

# [SOLUCIÓN

FACULTAD DE INGENIERÍA  
ECUACIONES DIFERENCIALES  
SEMESTRE 2012-2  
SEGUNDO EXAMEN PARCIAL

2012-04-30

> restart

**1) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)**

$$x y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2 x y(x) + x^2 + 2 y(x) - 2 x + 2) \left( \frac{d}{dx} y(x) \right) = 0 \quad (1)$$

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**RESPUESTA 1)**

> Ecuacion :=  $x y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2 x y(x) + x^2 + 2 y(x) - 2 x + 2) \left( \frac{d}{dx} y(x) \right) = 0$

Ecuacion :=  $x y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2 x y(x) + x^2 + 2 y(x) - 2 x + 2) \left( \frac{d}{dx} y(x) \right) = 0$  (2)

> with(DEtools) :

> odeadvisor(Ecuacion)

[\_separable] (3)

>  $M(x, y) := \text{factor}(x y^2 - y^2 + x - 1); N(x, y) := \text{factor}(x^2 y - 2 x y + x^2 + 2 y - 2 x + 2)$   
 $M(x, y) := (y^2 + 1) (x - 1)$

$N(x, y) := (x^2 - 2 x + 2) (1 + y)$  (4)

>  $P(x) := x - 1; Q(y) := y \cdot 2 + 1; R(x) := x \cdot 2 - 2 \cdot x + 2; S(y) := y + 1;$   
 $P(x) := x - 1$

$Q(y) := y^2 + 1$

$R(x) := x^2 - 2 x + 2$

$S(y) := 1 + y$  (5)

>  $\text{SolucionGeneral} := \text{int}\left(\frac{P(x)}{R(x)}, x\right) + \text{int}\left(\frac{S(y)}{Q(y)}, y\right) = C1$

$\text{SolucionGeneral} := \frac{1}{2} \ln(x^2 - 2 x + 2) + \frac{1}{2} \ln(y^2 + 1) + \arctan(y) = C1$  (6)

**FIN RESPUESTA 1)**

> restart

**2) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)**

$$\sin(x y(x)) + x y(x) \cos(x y(x)) + x^2 \cos(x y(x)) \left( \frac{d}{dx} y(x) \right) = 0 \quad (7)$$

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**RESPUESTA 2)**

$$> \text{Ecuacion} := \sin(xy(x)) + xy(x) \cos(xy(x)) + x^2 \cos(xy(x)) \left( \frac{d}{dx} y(x) \right) = 0$$

$$\text{Ecuacion} := \sin(xy(x)) + xy(x) \cos(xy(x)) + x^2 \cos(xy(x)) \left( \frac{d}{dx} y(x) \right) = 0 \quad (8)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

$$[[_{\text{homogeneous}}, \text{class } G], \text{\_exact}] \quad (9)$$

$$> M(x, y) := \sin(xy) + xy \cos(xy); N(x, y) := x^2 \cos(xy);$$

$$M(x, y) := \sin(xy) + xy \cos(xy)$$

$$N(x, y) := x^2 \cos(xy) \quad (10)$$

$$> \text{comprobacion}_1 := \text{simplify}(\text{diff}(M(x, y), y) - \text{diff}(N(x, y), x)) = 0$$

$$\text{comprobacion}_1 := 0 = 0 \quad (11)$$

$$> \text{IntMx} := \text{simplify}(\text{int}(M(x, y), x))$$

$$\text{IntMx} := \sin(xy) x \quad (12)$$

$$> \text{SolucionGeneral} := \text{IntMx} + \text{int}((N(x, y) - \text{diff}(\text{IntMx}, y)), y) = C1$$

$$\text{SolucionGeneral} := \sin(xy) x = C1 \quad (13)$$

**FIN RESPUESTA 2)**

> restart

**3) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)**

$$x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left( \frac{d}{dx} y(x) \right) = 0 \quad (14)$$

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**RESPUESTA 3)**

$$> \text{Ecuacion} := x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left( \frac{d}{dx} y(x) \right) = 0$$

$$\text{Ecuacion} := x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left( \frac{d}{dx} y(x) \right) = 0 \quad (15)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

$$[_{\text{Bernoulli}}] \quad (16)$$

> FactInt := infactor(Ecuacion)

$$\text{FactInt} := \frac{1}{x^4} \quad (17)$$

$$> M(x, y) := x^4 \ln(x) - 2xy^3; N(x, y) := 3x^2y^2;$$

$$M(x, y) := x^4 \ln(x) - 2xy^3$$

$$N(x, y) := 3y^2x^2 \quad (18)$$

$$> \text{comprobacion}_2 := \text{simplify}(\text{diff}(M(x, y), y) - \text{diff}(N(x, y), x)) = 0$$

$$\text{comprobacion}_2 := -6y^2x - 6yx^2 = 0 \quad (19)$$

$$> \text{MM}(x, y) := \text{expand}(\text{FactInt} \cdot M(x, y)); \text{NN}(x, y) := \text{simplify}(\text{FactInt} \cdot N(x, y));$$

$$MM(x, y) := \ln(x) - \frac{2y^3}{x^3}$$

$$NN(x, y) := \frac{3y^2}{x^2} \quad (20)$$

$$> comprobacion_3 := simplify(diff(MM(x, y), y) - diff(NN(x, y), x)) = 0$$

$$comprobacion_3 := 0 = 0 \quad (21)$$

$$> IntNNy := int(NN(x, y), y)$$

$$IntNNy := \frac{y^3}{x^2} \quad (22)$$

$$> SolucionGeneral := IntNNy + int((MM(x, y) - diff(IntNNy, x)), x) = CI$$

$$SolucionGeneral := \frac{y^3}{x^2} + x \ln(x) - x = CI \quad (23)$$

**FIN RESPUESTA 3)**

> restart

**4) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)**

$$2xy(x)^2 - 3y(x)^3 + (8 - 3xy(x)^2) \left( \frac{d}{dx} y(x) \right) = 0 \quad (24)$$

