>

SOLUCIÓN

ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL COLEGIADO

2012 JUNIO 2012

> *restart*

1) Resuelva la ecuación para a) A=1; b) A=0

> $Ecuacion := (4\,x\cdot\cdot 2 - A\cdot y(x)\cdot\cdot 2) - 2\cdot x\cdot y(x)\cdot diff(y(x), x) = 0$

$$Ecuacion := 4\,x^2 - A\,y(x)^2 - 2\,x\,y(x)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(1)}$$

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RESPUESTA 1a)

> $EcuacionA := subs(A = 1, Ecuacion)$

$$EcuacionA := 4\,x^2 - y(x)^2 - 2\,x\,y(x)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(2)}$$

> *with(DEtools)* :

> *odeadvisor(EcuacionA)*

[[_homogeneous, class A], _exact, _rational, _Bernoulli] **(3)**

>

Opción 1: EXACTA

> $M(x, y) := 4\,x^2 - y^2;\, N(x, y) := -2\,x\,y$

$$M(x, y) := 4\,x^2 - y^2$$
$$N(x, y) := -2\,y\,x \quad \textbf{(4)}$$

> $comprobacion := simplify(diff(M(x, y), y) - diff(N(x, y), x)) = 0$

$$comprobacion := 0 = 0 \quad \textbf{(5)}$$

> $IntMx := int(M(x, y), x)$

$$IntMx := \frac{4}{3}\,x^3 - y^2\,x \quad \textbf{(6)}$$

> $SolucionGeneralA := -3\cdot(IntMx + int((N(x, y) - diff(IntMx, y)), y)) = C1$

$$SolucionGeneralA := -4\,x^3 + 3\,y^2\,x = C1 \quad \textbf{(7)}$$

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Opcion 2: COEFICIENTES HOMOGÉNEOS

> $EcuacionSeparableA := simplify(eval(subs(y(x) = x\cdot u(x), EcuacionA)))$

$$EcuacionSeparableA := -x^2\left(-4 + 3\,u(x)^2 + 2\,u(x)\,x\left(\frac{d}{dx}\,u(x)\right)\right) = 0 \quad \textbf{(8)}$$

> $EcuacionIntermediaA := isolate(EcuacionSeparableA, diff(u(x), x))$

$$EcuacionIntermediaA := \frac{d}{dx}\,u(x) = \frac{1}{2}\,\frac{4 - 3\,u(x)^2}{x\,u(x)} \quad \textbf{(9)}$$

> $EcuacionSeparadaA := \dfrac{lhs(EcuacionIntermediaA)}{\frac{1}{2}\,\frac{4 - 3\,u(x)^2}{u(x)}} - \dfrac{rhs(EcuacionIntermediaA)}{\frac{1}{2}\,\frac{4 - 3\,u(x)^2}{u(x)}} = 0$

$$EcuacionSeparadaA := \frac{2\left(\frac{d}{dx}\,u(x)\right)u(x)}{4 - 3\,u(x)^2} - \frac{1}{x} = 0 \quad \textbf{(10)}$$

> $P(u) := \frac{2\,u}{4-3\,u^2}; Q(x) := \frac{-1}{x};$

$$P(u) := \frac{2\,u}{4-3\,u^2}$$
$$Q(x) := -\frac{1}{x} \quad \textbf{(11)}$$

> $SolucionInicialA := int(P(u), u) + int(Q(x), x) = C1$

$$SolucionInicialA := -\frac{1}{3}\ln\left(-4+3\,u^2\right) - \ln(x) = C1 \quad \textbf{(12)}$$

> $SolucionFinalA := expand\left(\left(simplify\left(\frac{1}{\left(\exp\left(subs\left(u=\frac{y}{x}, lhs(SolucionInicialA)\right)\right)\right)}\right)\right)\cdot 3\right) = C1$

$$SolucionFinalA := -4\,x^3 + 3\,y^2\,x = C1 \quad \textbf{(13)}$$

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RESPUESTA 1b)

> $EcuacionB := subs(A=0, Ecuacion)$

$$EcuacionB := 4\,x^2 - 2\,x\,y(x)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(14)}$$

> $odeadvisor(EcuacionB)$

$$[\_separable] \quad \textbf{(15)}$$

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Opcion 1) VARIABLES SEPARABLES

> $M(x,y) := 4\,x^2; N(x,y) := -2\,x\,y$

$$M(x,y) := 4\,x^2$$
$$N(x,y) := -2\,y\,x \quad \textbf{(16)}$$

> $P(x) := x\cdot\cdot 2; Q(y) := 4; R(x) := -2\,x; S(y) := y$

$$P(x) := x^2$$
$$Q(y) := 4$$
$$R(x) := -2\,x$$
$$S(y) := y \quad \textbf{(17)}$$

