(0) = 4$

$$CondIniCuatro := y(0) = 3, D(y)(0) = 4 \quad (60)$$

> $SolIniCuatro := simplify(subs(x = 0, rhs(SolGralDos) = 3))$

$$SolIniCuatro := \_C1 + \_C2 - \frac{3}{2} = 3 \quad (61)$$

> $DerSolIniCuatro := simplify(subs(x = 0, rhs(diff(SolGralDos, x)) = 4))$

$$DerSolIniCuatro := 2\,\_C2 - 2 = 4 \quad (62)$$

> $SistemaCuatro := SolIniCuatro, DerSolIniCuatro : SistemaCuatro[1]; SistemaCuatro[2];$

$$\_C1 + \_C2 - \frac{3}{2} = 3$$

$$2\,\_C2 - 2 = 4 \quad (63)$$

> $ParaCuatro := solve([SistemaCuatro])$

$$ParaCuatro := \left\{\_C1 = \frac{3}{2}, \_C2 = 3\right\} \quad (64)$$

> $SolPartCuatro := expand(subs(\_C1 = rhs(ParaCuatro[1]), \_C2 = rhs(ParaCuatro[2]), SolGralDos))$

$$SolPartCuatro := y(x) = \frac{1}{2} + \frac{5\left(e^{x}\right)^2}{2} + \left(e^{x}\right)^2 x - 2x \quad (65)$$

>

> $ComprobarCuatro := simplify(eval(subs(y(x) = rhs(SolPartCuatro), Ecua)))$

$$ComprobarCuatro := 2\,e^{2x} + 4 = 2\,e^{2x} + 4 \quad (66)$$

> *SolPart*

$$y(x) = \frac{1}{2} + \frac{(e^x)^2}{2} + (e^x)^2 x - 2x \quad (67)$$

> *SolPartTres*

$$y(x) = 1 + (e^x)^2 + (e^x)^2 x - 2x \quad (68)$$

> *SolPartCuatro*

$$y(x) = \frac{1}{2} + \frac{5\,(e^x)^2}{2} + (e^x)^2 x - 2x \quad (69)$$

> *restart*

5)

> $SolGral := y(t) = \_C1 \cdot \exp(-t) \cdot \cos(t) + \_C2 \cdot \exp(-t) \cdot \sin(t) + 3 \cdot \sin(t) - \cos(t)$

$$SolGral := y(t) = \_C1\, e^{-t} \cos(t) + \_C2\, e^{-t} \sin(t) + 3 \sin(t) - \cos(t) \quad (70)$$

> $SolGralHom := y(t) = \_C1 \cdot \exp(-t) \cdot \cos(t) + \_C2 \cdot \exp(-t) \cdot \sin(t)$

$$SolGralHom := y(t) = \_C1\, e^{-t} \cos(t) + \_C2\, e^{-t} \sin(t) \quad (71)$$

> $SolPartNoHom := y(t) = 3 \sin(t) - \cos(t)$

$$SolPartNoHom := y(t) = 3 \sin(t) - \cos(t) \quad (72)$$

> $yy[1] := e^{-t} \cos(t); yy[2] := e^{-t} \sin(t)$

$$yy_1 := e^{-t} \cos(t)$$

$$yy_2 := e^{-t} \sin(t) \quad (73)$$

> $EcuaCarac := expand((m - (-1 + I)) \cdot (m - (-1 - I))) = 0$

$$EcuaCarac := m^2 + 2m + 2 = 0 \quad (74)$$

> $EcuaHom := diff(y(t), t\$2) + 2 \cdot diff(y(t), t) + 2 \cdot y(t) = 0$

$$EcuaHom := \frac{d^2}{dt^2} y(t) + 2 \frac{d}{dt} y(t) + 2\, y(t) = 0 \quad (75)$$

> $Q := simplify(eval(subs(y(t) = rhs(SolPartNoHom), lhs(EcuaHom))))$

$$Q := 5 \sin(t) + 5 \cos(t) \quad (76)$$

> $Ecua := lhs(EcuaHom) = Q$

$$Ecua := \frac{d^2}{dt^2} y(t) + 2 \frac{d}{dt} y(t) + 2\, y(t) = 5 \sin(t) + 5 \cos(t) \quad (77)$$

> *SolGral*

$$y(t) = \_C1\, e^{-t} \cos(t) + \_C2\, e^{-t} \sin(t) + 3 \sin(t) - \cos(t) \quad (78)$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(SolGral), Ecua)))$

$$Comprobar := 5 \sin(t) + 5 \cos(t) = 5 \sin(t) + 5 \cos(t) \quad (79)$$

> *restart*

>

>

>