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**RESPUESTA 4)**

$$> Ecuacion := 2xy(x)^2 - 3y(x)^3 + (8 - 3xy(x)^2) \left( \frac{d}{dx} y(x) \right) = 0$$

$$Ecuacion := 2xy(x)^2 - 3y(x)^3 + (8 - 3xy(x)^2) \left( \frac{d}{dx} y(x) \right) = 0 \quad (25)$$

> with(DEtools) :

$$> odeadvisor(Ecuacion)$$

$$[_rational] \quad (26)$$

$$> intfactor(Ecuacion)$$

$$\frac{1}{y(x)^2} \quad (27)$$

$$> FactInt := \frac{1}{y^2}$$

$$FactInt := \frac{1}{y^2} \quad (28)$$

$$> M(x, y) := 2xy^2 - 3y^3; N(x, y) := 8 - 3xy^2;$$

$$M(x, y) := 2xy^2 - 3y^3$$

$$N(x, y) := 8 - 3xy^2 \quad (29)$$

$$> comprobacion_4 := simplify(diff(M(x, y), y) - diff(N(x, y), x)) = 0$$

$$comprobacion_4 := 4xy - 6y^2 = 0 \quad (30)$$

$$> MM(x, y) := simplify(FactInt \cdot M(x, y)); NN(x, y) := expand(FactInt \cdot N(x, y));$$

$$MM(x, y) := 2x - 3y$$

$$NN(x, y) := \frac{8}{y^2} - 3x \quad (31)$$

$$> comprobacion_5 := simplify(diff(MM(x, y), y) - diff(NN(x, y), x)) = 0$$

$$comprobacion_5 := 0 = 0 \quad (32)$$

$$> IntMMx := int(MM(x, y), x)$$

$$IntMMx := x^2 - 3xy \quad (33)$$

$$> SolucionGeneral := IntMMx + int((NN(x, y) - diff(IntMMx, y)), y) = C1$$

$$SolucionGeneral := x^2 - 3xy - \frac{8}{y} = C1 \quad (34)$$

**FIN RESPUESTA 4)**

> restart

**5) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)**

$$y(x) (y(x)^2 + 2x^2) - 2x (x^2 + y(x)^2) \left( \frac{d}{dx} y(x) \right) = 0 \quad (35)$$

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**RESPUESTA 5)**

$$> Ecuacion := y(x) (y(x)^2 + 2x^2) - 2x (x^2 + y(x)^2) \left( \frac{d}{dx} y(x) \right) = 0$$

$$Ecuacion := y(x) (y(x)^2 + 2x^2) - 2x (x^2 + y(x)^2) \left( \frac{d}{dx} y(x) \right) = 0 \quad (36)$$

> with(DEtools) :

$$> odeadvisor(Ecuacion)$$

$$[[_homogeneous, class A], _rational, _dAlembert] \quad (37)$$

$$> EcuacionSeparable := factor(isolate(simplify(eval(subs(y(x) = x*u(x), Ecuacion))), diff(u(x), x)))$$

$$EcuacionSeparable := \frac{d}{dx} u(x) = -\frac{1}{2} \frac{u(x)^3}{x(1+u(x)^2)} \quad (38)$$

$$> EcuacionSeparada := lhs(EcuacionSeparable) - rhs(EcuacionSeparable) = 0$$

$$EcuacionSeparada := \frac{d}{dx} u(x) + \frac{1}{2} \frac{u(x)^3}{x(1+u(x)^2)} = 0 \quad (39)$$

$$> M(x, u) := \frac{1}{2} \frac{u^3}{x(1+u^2)}; N(x, u) := 1;$$

$$M(x, u) := \frac{1}{2} \frac{u^3}{x(1+u^2)}$$

$$N(x, u) := 1 \quad (40)$$

$$> P(x) := \frac{1}{x}; Q(u) := \frac{1}{2} \frac{u^3}{(1+u^2)}; R(x) := 1; S(u) := 1;$$

$$P(x) := \frac{1}{x}$$

$$Q(u) := \frac{1}{2} \frac{u^3}{1+u^2}$$

$$R(x) := 1$$

$$S(u) := 1$$

(41)

$$\text{> Solucion} := \text{int}\left(\frac{P(x)}{R(x)}, x\right) + \text{int}\left(\frac{S(u)}{Q(u)}, u\right) = CI$$

$$\text{Solucion} := \ln(x) + 2 \ln(u) - \frac{1}{u^2} = CI$$

(42)

$$\text{> SolucionGeneral} := \text{expand}\left(\text{subs}\left(u = \frac{y}{x}, \text{Solucion}\right)\right)$$

$$\text{SolucionGeneral} := \ln(x) + 2 \ln\left(\frac{y}{x}\right) - \frac{x^2}{y^2} = CI$$

(43)

FIN RESPUESTA 5)

> restart

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FIN DEL EXAMEN