> $SolucionB := int\left(\frac{P(x)}{R(x)}, x\right) + int\left(\frac{S(y)}{Q(y)}, y\right) = C1$

$$SolucionB := -\frac{1}{4}\,x^2 + \frac{1}{8}\,y^2 = C1 \quad \textbf{(18)}$$

> $SolucionGeneralB := -8\cdot lhs(SolucionB) = C1$

$$SolucionGeneralB := 2\,x^2 - y^2 = C1 \quad \textbf{(19)}$$

>

Opcion 2) COEFICIENTES HOMOGENEOS

> $EcuacionSeparableB := simplify(eval(subs(y(x) = x\cdot u(x), EcuacionB)))$

(20)

$$EcuacionSeparableB := -2\,x^2\left(-2+u(x)^2+u(x)\,x\left(\frac{d}{dx}\,u(x)\right)\right)=0 \quad \textbf{(20)}$$

> $EcuacionIntermediaB := isolate(EcuacionSeparableB, diff(u(x), x))$

$$EcuacionIntermediaB := \frac{d}{dx}\,u(x) = \frac{2-u(x)^2}{x\,u(x)} \quad \textbf{(21)}$$

> $EcuacionSeparadaB := \dfrac{lhs(EcuacionIntermediaB)}{\dfrac{2-u(x)^2}{u(x)}} - \dfrac{rhs(EcuacionIntermediaB)}{\dfrac{2-u(x)^2}{u(x)}} = 0$

$$EcuacionSeparadaB := \frac{\left(\frac{d}{dx}\,u(x)\right)u(x)}{2-u(x)^2} - \frac{1}{x} = 0 \quad \textbf{(22)}$$

> $R(u) := \dfrac{u}{2-u^2};\ S(x) := -\dfrac{1}{x};$

$$R(u) := \frac{u}{2-u^2}$$

$$S(x) := -\frac{1}{x} \quad \textbf{(23)}$$

> $SolucionInicialB := int(R(u), u) + int(S(x), x) = C1$

$$SolucionInicialB := -\frac{1}{2}\ln\left(-2+u^2\right) - \ln(x) = C1 \quad \textbf{(24)}$$

> $SolucionFinalB := -\left(simplify\left(\dfrac{1}{\exp\left(subs\left(u=\dfrac{y}{x}, lhs(SolucionInicialB)\right)\right)}\right)\right)\cdot 2 = C1$

$$SolucionFinalB := 2\,x^2 - y^2 = C1 \quad \textbf{(25)}$$

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FIN RESPUESTA 1)

> *restart*

2) Obtenga la solución de la ecuación diferencial siguiente que satisfaga la condición dada

> $Ecuacion := y' - 3\,y = \dfrac{(1-\exp(4\,x))}{\exp(x)};\ Condicion := y(0) = 0$

$$Ecuacion := \frac{d}{dx}\,y(x) - 3\,y(x) = \frac{1-e^{4\,x}}{e^x}$$

$$Condicion := y(0) = 0 \quad \textbf{(26)}$$

RESPUESTA 2)

Opción 1: SOLUCIÓN DIRECTA

> $SolucionParticular := simplify(expand(dsolve(\{Ecuacion, Condicion\})))$

$$SolucionParticular := y(x) = -\frac{1}{4}\,e^{-x} - e^{3\,x}\,x + \frac{1}{4}\,e^{3\,x} \quad \textbf{(27)}$$

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Opción 2: LINEAL

> $EcuacionHomogenea := lhs(Ecuacion) = 0$

$$EcuacionHomogenea := \frac{d}{dx}\,y(x) - 3\,y(x) = 0 \quad \textbf{(28)}$$

> $p(x) := -3; q(x) := simplify(expand(rhs(Ecuacion)))$

$$p(x) := -3$$

$$q(x) := e^{-x} - e^{3x} \quad \textbf{(29)}$$

> $SolucionInicial := y(x) = simplify(C1 \cdot \exp(int(-p(x), x)) + \exp(int(-p(x), x)) \cdot int(\exp(int(p(x), x)) \cdot q(x), x))$

$$SolucionInicial := y(x) = C1\, e^{3x} - \frac{1}{4}\, e^{-x} - e^{3x}\, x \quad \textbf{(30)}$$

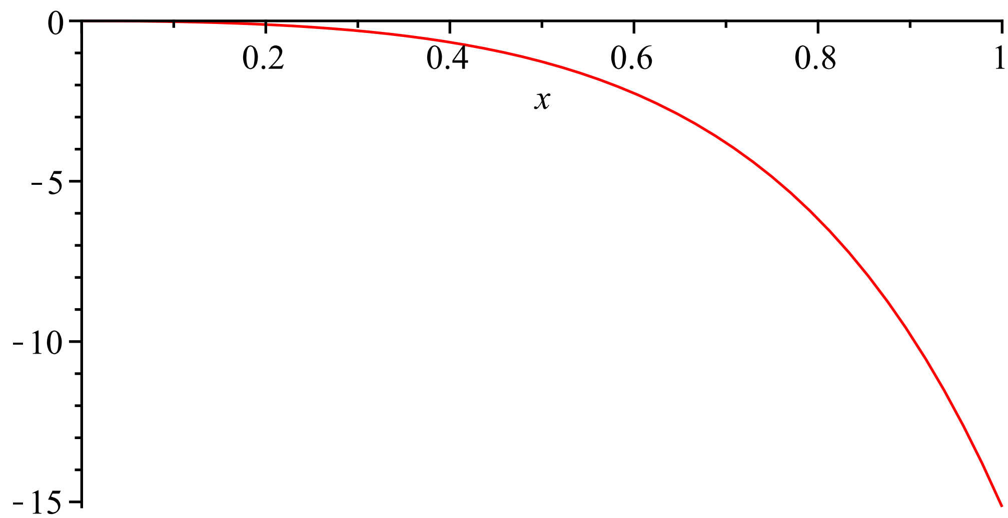
> $parametro := isolate(eval(subs(x = 0, rhs(SolucionInicial) = 0)), C1)$

$$parametro := C1 = \frac{1}{4} \quad \textbf{(31)}$$

> $SolucionParticularB := subs(C1 = rhs(parametro), SolucionInicial)$

$$SolucionParticularB := y(x) = -\frac{1}{4}\, e^{-x} - e^{3x}\, x + \frac{1}{4}\, e^{3x} \quad \textbf{(32)}$$

> $plot(rhs(SolucionParticularB), x = 0..1)$



FIN RESPUESTA 2)

> $restart$

3) Resuelva la ecuación diferencial

> $Ecuacion := 2\, y'' + 8\, y = \csc(2\, x)$

$$Ecuacion := 2 \left(\frac{d^2}{dx^2}\, y(x) \right) + 8\, y(x) = \csc(2\, x) \quad \textbf{(33)}$$

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RESPUESTA 3)

Opción 1): DIRECTA

> $SolucionGeneral := simplify(dsolve(Ecuacion))$

$$SolucionGeneral := y(x) = \sin(2\, x)\, \_C2 + \cos(2\, x)\, \_C1 + \frac{1}{8} \ln(\sin(2\, x)) \sin(2\, x) - \frac{1}{4}\, x \cos(2\, x) \quad \textbf{(34)}$$

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Opción 2) PARÁMETROS VARIABLES

> $EcuacionNormal := \frac{lhs(Ecuacion)}{2} = \frac{rhs(Ecuacion)}{2}$

$$EcuacionNormal := \frac{d^2}{dx^2}\, y(x) + 4\, y(x) = \frac{1}{2}\csc(2\, x) \quad \textbf{(35)}$$

> $EcuacionHomogenea := lhs(EcuacionNormal) = 0$

$$EcuacionHomogenea := \frac{d^2}{dx^2}\, y(x) + 4\, y(x) = 0 \quad \textbf{(36)}$$

> $Q(x) := rhs(EcuacionNormal)$

$$Q(x) := \frac{1}{2}\csc(2\, x) \quad \textbf{(37)}$$

> $EcuacionCaracteristica := m \cdot \cdot 2 + 4 = 0$

$$EcuacionCaracteristica := m^2 + 4 = 0 \quad \textbf{(38)}$$

> $Raiz := solve(EcuacionCaracteristica)$

$$Raiz := 2\, I, -2\, I \quad \textbf{(39)}$$

> $Solucion_1 := y(x) = \cos(\mathrm{Im}(Raiz_1) \cdot x); Solucion_2 := y(x) = \sin(\mathrm{Im}(Raiz_1) \cdot x)$

$$Solucion_1 := y(x) = \cos(2\, x)$$

$$Solucion_2 := y(x) = \sin(2\, x) \quad \textbf{(40)}$$

> $with(linalg):$

> $AA := wronskian([rhs(Solucion_1), rhs(Solucion_2)], x)$

$$AA := \begin{bmatrix} \cos(2\, x) & \sin(2\, x) \\ -2\sin(2\, x) & 2\cos(2\, x) \end{bmatrix} \quad \textbf{(41)}$$

> $BB := array([0, Q(x)])$

$$BB := \begin{bmatrix} 0 & \frac{1}{2}\csc(2\, x) \end{bmatrix} \quad \textbf{(42)}$$

> $SOL := simplify(linsolve(AA, BB))$

$$SOL := \begin{bmatrix} -\frac{1}{4} & \frac{1}{4}\frac{\cos(2\, x)}{\sin(2\, x)} \end{bmatrix} \quad \textbf{(43)}$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := -\frac{1}{4}$$

$$Bprima := \frac{1}{4}\frac{\cos(2\, x)}{\sin(2\, x)} \quad \textbf{(44)}$$

> $A(x) := int(Aprima, x) + C1; B(x) := int(Bprima, x) + C2;$

$$A(x) := -\frac{1}{4}\, x + C1$$

$$B(x) := \frac{1}{8}\ln(\sin(2\, x)) + C2 \quad \textbf{(45)}$$

> $SolucionGeneral := y(x) = simplify(A(x) \cdot rhs(Solucion_1) + B(x) \cdot rhs(Solucion_2))$

$$SolucionGeneral := y(x) = -\frac{1}{4}\, x\cos(2\, x) + \cos(2\, x)\, C1 + \frac{1}{8}\ln(\sin(2\, x))\sin(2\, x) \quad \textbf{(46)}$$

$$+\sin(2x)\ \_C2$$

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FIN RESPUESTA 3)

> *restart*

4)Resuelva el sistema de ecuaciones diferenciales

> $Sistema := diff(x(t), t) - 2\,y(t) = 2, 2\,x(t) + diff(y(t), t) = 0 : Sistema_1; Sistema_2$

$$\frac{d}{dt}x(t) - 2\,y(t) = 2$$

$$2\,x(t) + \frac{d}{dt}y(t) = 0 \tag{47}$$

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RESPUESTA 4)

> $Solucion := dsolve(\{Sistema\}) : Solucion_1; Solucion_2;$

$$x(t) = \_C1\sin(2\,t) + \_C2\cos(2\,t)$$

$$y(t) = \_C1\cos(2\,t) - \_C2\sin(2\,t) - 1 \tag{48}$$

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FIN RESPUESTA 4)

> *restart*

5) Resuelva la ecuación diferencial siguientes que satisfaga la condición dada

> $Ecuacion := diff(y(t), t) + 3\,y(t) = 4\,\mathrm{Heaviside}(t-1); Condicion := y(0) = 1;$

$$Ecuacion := \frac{d}{dt}y(t) + 3\,y(t) = 4\,\mathrm{Heaviside}(t-1)$$

$$Condicion := y(0) = 1 \tag{49}$$

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RESPUESTA 5)

> $with(inttrans) :$

> $TransLapEcuacion := subs(Condicion, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s\,laplace(y(t), t, s) - 1 + 3\,laplace(y(t), t, s) = \frac{4\,e^{-s}}{s} \tag{50}$$

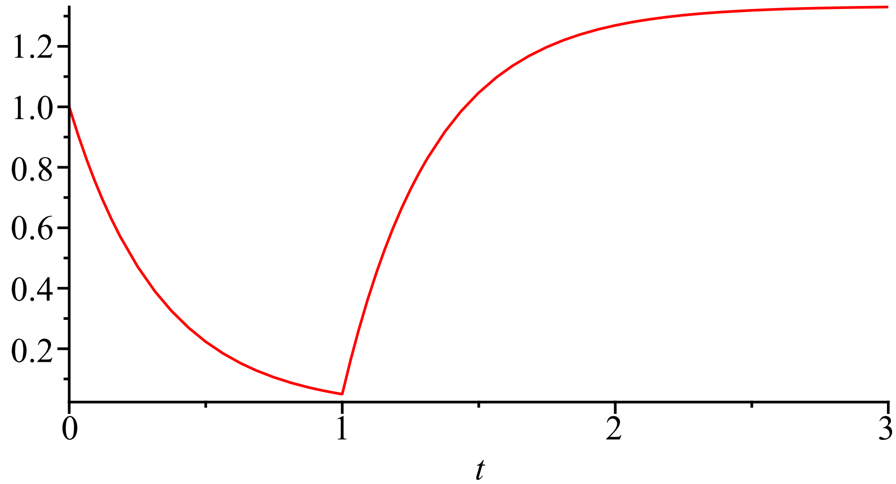
> $TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))$

$$TransLapSolucion := laplace(y(t), t, s) = \frac{4\,e^{-s} + s}{s\,(s+3)} \tag{51}$$

> $Solucion := invlaplace(TransLapSolucion, s, t)$

$$Solucion := y(t) = \frac{4}{3}\,\mathrm{Heaviside}(t-1)\,\left(1 - e^{-3\,t+3}\right) + e^{-3\,t} \tag{52}$$

> $plot(rhs(Solucion), t = 0\,..3)$



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FIN RESPUESTA 5)

> *restart*

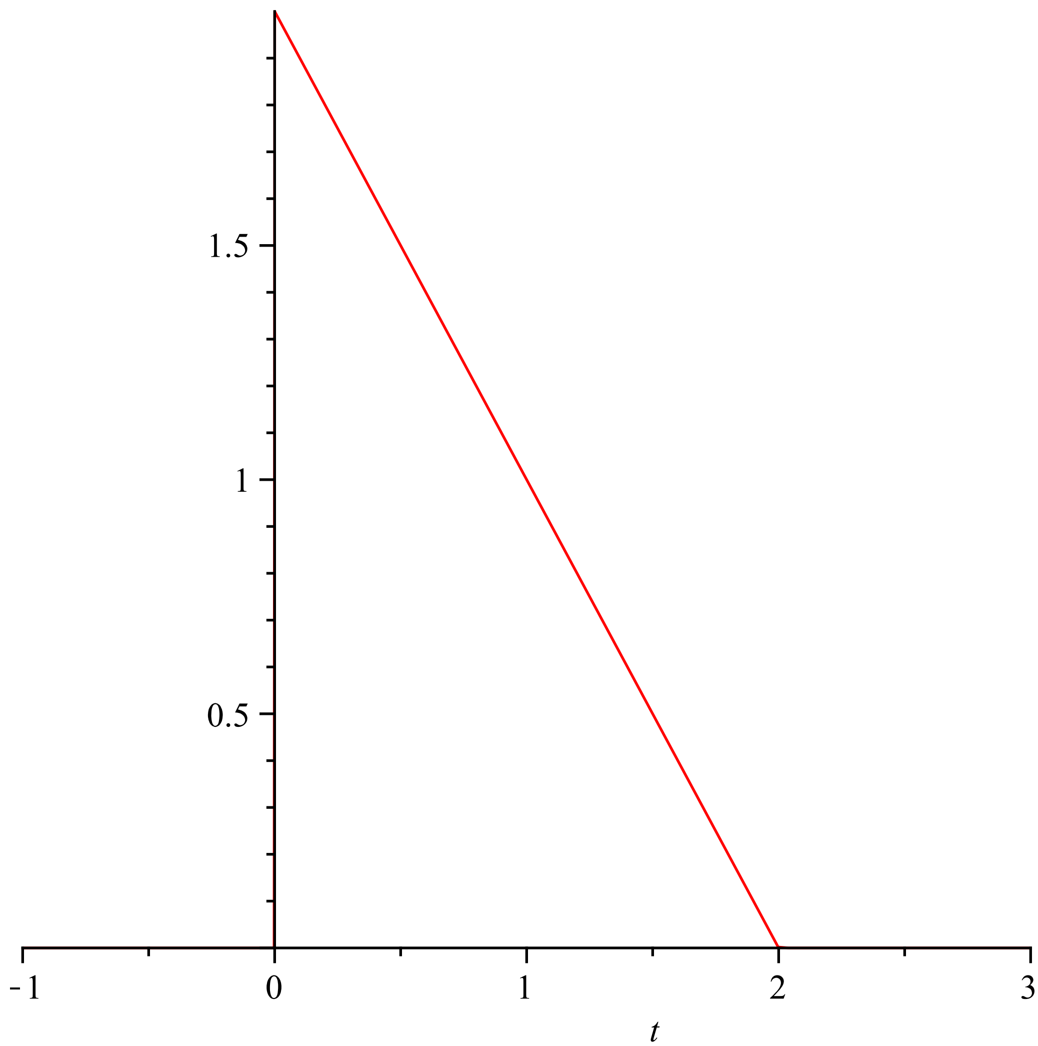
6) Determine la serie seno de Fourier para la función

> $f(t) := (-t+2)\cdot\text{Heaviside}(t) + (t-2)\cdot\text{Heaviside}(t-2);\ intervalo := 0..2$

$$f(t) := (-t+2)\ \text{Heaviside}(t) + (t-2)\ \text{Heaviside}(t-2)$$

$$intervalo := 0..2 \qquad \textbf{(53)}$$

> $plot(f(t), t=-1..3)$



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RESPUESTA 6)

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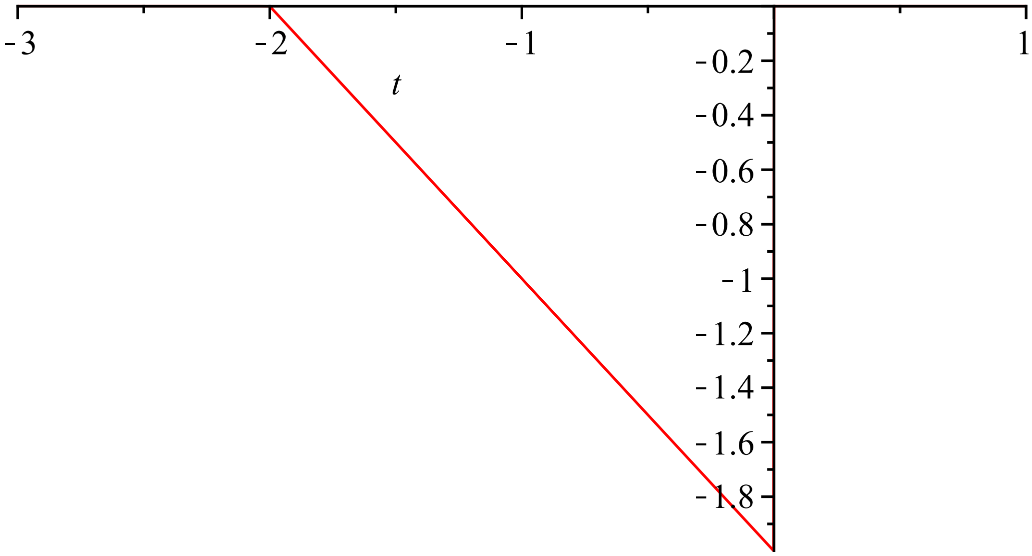
> $g(t) := (-(t+2))\cdot\text{Heaviside}(t+2) + t\cdot\text{Heaviside}(t) + 2\,\text{Heaviside}(t);$

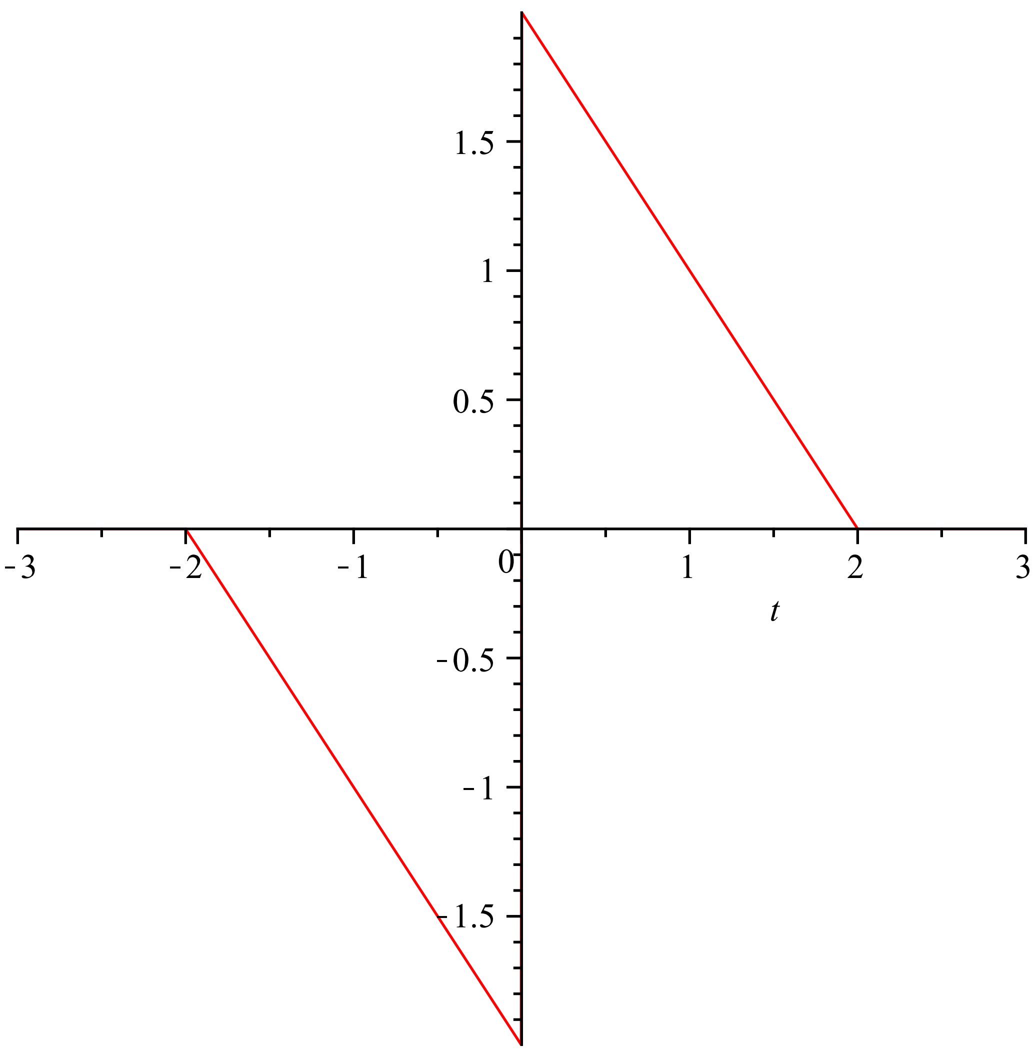
$$g(t) := -(t+2)\,\text{Heaviside}(t+2) + t\,\text{Heaviside}(t) + 2\,\text{Heaviside}(t) \quad \textbf{(54)}$$

> $plot(g(t), t=-3..1)$



> $h(t) := f(t) + g(t); plot(h(t), t=-3..3)$

$$h(t) := (-t+2)\ \mathrm{Heaviside}(t) + (t-2)\ \mathrm{Heaviside}(t-2) - (t+2)\ \mathrm{Heaviside}(t+2) + t\ \mathrm{Heaviside}(t) + 2\ \mathrm{Heaviside}(t)$$



> $L := 2$

$$L := 2 \quad \textbf{(55)}$$

> $b_n := subs\left(\sin(n\cdot\text{Pi}) = 0, \cos(n\cdot\text{Pi}) = (-1)^{n}, \left(\frac{1}{L}\right)\cdot int\left(h(t)\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot t}{L}\right), t=-L..L\right)\right)$

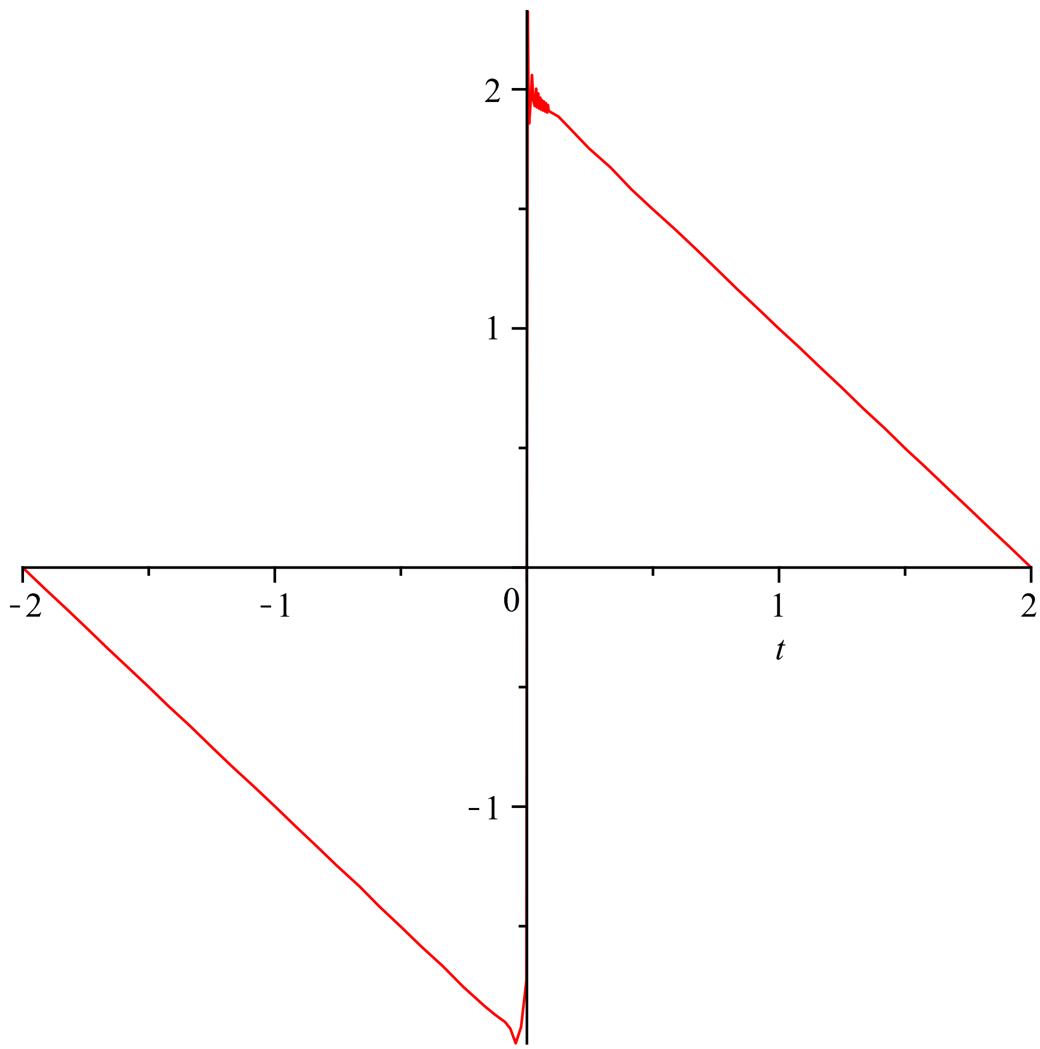
$$b_n := \frac{4}{n\,\pi} \quad \textbf{(56)}$$

> $STF := Sum\left(b_n\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot t}{L}\right), n = 1..infinity\right)$

$$STF := \sum_{n=1}^{\infty} \frac{4\sin\left(\frac{1}{2}\,n\,\pi\,t\right)}{n\,\pi} \quad \textbf{(57)}$$

> $STF_{500} := sum\left(b_n\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot t}{L}\right), n = 1..500\right):$

> $plot(STF_{500}, t=-2..2)$



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FIN RESPUESTA 6)

> *restart*

7) Obtenga la ecuación diferencial cuya solución es de la forma

> $Solucion := u(x,y) = f(y) + g\left(-\frac{2}{5}\cdot x + y\cdot\cdot 2\right)$

$$Solucion := u(x,y) = f(y) + g\left(-\frac{2}{5}\, x + y^2\right) \quad \textbf{(58)}$$

> $PrimeraDer := diff(Solucion, x\$2); SegundaDer := diff(Solucion, x, y);$

$$PrimeraDer := \frac{\partial^2}{\partial x^2}\, u(x,y) = \frac{4}{25}\, D^{(2)}(g)\left(-\frac{2}{5}\, x + y^2\right)$$

$$SegundaDer := \frac{\partial^2}{\partial y\, \partial x}\, u(x,y) = -\frac{4}{5}\, D^{(2)}(g)\left(-\frac{2}{5}\, x + y^2\right) y \quad \textbf{(59)}$$

> $FuncArbitraria_1 := isolate\left(PrimeraDer, D^{(2)}(g)\left(-\frac{2}{5}\, x + y^2\right)\right)$

$$FuncArbitraria_1 := D^{(2)}(g)\left(-\frac{2}{5}\, x + y^2\right) = \frac{25}{4}\, \frac{\partial^2}{\partial x^2}\, u(x,y) \quad \textbf{(60)}$$

> $FuncArbitraria_2 := isolate\left(SegundaDer, D^{(2)}(g)\left(-\frac{2}{5}\, x + y^2\right)\right)$

$$FuncArbitraria_2 := D^{(2)}(g)\left(-\frac{2}{5}\, x + y^2\right) = -\frac{5}{4}\, \frac{\frac{\partial^2}{\partial y\, \partial x}\, u(x,y)}{y} \quad \textbf{(61)}$$

> $EcuacionInicial := rhs(FuncArbitraria_1) = rhs(FuncArbitraria_2)$

$$EcuacionInicial := \frac{25}{4}\, \frac{\partial^2}{\partial x^2}\, u(x,y) = -\frac{5}{4}\, \frac{\frac{\partial^2}{\partial y\, \partial x}\, u(x,y)}{y} \quad \textbf{(62)}$$

> $EcuacionFinal := \frac{(lhs(EcuacionInicial)\cdot 4\cdot y)}{5} - \frac{(rhs(EcuacionInicial)\cdot 4\cdot y)}{5} = 0$

$$EcuacionFinal := 5\left(\frac{\partial^2}{\partial x^2}\, u(x,y)\right) y + \frac{\partial^2}{\partial y\, \partial x}\, u(x,y) = 0 \quad \textbf{(63)}$$

> $with(PDEtools):$

> $pdsolve(EcuacionFinal)$

$$u(x,y) = \_F1(y) + \_F2\left(-\frac{2}{5}\, x + y^2\right) \quad \textbf{(64)}$$

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FIN RESPUESTA 7

> restart

FIN EXAMEN SEGUNDO FINAL

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